

Bridging Accuracy and Efficiency in Uncertainty-Aware Opto-Mechanical Design

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ABSTRACT

The dynamic response of opto-mechanical systems is a determining factor in achieving the required performance under disturbed environmental conditions. Vibration attenuation, in particular, drives the allocation of pointing and wavefront-error budgets. In the early design phase, standard practice adopts nominal values for material properties, geometric dimensions, and interface characteristics, producing a single deterministic set of frequency response functions. The resulting design is not robust: it commonly requires substantial rework after the first test campaigns, or the introduction of conservative margin factors. Fully statistical treatments of parameter dispersion, by contrast, are computationally demanding and are frequently obstructed by confidentiality restrictions on the exchange of models between partners.

This paper reviews the principal techniques employed to account for uncertainty in material properties. In addition, a computationally efficient method is proposed for the construction of a robust family of system responses through a simplified relationship between material-property dispersion and dynamic response. The formulation is compatible with representations in modal coordinates.

Several approaches to the rigorous treatment of material property uncertainty are compared and applied to a representative opto-mechanical system from ground-based telescope. The results demonstrate that the simplified method yields outcomes comparable to those obtained with more elaborate algorithms, providing a practical instrument for use in the early design phase.

Keywords: uncertainty propagation, early design, optomechanical systems, vibrations, material properties, model uncertainty factor

1. INTRODUCTION

Opto-mechanical assemblies constitute critical subsystems in high-precision instrumentation for astronomy, interferometry, atmospheric remote sensing, and optical communication.¹⁻⁴ The integration of optical trains with load-bearing structures imposes stringent tolerances that must be maintained under a range of operating conditions. Moreover, parametric dispersion in geometry, material properties, boundary conditions, and operational loads remains a dominant contributor to performance variability and constitutes a principal design driver.

Consider an ensemble of nominally identical structures. Their frequency response functions (FRFs) exhibit low dispersion in the low-frequency regime and increasing dispersion as modal density grows.⁵ The high-frequency band is conventionally addressed via Statistical Energy Analysis,⁶ whereas the low-frequency band admits a deterministic treatment when the statistics of the physical parameters are known. In principle, the Finite Element Method (FEM) enables such a treatment through Monte Carlo sampling; in practice, stochastic campaigns on high-fidelity models incur prohibitive computational cost.⁷ This limitation is compounded in multi-partner programmes, in which dynamic sub-models are exchanged in reduced form and the explicit dependence of the response on the physical parameters is no longer accessible.

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The prevailing engineering solution is the application of margin factors: analyses are conducted with nominal parameters, and residual uncertainty is absorbed by conservative allowances. Although computationally efficient, this practice lacks statistical foundation and typically yields sub-optimal designs, particularly for hardware subject to tight constraints on optical path length or modal separation. A representative case is reported in Ref. 8 for a subsystem of the Extremely Large Telescope (ELT⁹), in which frequency-dependent compliance varies by a factor of approximately three as eigenfrequencies traverse control-loop cut-offs. Such behaviour motivates the adoption of robust design and analysis procedures.

Several methodological developments have addressed subsets of this problem. In the context of spacecraft micro-vibration, modifications of the Craig-Bampton reduction¹⁰ (which is widely adopted in aerospace applications where sub-model exchange is subject to confidentiality constraints) introduce randomisation of subsystem eigenfrequencies and modal participation factors,^{11–14} enabling efficient generation of FRF ensembles. Ref. 15 quantifies the propagation of material and geometric dispersion to transfer-function variability on a canonical structure but does not formulate corresponding design-stage strategies. Launch-induced dynamic environments further amplify the relevance of uncertainty for space hardware, as discussed in Refs. 16–18.

At the system-of-systems level, Ref. 19 addresses parametric uncertainty in the design of the James Webb Space Telescope by combining models of heterogeneous fidelity within a Monte Carlo framework, achieving substantially improved efficiency relative to a brute-force high-fidelity implementation while retaining a dependence on the availability of underlying physical models. The conventional formulation of the Model Uncertainty Factor is described in Ref. 20; its calibration relies on prior engineering experience that is not consistently available. General surveys of uncertainty quantification and propagation^{21–23} and large-scale Monte Carlo implementations^{19, 24} share two common limitations: dependence on detailed physical models and significant computational demand. The deployment of more advanced schemes at programme scale has been shown to be largely impractical.²⁵

The present work aims to bridge the methodological gap between margin-based practice and computationally intensive state-of-the-art approaches. The objective is a formulation that requires neither detailed physical models nor extended simulation campaigns. The starting point is Ref. 26, in which a local expansion is employed to propagate parametric perturbations to eigenfrequency and eigenmode variations, and subsequently to FRF realisations. This formulation is revisited and simplified. A novel approach is then proposed, in which a compact relationship between material-property dispersion and structural response yields an ensemble of FRFs from a single nominal FRF, without additional FEM analyses and with limited a priori knowledge of the system. The method is compatible with reduced and modal representations and is unaffected by the confidentiality constraints that restrict model availability.

