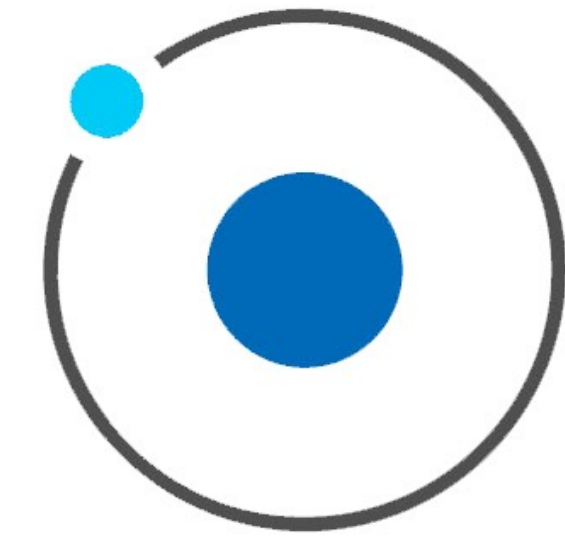


# An estimation of the Yarkovsky effect for (99942) Apophis via semi-analytical methods: implications for orbital evolution and uncertainty propagation beyond 2029

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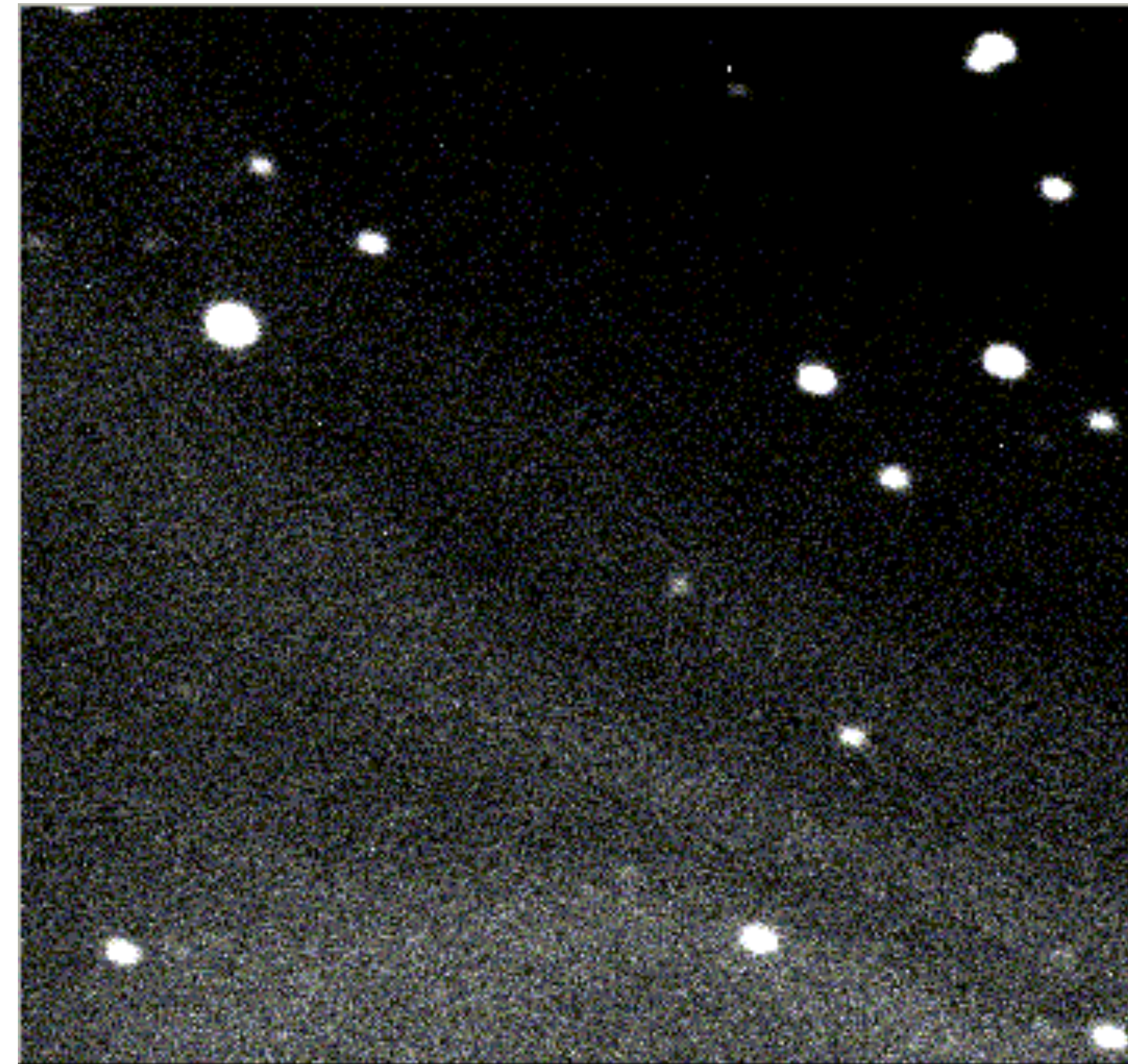
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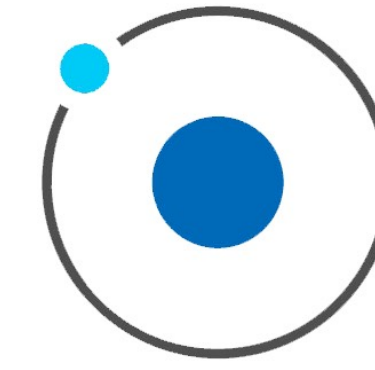
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- Discovered in 2004 by R.A. Tucker, D.J. Tholen, and F. Bernardi at the Kitt Peak National Observatory. It has a diameter  $\sim 340$  m.
- On April 13th, 2029, it will have a deep close encounter with Earth ( $\sim 6$  Earth radii).
- Largest source of uncertainty for predictions of its orbital motion is due to anisotropic re-emission of absorbed solar radiation (Yarkovsky effect).
- Yet, previous attempts to estimate the Yarkovsky effect from astrometry for Apophis have only yielded marginal detections (Vokrouhlický et al. 2015, Brozović et al. 2018, Del Vigna et al. 2018, Greenberg et al. 2020).
- During the recent close approach to Earth (March 2021), new radar astrometric observations have been obtained at Goldstone. This, together with high-quality optical astrometry, should allow a more strong detection of the Yarkovsky acceleration acting upon Apophis.