The focus on opto-mechanical systems is motivated by two considerations: their high sensitivity to vibratory excitation, both terrestrial and orbital, and the recurrence of characteristic topologies within the family. In particular, a mirror cell can frequently be represented as a cantilever beam terminated by a concentrated mass. This topological analogy underpins the formulation and validation of a general relationship between material-property dispersion and the response envelope, which is of particular utility in early design phases, when measured data are unavailable.

The remainder of the paper is organised as follows. Section 2 reviews four methods for the generation of random system realisations from parametric uncertainty, including the two proposed in this work. Section 3 introduces three opto-mechanical designs drawn from operational programmes. Section 4 applies the four methods to these test cases and evaluates their performance. Section 5 summarises the conclusions and outlines directions for further investigation.

2. METHODS

The uncertainty in the material properties and geometrical parameters of the system affects the system dynamics and its FRF. A robust design approach shall account for the source error in the simulation of the system due to the uncertainties over the model parameters. A conservative approach is to inflate the value of the FRF of $+1\sigma$, where σ is the FRF standard deviation, propagated from the parameters uncertainties.

In the following, the approaches already discussed in Ref. 27 are presented, extending the notation to the case in which more parameters are involved in the analysis.

2.1 Deterministic Finite Element Monte Carlo Method (DFEM)

Finite element analyses offer a direct evaluation of the system response in the low-mid frequency range, where the behaviour of the system is deterministic. The deterministic finite element Monte Carlo method (DFEM) is based on evaluating the variation of the system response by perturbing the model parameters, within their uncertainty.

Considering the vector $\mathbf{p}$ as the vector of the model parameters, different modal analyses are run by randomizing the values in $\mathbf{p}$,⁷ assuming them to be normally distributed. At each step of the Monte Carlo, corresponding to a different set of parameters, the FRF is computed from the corresponding modal analysis result, by using a classical modal superposition approach (Ref. 28, 29).

The solution to a mechanical system $\mathbf{y}$, computed at the point $\mathbf{x}_0$, can be written as follow.

$$\mathbf{y}(\mathbf{x}_0, t) = \sum_n \mathbf{W}_n(\mathbf{x}_0) \cdot q_n(t) = \mathbf{\Phi} \cdot \mathbf{q} \quad (1)$$

Where $\mathbf{W}_n(\mathbf{x}_0)$ is the n -th modal shape evaluated at $\mathbf{x}_0$, q_n is the n -th modal coordinate. Written in matrix form, the matrix $\mathbf{\Phi}$ collects the modal shapes in its columns, and the column vector $\mathbf{q}$ collects the modal coordinates. Each modal coordinate behaves as a second order system, which equations can be written, in matrix form, as reported in the following.

$$\ddot{\mathbf{q}} + [2\zeta\omega] \dot{\mathbf{q}} + [\omega^2] \mathbf{q} = \mathbf{\Phi}^T \mathbf{f} \quad (2)$$

By defining the modal damping coefficient ζ_n and the modal frequency ω_n , $[2\zeta\omega]$ and $[\omega^2]$ are diagonal matrices, with respectively $2\zeta_n\omega_n$ and ω_n^2 in the diagonal. The parameter $\mathbf{f}$ is the force input to the system.

The response of the system can be written in state space representation, being $\mathbf{x}$ the state variable defined as follow.

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{\Phi}^T \mathbf{f} - [2\zeta\omega] \dot{\mathbf{q}} - [\omega^2] \mathbf{q} \end{bmatrix} \quad (3)$$

The output of the state space is the solution $\mathbf{y}$ of the mechanical system (see Eq. 2). To simulate seismic response, the input is typically a displacement $\mathbf{u}$, applied to the base node. Considering the compliancy of the interface, the input node is connected to the system with a compliant element, with stiffness k .

Then, the state space representation is reported as follows.

$$\begin{aligned} \dot{\mathbf{x}} &= A \mathbf{x} + B k \mathbf{u} \\ \mathbf{y} &= C \mathbf{x} \end{aligned} \quad (4)$$

$$A = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -[\omega^2] & -[2\zeta\omega] \end{bmatrix} \quad ; \quad B = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^T \end{bmatrix} \quad ; \quad C = [\mathbf{\Phi} \quad \mathbf{0}] \quad (5)$$

The matrices $\mathbf{0}$ and $\mathbf{I}$ are respectively a zero and a identity matrix.

By exploiting the Laplace transform (see Eq. 6), the transfer function between input and output can be computed as shown in the following expression.

$$\begin{aligned} s \mathbf{X}(s) &= A \mathbf{X}(s) + B k \mathbf{U}(s) \\ \mathbf{Y}(s) &= C \mathbf{X}(s) \end{aligned} \quad (6)$$

$$\mathbf{H}(s) = \frac{\mathbf{Y}(s)}{\mathbf{U}(s)} = C (s \mathbf{I} - A)^{-1} B k \quad (7)$$

The functions $\mathbf{X}(s)$, $\mathbf{U}(s)$, and $\mathbf{Y}(s)$ are the Laplace transforms of state variable, input, and output respectively. For each iteration of the Monte Carlo, the results of the modal analysis are used to fill the state matrices. Then, the FRF can be computed as the absolute value of the transfer function $H(i\omega)$.

Given the hypothesis that the system is deterministic at the frequency of interest, the DFEM approach is considered as a benchmark. However, typically, designers shall consider also uncertainties coming from the model itself, for example meshing and boundary conditions.

Moreover, although, the DFEM offers a solid tool to propagate the parameters uncertainty over the system FRF,

it needs to run many simulations, requiring a large computational burden.⁷

2.2 Modal propagation (MP)

By focusing on the modal representation of the system solution, a variation in the parameters vector $\mathbf{p}$ affects both the values of the modal frequencies ω_n and the modal shapes, collected in Φ . The expression for the variation of ω_n and Φ , respectively $\delta\omega_n$ and $\delta\Phi$, can be linearized as shown in Eq. 8 and 9 (Ref. 26), as a function of the parameter uncertainty $\delta\mathbf{p}$.

$$\delta\omega_n = J_\omega \delta\mathbf{p} \quad (8)$$

$$\delta\Phi = J_\Phi \delta\mathbf{p} \quad (9)$$

The terms J_ω and J_Φ are the Jacobian respectively of the natural frequencies and the modal shapes with respect to the parameter vector. While J_ω is a matrix, J_Φ is a tensor.

Both the Jacobian can be approximated by finite differences, by perturbing one parameter at a time across its nominal value (for example, one at $\pm 0.25\%$). Then, the propagation to the system FRF is trivial.

The modal propagation (MP) approach requires a number of simulations proportional to the number of parameter (considering a number of parameters N_p , MP requires $2N_p + 1$ simulations). Then, it saves computational time only if the number of parameters to investigate is small.

2.3 Frequency propagation (FP)

As already discussed in Ref. 27, the effects of variation of the modal shapes can be typically neglected ($\delta\Phi \approx 0$), considering only the effect of the frequency propagation (FP).

Since the modal frequency appears only in the matrix A of the state space representation, it is the only component which need to be updated to propagate the uncertainty. This simplifies the computation. However, as already discussed for MP, FP requires to run $2N_p + 1$ to compute the Jacobian of the frequencies.

2.4 Frequency envelope perturbation (FEP)

The frequency envelope perturbation (FEP²⁷) take advantage of some considerations over the static deflection of a beam to formulate a physics-based propagation approach.

The static deformation y at the end of a beam, due to gravity, is equal to:^{28,29}

$$y(l) = \frac{m g l^3}{3 E I} \quad (10)$$

where m is the beam mass, g is the gravitational acceleration, l is the beam length, E and I are the beam Young's modulus and moment of inertia respectively.

The n -th natural frequency of the beam is:^{28,29}

$$\omega_n = n \sqrt{\frac{3 E I}{m l^3}} \quad (11)$$

The variation of a given system parameter causes a change in the value of ω_n .

The derivative of the modal frequency with respect to m is equal to:

$$\frac{\delta\omega_n}{\delta m} = -\frac{n}{2m} \sqrt{\frac{3 E I}{m l^3}} \quad (12)$$

By ordering the terms in Eq. 12, the relative variation of the natural frequency, due to a mass variation, is equal to half of the relative variation of the mass, as shown in Eq. 13. The notation $\delta\omega_{n,\delta p}$ stands for the variation of ω_n due to a variation of the generic parameter p (δm in Eq. 12). This holds also for the density ρ .

$$\frac{\delta\omega_{n,\delta m}}{\omega_n} = -\frac{1}{2} \frac{\delta m}{m} \quad (13)$$

158 Similarly, the relative variation of the natural frequency, due to a Young's modulus variation, is equal to half of
 159 the relative variation of the E (see Eq. 14).

$$\frac{\delta\omega_{n,\delta E}}{\omega_n} = \frac{1}{2} \frac{\delta E}{E} \quad (14)$$

160 Then, by knowing the relative uncertainty on the model parameter, the relative uncertainty on the modal
 161 frequency can be directly computed from Eq. 13 and 14. FEP provides a clear and intuitive relation between
 162 the variation of a model parameter and the variation of the FRF, which can be evaluated by running a single
 163 modal analysis.