Credit: Osservatorio Astronomico  
Sormano via Wikipedia



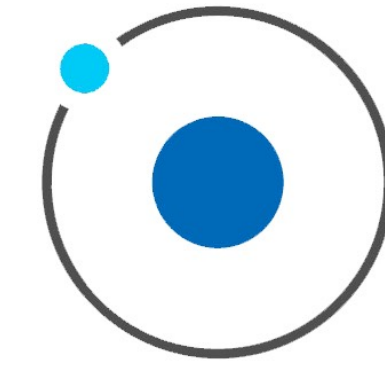


## Apophis orbital motion model

Our dynamical model for Apophis orbital motion includes:

- Post-Newtonian interactions with the Sun, the eight planets, the Moon and Pluto.
- Newtonian interactions with 16 most-massive asteroids.
- Acceleration due to Earth's  $J_2$ .
- We model the Yarkovsky effect as an acceleration in the transverse direction:  
 $\mathbf{a}_t = A_2/r^2 \hat{\mathbf{t}}$ .
- We integrate the equations of motion exploiting jet transport techniques, as implemented in `TaylorIntegration.jl`. Jet transport allows us to compute the  $q$ -th order Taylor expansion of the orbit around a given set of orbital parameters  $\mathbf{x}_0$ :

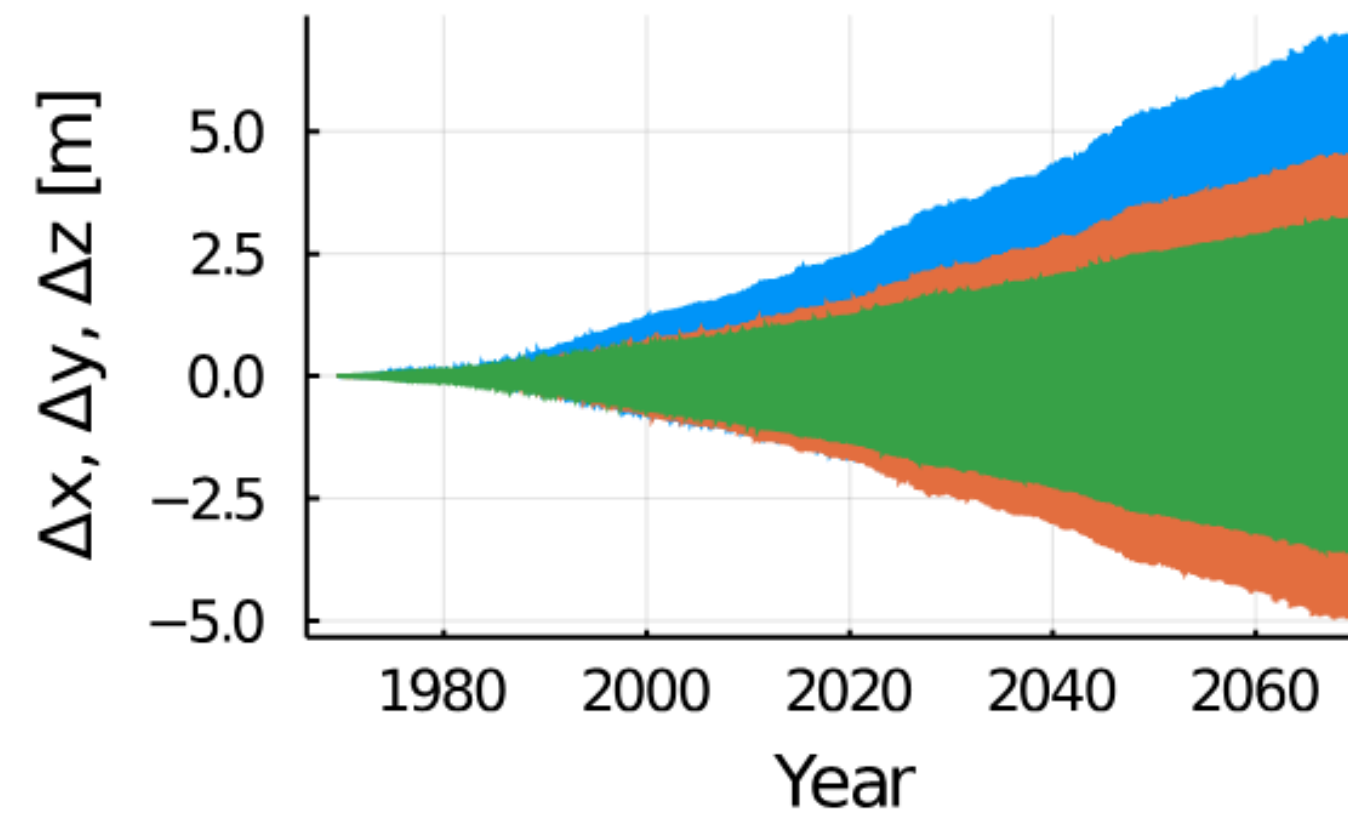
$$\tilde{\mathbf{x}}(\delta\mathbf{x}_0, t) = \mathbf{x}^{[0]}(t) + \sum_{l=1}^q \mathbf{x}^{[l]}(t)(\delta\mathbf{x}_0)^l.$$



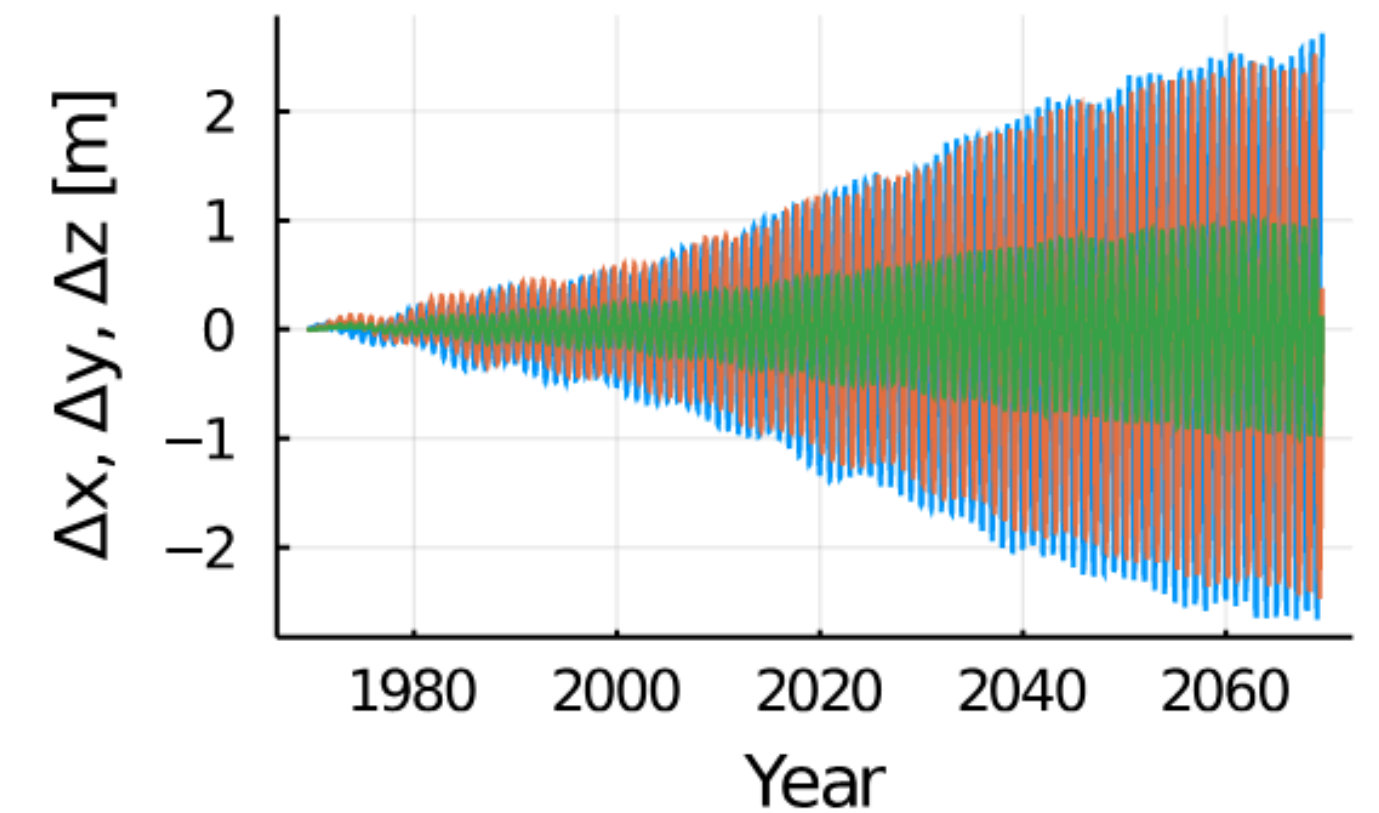
## Planetary ephemeris

- Using `TaylorIntegration.jl`, we implemented our own ephemeris integrator, based on the JPL DE430 dynamical model (Folkner et al., 2014).
- In the 1969-2069 time-span, we are able to reproduce the planetary positions and lunar geocentric range below the ~5 m level.