164 In conclusion, the variation of the n -th modal frequency due to the variation of multiple parameters can be
 165 written as follows:

$$\delta\omega_n = \sum_{i=1}^{N_p} \delta\omega_{n,\delta p_i} \quad (15)$$

166 where N_p is the number of parameters under investigation and p_i is the i -th parameter, and $\delta\omega_{n,\delta p_i}$ is the
 167 contribution to the frequency variation given by p_i , computed according to Eq. 13 or Eq. 14. Then, as already
 168 done for FP, the matrix A shall be updated in Eq. 7 for the computation of the FRF.

169 3. CASE STUDY SYSTEMS

170 A case study for testing different uncertainty propagation techniques is taken from MORFEO:³⁰ an advanced
 171 optical system, used in ELT for compensating the image distortion from the turbulence in the atmosphere. In
 172 particular, the focus is on the MORFEO Calibration Unit Folding Mirror (CUFM). The approach for modeling
 173 the mirror is the same used in Ref. 27. The materials are represented in Fig. 1 and collected in Tab. 1. The
 174 input $u(t)$ is the displacement of a node, connected to the base of the structure with a compliant element of
 175 stiffness k . The excitation is provided in the vertical direction (Z in Fig. 1). This simulates the effect of seismic
 176 noise, injected at the base of the structure. The output $y(t)$ corresponds to the mirror rotation around Y.
 177 The damping ratio ζ is assumed to be 0.75% for all modes.

178
 179 The mirror support structure is made of steel. To account for small variations in material properties arising
 180 from the manufacturing process, the structural components are divided into three groups: (i) beams located to
 181 the left of the mirror, (ii) beams located to the right of the mirror, and (iii) all remaining components.
 182 To present the approach for modal uncertainty propagation, four parameters have been selected to propagate
 183 the uncertainty:

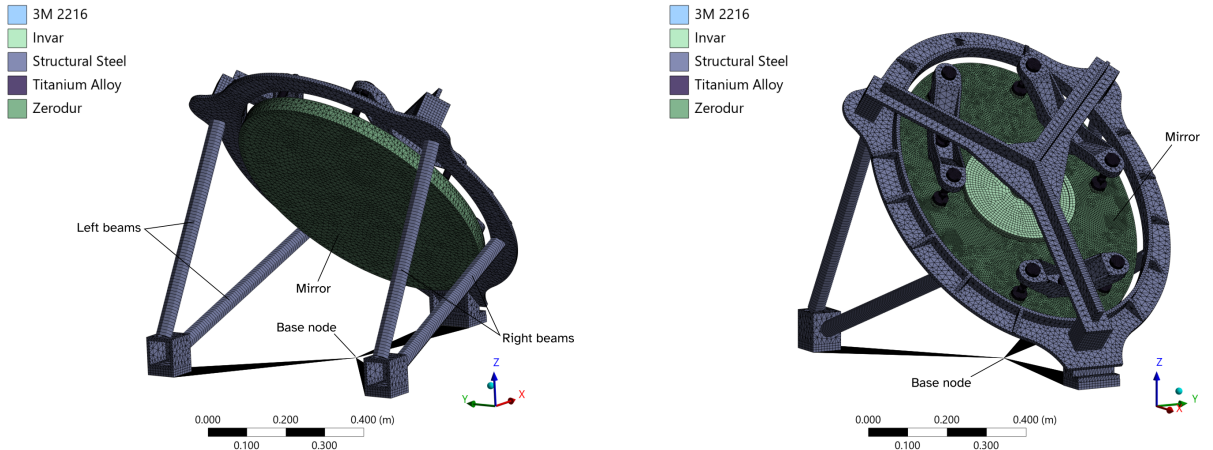


Figure 1: FEM model of the MORFEO CUFM. The mesh is represent in the figure, color coded by material. The material properties are reported in Tab. 1.

Table 1: Material data of MORFEO's CUFM: reference to Fig. 1.

Material	Density	Young's modulus	Poisson ratio
3M 2216 glue	1.33 g/cm ³	2.95 GPa	0.43
Invar	8.00 g/cm ³	150 GPa	0.30
Structural steel	7.85 g/cm ³	202 GPa	0.30
Titanium alloy	4.62 g/cm ³	96 GPa	0.36
Zerodur	2.50 g/cm ³	91 GPa	0.24

- **Parameter 1:** Young's modulus of the steel of the beams at the left side of the mirror;
- **Parameter 2:** Young's modulus of the steel of the beams at the right side of the mirror;
- **Parameter 3:** Young's modulus of the titanium alloy (material of the mirror suspension system);
- **Parameter 4:** density of the Zerodur, which is the material of the mirror (suspended mass);

The effects of each parameter is analyzed separately by following the approach presented in Ref. 27. Then, the combined effect of them are analyzed as different contributions to the overall uncertainty of the system FRF.

3.1 Metrics for comparison between FRFs

As already done in Ref. 27, for each proposed method, the $+1\sigma$ FRF is computed as the result of uncertainty propagation. The generated FRFs are graphed together to identify differences between them. However, the tiny differences among the different FRF curves make them difficult to be identified. For this reason, the differences between each method (MP, FP, and FEP) and the benchmark (DFEM) are plotted.

Moreover, two metrics are evaluated to access quantitative the difference between the curves: the root mean square error and the Pearson correlation coefficient.²⁷

Root mean square errors

The root mean square error (RMSE) quantifies the difference between the curves^{31,32} and it is defined as follows.

$$\text{RMSE} = \sqrt{\frac{\sum_{n=1}^L (a_n - b_n)^2}{L}} \quad (16)$$

where a_n and b_n are the n -th sample of two different curves, having L samples each.

Pearson correlation coefficient

The Pearson correlation coefficient^{11,31,33} (ϱ) gives a metric related to the relative trend between two curves, quantifying the level of congruence between the two.

$$\varrho = \frac{L \sum_{n=1}^L a_n b_n - \sum_{n=1}^L a_n \sum_{n=1}^L b_n}{\sqrt{L \sum_{n=1}^L a_n^2 - (\sum_{n=1}^L a_n)^2} \sqrt{L \sum_{n=1}^L b_n^2 - (\sum_{n=1}^L b_n)^2}} \quad (17)$$

The value of ϱ is defined between one and minus one, being 1 the value related to the best correlation between the curves.³²

The correlation coefficient is verified by a hypothesis test with the aim of confirming its significance.³²⁻³⁵ The test's null hypothesis (H_0) is defined as the case in which the two compared curves are not correlated. The test statistic t_m , associated with the ϱ_m coefficient, is defined as follows.

$$t_m = \varrho_m \sqrt{\frac{L-2}{1-\varrho_m^2}} \quad (18)$$

By considering a significance level α , to reject the null hypothesis, the value of t_m shall be larger than the two-sided t-student's distribution ($t_{1-\frac{\alpha}{2}, \nu}$). The value of ν is the number of degrees of freedom of the distribution, equal to $L - 2$.

For all the cases reported in the following, by considering α equal to 1%, there exist a strong statistical evidence of rejecting the null hypothesis, always confirming the statistical relevance of the correlation coefficient.

4. SUMMARY OF NUMERICAL RESULTS

4.1 Single parameter - uncertainty propagation

The propagation of uncertainty for each parameter considered individually has been previously investigated in Ref. 27. Nevertheless, for completeness, the following section briefly compares the influence of the parameters of interest and evaluates the performance of the proposed uncertainty propagation methods. In Ref. 27, 4 different levels of uncertainty are evaluated ($\pm 2\%$, $\pm 3\%$, $\pm 5\%$, and $\pm 7\%$). In the following, just the $\pm 5\%$ is considered. Fig. 2 shows that the effect of parameters 1 and 2 (respectively Fig. 2a and 2b) does not strongly affect the system behaviour. The effect of the titanium Young's modulus gets relevant at high frequency. The Zerodur density, instead, produces a larger dispersion of the FRF curves, as expected, being the mass parameter of the suspended object, focus of the present analysis.