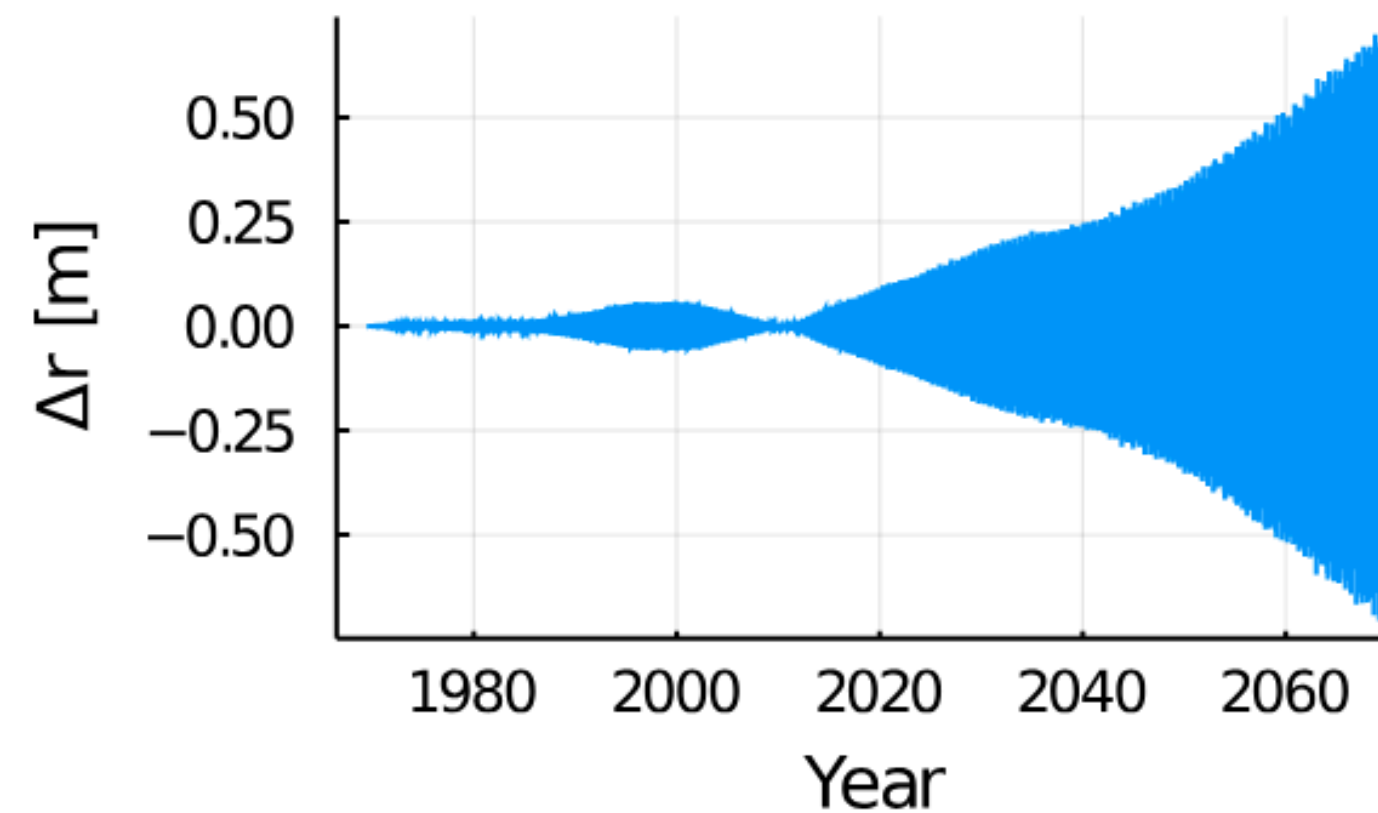
Mercury heliocentric position



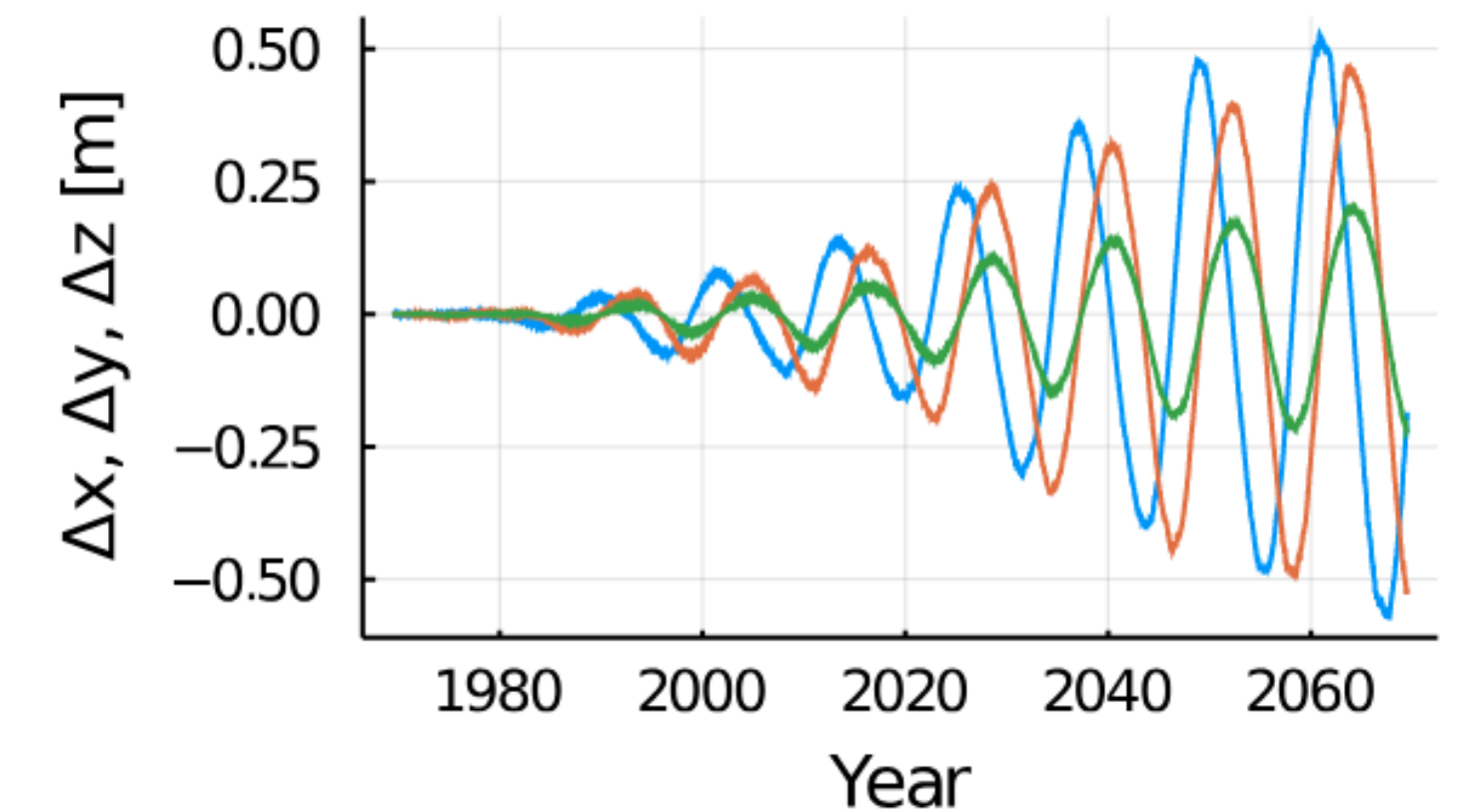
Earth heliocentric position



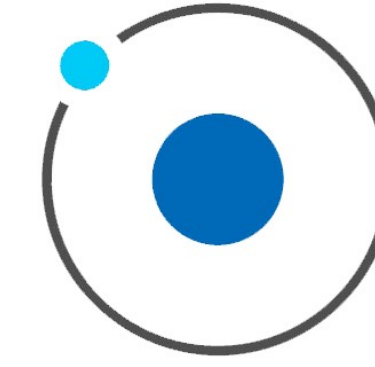
Moon geocentric range



Jupiter heliocentric position







## Results

- Using `TaylorIntegration.jl`<sup>1</sup>, we propagate the orbit of Apophis as a Taylor polynomial w.r.t. time  $t$ , changes in the initial position/velocity  $\delta\mathbf{x}_0$  and the Yarkovsky parameter  $A_2$ . This technique is known as *jet transport*.
- From this, we compute the (weighted) mean square residual  $Q(\delta x_0, A_2) = \chi^2/n_{\text{obs}}$  as a Taylor polynomial in the unknown orbital parameters. We use all available radar astrometry for this object (17 delay, 29 Dopplers) and selected optical astrometry (472 RA/Dec pairs) following Vokrouhlický et al. (2015). The observational arc spans from 2004 to 2014.
- We obtain two orbital fits to radar+optical data: a gravity only 6-DOF (**OR6**) and a 7-DOF fit (**OR7**) which accounts for Yarkovsky. We find local minima of the Taylor expansion of  $Q$  via Newton method.
- We find a mean semi major axis drift  $\langle \dot{a} \rangle = (-341 \pm 158) \text{ m/yr}$ , corresponding to  $A_2 = (-5.0 \pm 2.8) \times 10^{-14} \text{ au/d}^2$ .