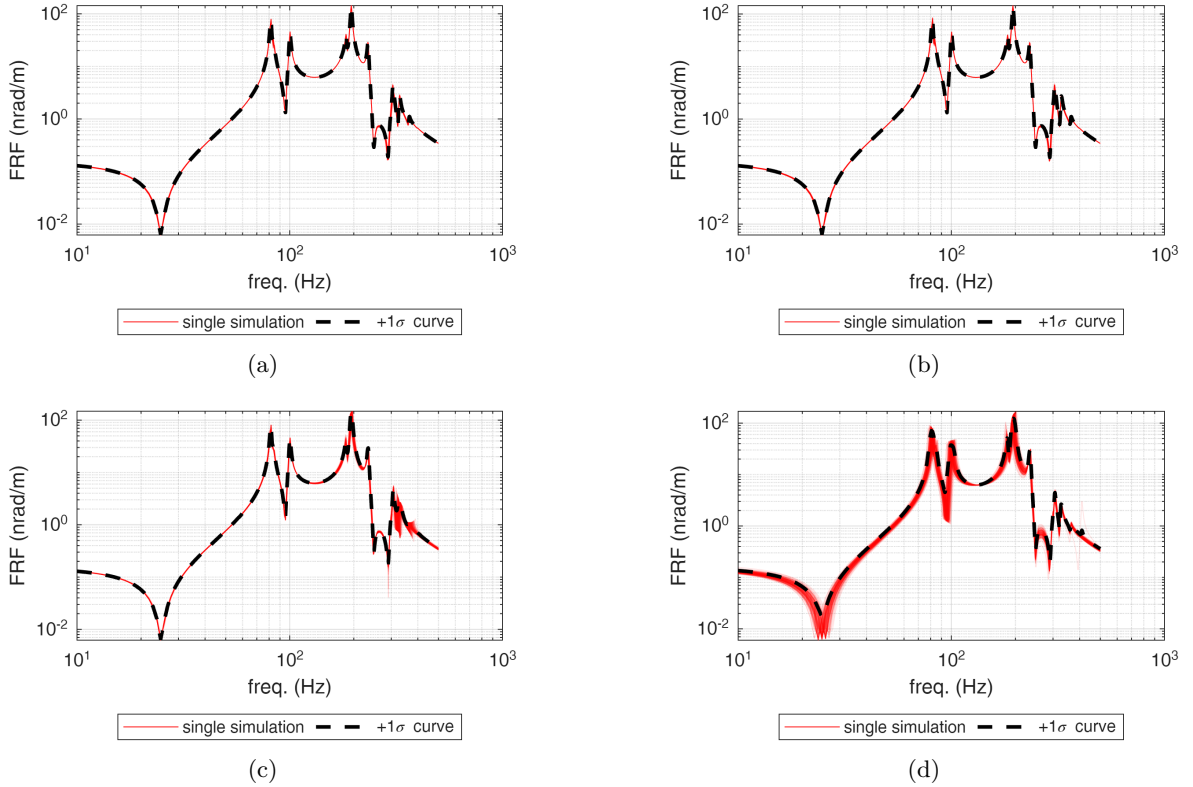


Figure 2: Results of DFEM method run over 150 simulations run by randomizing one parameter at a time, assuming a normal distribution with $\pm 5\%$ standard deviation. Both the FRF computed by every single simulation and the $+1\sigma$ curve are reported. (a) reports the results by varying the Young's modulus of the steel of the beams at the left side of the mirror; (b) shows the results related to the Young's modulus of the steel of the beams at the right side of the mirror; (c) refers to the Young's modulus of the titanium alloy; and (d) regards the density of the Zerodur.

Fig. 3 reports the absolute value of the relative error between the $+1\sigma$ FRF computed with each of the proposed methods and the benchmark (DFEM). It is worth noticing that all the methods produces a relative error