<sup>1</sup> URL: [github.com/PerezHz/TaylorIntegration.jl](https://github.com/PerezHz/TaylorIntegration.jl)

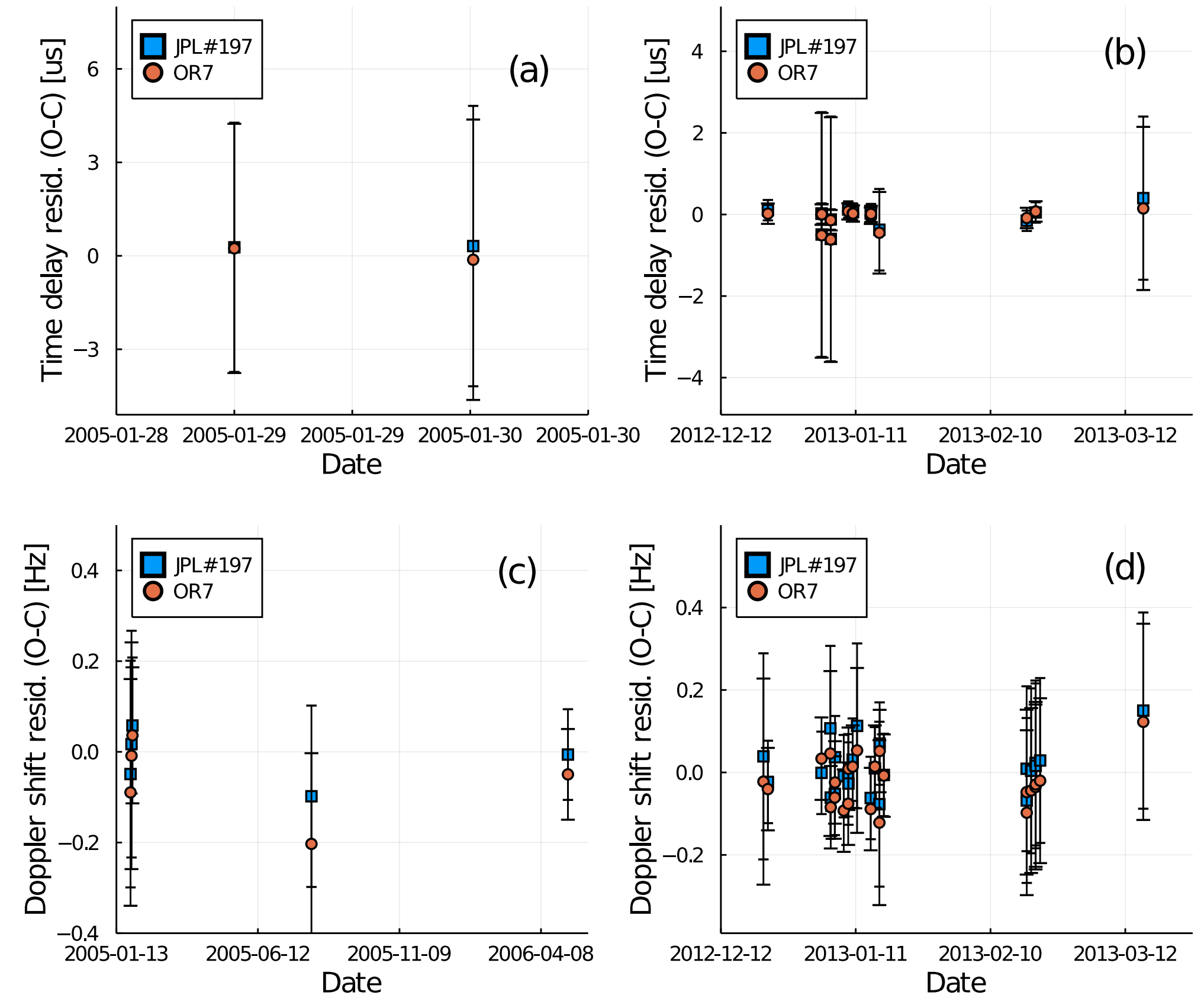
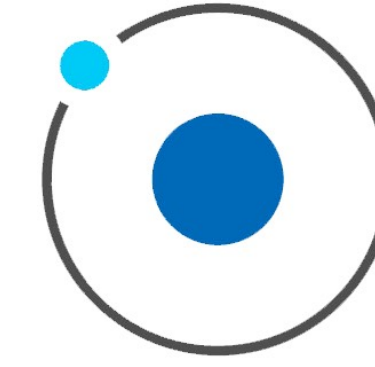


Fig: Post-fit delay/Doppler astrometric residuals (OR7).



## Optical astrometry postfit residuals

- Our optical astrometry error model accounts for biases present in star catalogs following Eggl et al. (2020), and accounts for other sources of systematic errors via an appropriate weighting scheme following Vokrouhlický et al. (2015).
- Using Eggl. et al. (2020) debiasing table, we find a mean RA/DEC postfit residual ( $0.011''$ ,  $-0.009''$ ) for the Tholen et al. (2013) astrometry.
- As a check on our implementation of the debiasing procedure, we compare the post-fit astrometric residuals for observations from Tholen et al. (2013), for two debiasing tables: Farnocchia et al. (2015) and Eggl et al. (2020).