226 of few percent, showing, in general, good performance in the prediction of the modal uncertainty propagation.
 227 The main error occurs in the proximity of a zero or a peak of the FRF, where the slope of the curve is more
 228 steep and a small error in the frequency can produce even a large error in the amplitude of the FRF.
 229 As already discussed, the Zerodur density is the parameter playing the bigger role in the FRF. By focusing to MP
 230 and FP, the error is much lower in the case of the Zerodur density. This is probably due to the lower numerical
 231 error introduced in the approximation of the Jacobian. The FEP, instead, is not sensitive to the errors in the
 232 Jacobian computation. For this reason, it shows the best performance in the Fig. 3a and 3b, where the error in
 233 the Jacobian is probably higher due to the small influence of the parameters in such cases.
 234 Although the values of the relative error around the peaks and the zeros can be relevant, the RMSE is not
 235 sensibly different among the different cases (Tab. 2). Marginal differences between the techniques can be seen
 236 also in terms of Pearson correlation coefficient (Tab. 3).
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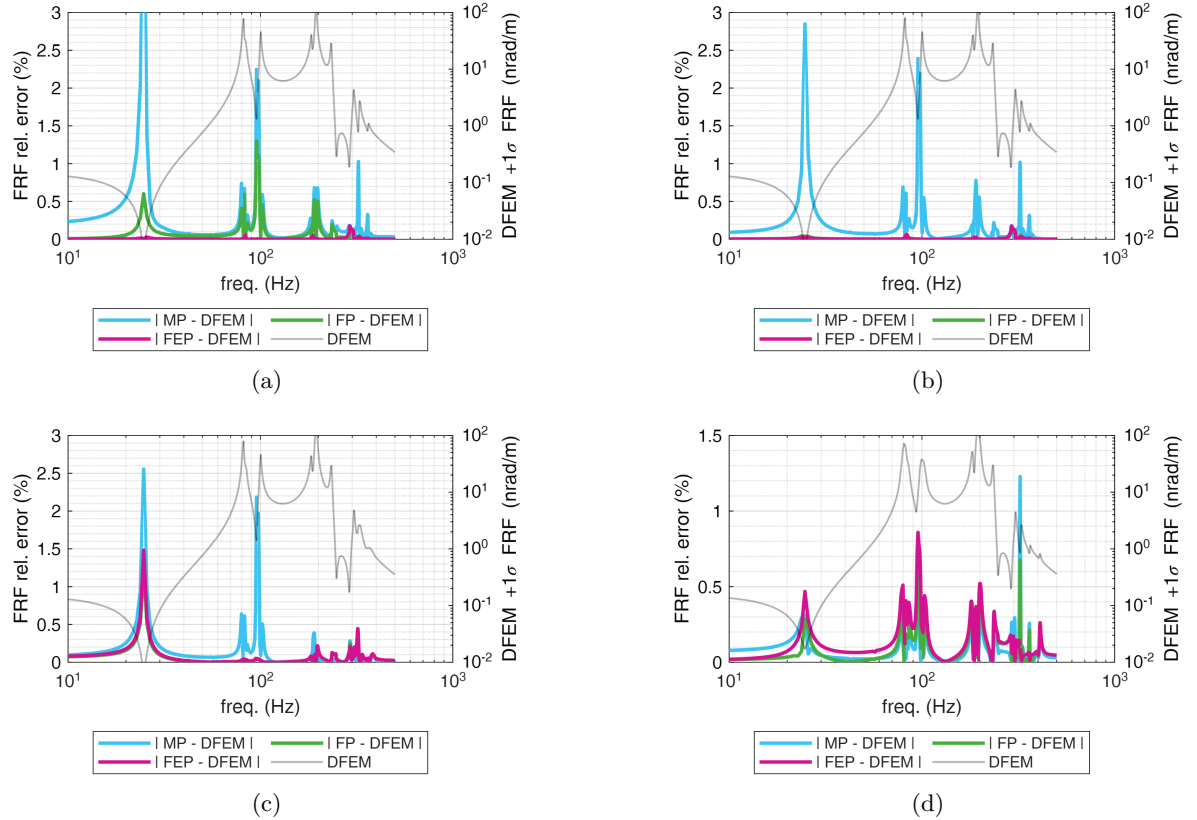


Figure 3: Absolute value of the relative error of the $+1\sigma$ FRF obtained by each method (FP, MP, and FEP) with respect to the one obtained by DFEM, used as a benchmark, varying one parameter at a time. Each parameter is assumed to be normally distributed around its nominal value, with a $\pm 5\%$ standard deviation. The DFEM curve is also reported as a reference. (a) reports the results by varying the Young's modulus of the steel of the beams at the left side of the mirror; (b) shows the results related to the Young's modulus of the steel of the beams at the right side of the mirror; (c) refers to the Young's modulus of the titanium alloy; and (d) regards the density of the Zerodur.

Table 2: Root mean square error (RMSE), computed as shown in Eq. 16, for all presented methods (FP, MP, and FEP).

Parameter	RMSE - MP (nrad m ⁻¹)	RMSE - FP (nrad m ⁻¹)	RMSE - FEP (nrad m ⁻¹)
1	5.2	4.1	0.2
2	4.9	0.2	0.3
3	2.8	1.7	1.8
4	3.3	5.1	6.6

Table 3: Pearson correlation coefficient (ϱ), computed as reported in Eq. 17, for all presented methods (FP, MP, and FEP).

Parameter	ϱ - MP (%)	ϱ - FP (%)	ϱ - FEP (%)
1	97.6	98.6	100.0
2	97.6	100.0	100.0
3	98.9	99.7	99.6
4	98.5	96.5	95.2

4.2 Multiple parameters - uncertainty propagation

In the following, the uncertainties of all parameters are propagated simultaneously to assess the capability of each method to accurately represent the variation in the system response resulting from the combined parameter uncertainties. By following the same approach of Ref. 27, 4 different values for the parameters' standard deviation are evaluated (2%, 3%, 5%, and 7%).