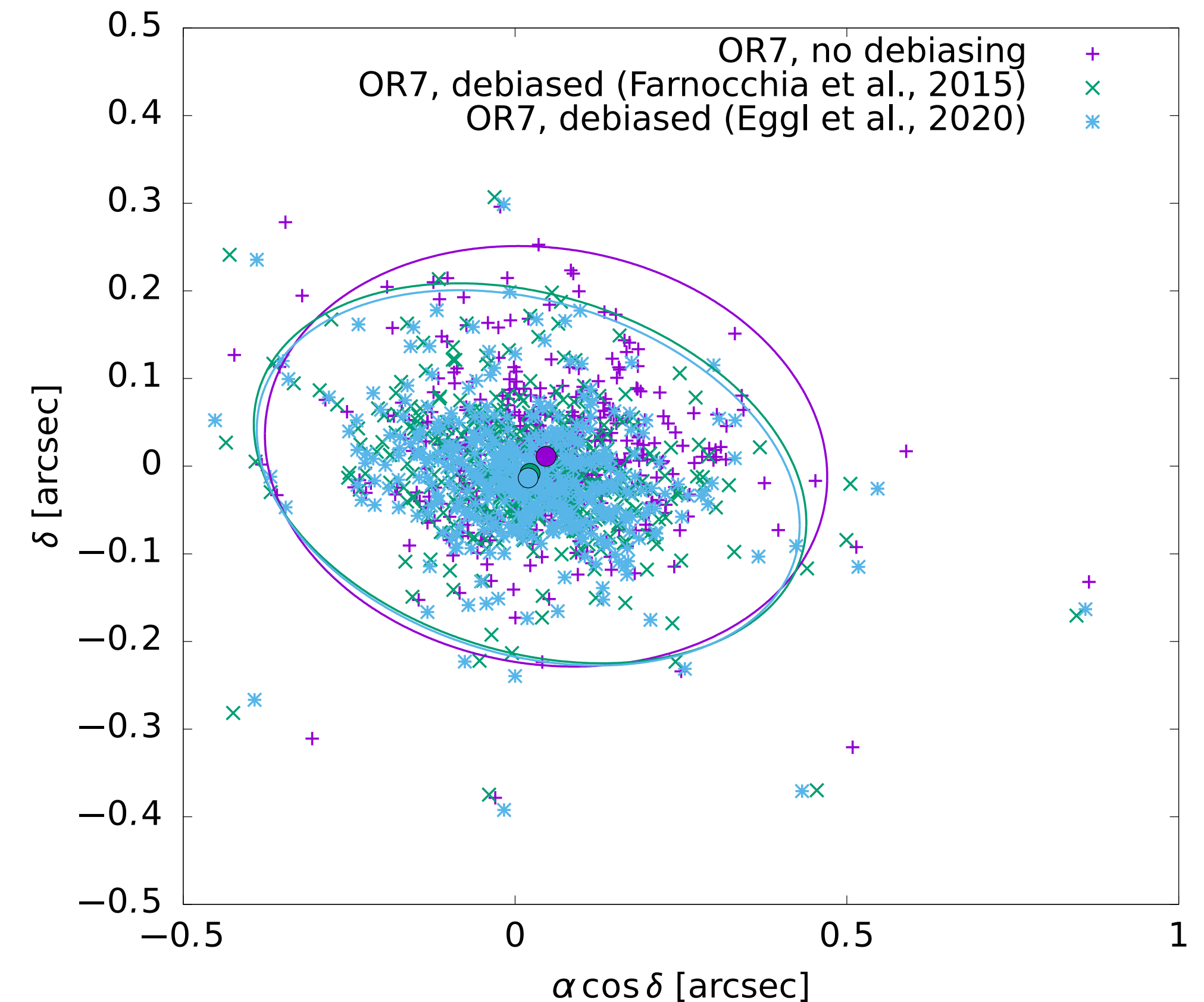
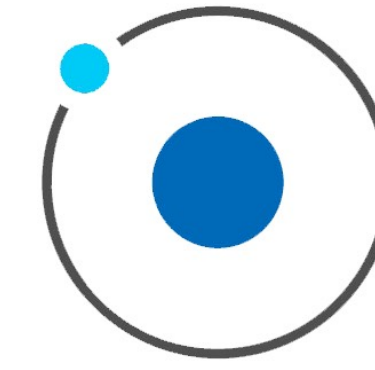


Fig: Post-fit residuals for Tholen et al. (2013) optical astrometry, for Farnocchia et al. (2015) and Eggl et al. (2020) debiasing schemes. Ellipses correspond to  $3\text{-}\sigma$  level.

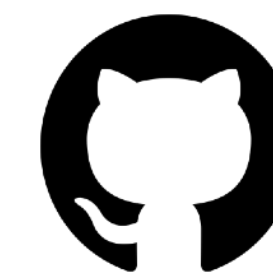
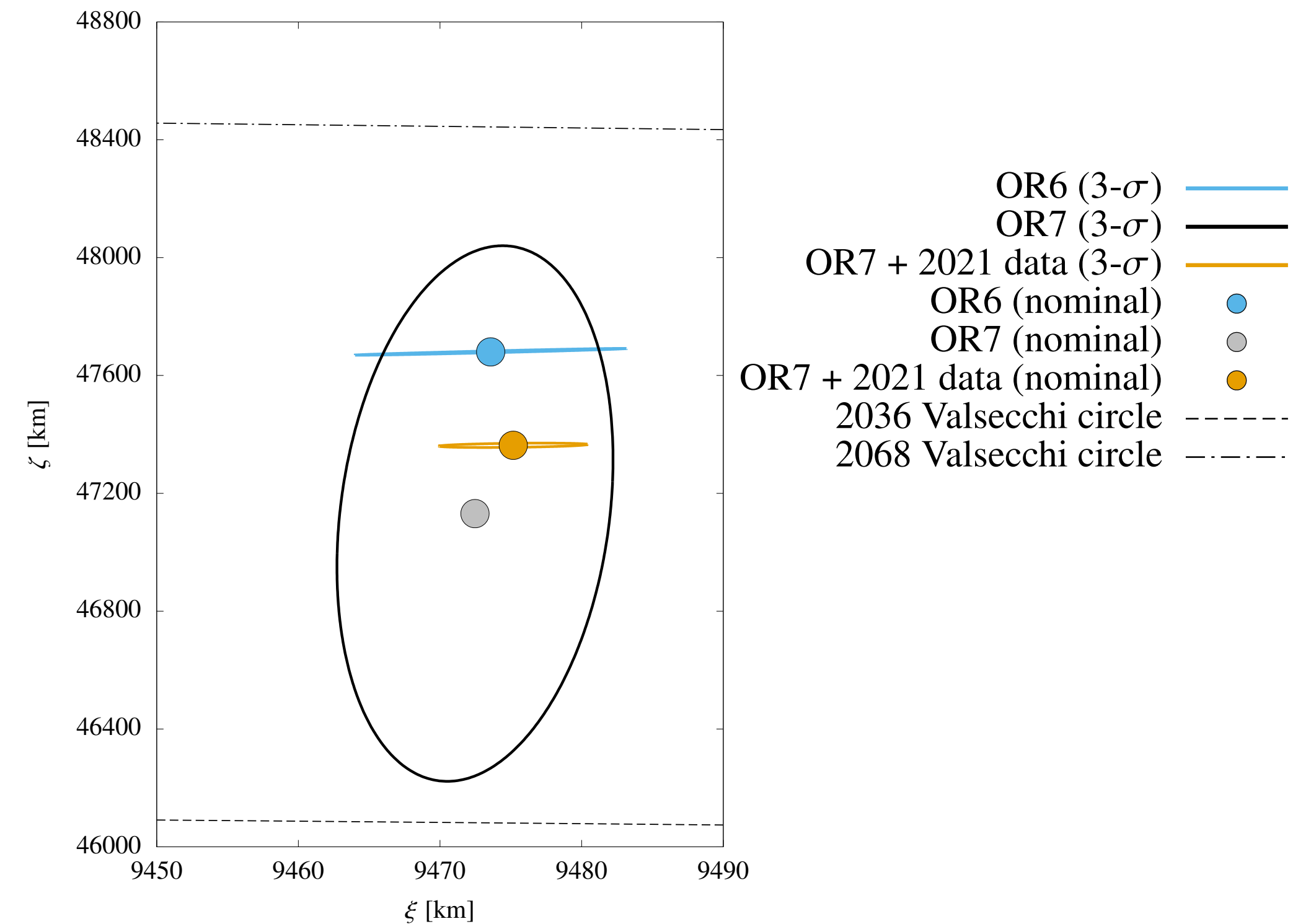




## Conclusions

- We implemented a software package for NEA orbit determination and uncertainty propagation based on high-order Taylor method. To be made public soon; stay tuned!
- We provide an independent (marginal) estimation of Yarkovsky effect for Apophis from data, consistent with previous results (Vokrouhlický et al., 2015). In particular, after performing an orbital fit to selected optical and radar astrometry spanning from 2004 to 2014 we find  $\langle \dot{a} \rangle = (-341 \pm 158) \text{ m/yr}$ , corresponding to  $A_2 = (-5.0 \pm 2.8) \times 10^{-14} \text{ au/d}^2$ .
- After updating our orbital solutions with new Goldstone radar astrometry from the current apparition (March 2021), as well as ~6,000 RA/Dec optical astrometry points spanning from 2004 to 2021, we find  $\langle \dot{a} \rangle = (-199 \pm 2) \text{ m/yr}$ , corresponding to  $A_2 = (-2.90 \pm 0.03) \times 10^{-14} \text{ au/d}^2$ .
- After projecting the orbital uncertainty region associated to our latest solution onto the 2029 B-plane, we find no significant intersection with the 2068 Valsecchi circle.

## 2029 B-plane (Öpik's frame)



**GitHub:**  
**@PerezHz**  
**[PerezHz/TaylorIntegration.jl](https://github.com/PerezHz/TaylorIntegration.jl)**

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