Fig. 4 shows the results of 150 simulations for each one of the 4 different levels of parameter uncertainties. The $+1\sigma$ FRF is also reported. As expected, increasing the parameter uncertainty leads to a larger dispersion of the FRFs.

The absolute value of the error in the estimation of the $+1\sigma$ FRF is evaluated for each of the proposed techniques (MP, FP, and FEP). The results are reported in Fig. 5. RMSE and Pearson correlation coefficient for each case are reported in Tab. 4 and 5.

As observed in the single-parameter uncertainty analysis, the error in the estimation of the $+1\sigma$ FRF exhibits localized peaks near the zeros and resonant peaks of the FRF. Nevertheless, even in the multivariate analysis, the error remains below a few percent.

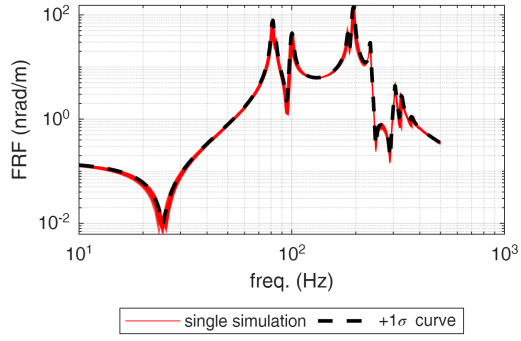
Although the MP method generally provides the best performance in terms of both RMSE and ϱ , the approximation introduced by the FEP method has only a limited impact on the results. In particular, the RMSE remains of the same order of magnitude, while the correlation coefficient consistently exceeds 90%.

Table 4: Root mean square error (RMSE), computed as shown in Eq. 16, for all presented methods (FP, MP, and FEP).

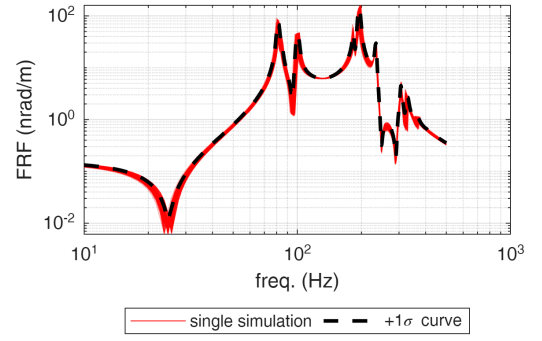
Par. STD	RMSE - MP (nrad m ⁻¹)	RMSE - FP (nrad m ⁻¹)	RMSE - FEP (nrad m ⁻¹)
2	1.5	3.4	4.6
3	1.7	3.5	5.2
5	3.6	4.8	7.0
7	3.8	5.1	8.0

Table 5: Pearson correlation coefficient (ϱ), computed as reported in Eq. 17, for all presented methods (FP, MP, and FEP).

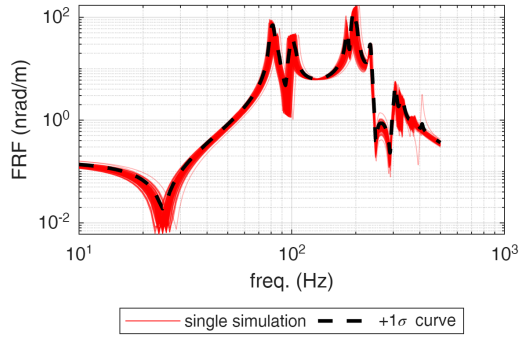
Par. STD	ϱ - MP (%)	ϱ - FP (%)	ϱ - FEP (%)
2	99.7	99.0	98.5
3	99.6	98.9	97.9
5	98.2	97.6	95.4
7	98.1	97.0	92.9



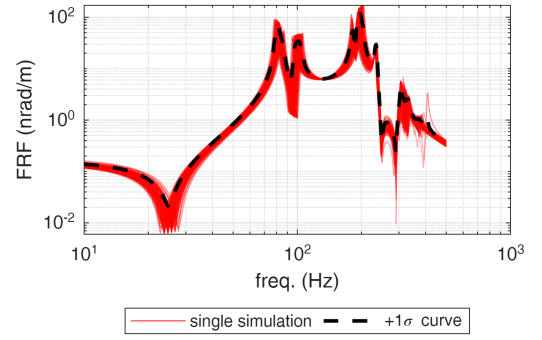
(a)



(b)



(c)



(d)

Figure 4: Results of DFEM method run over 150 simulations run by randomizing all the parameters at the same time, assuming a normal distribution with standard deviation respectively equal to 2%, 3%, 5%, and 7%, for (a), (b), (c), and (d). Both the FRF computed by every single simulation and the $+1\sigma$ curve are reported.

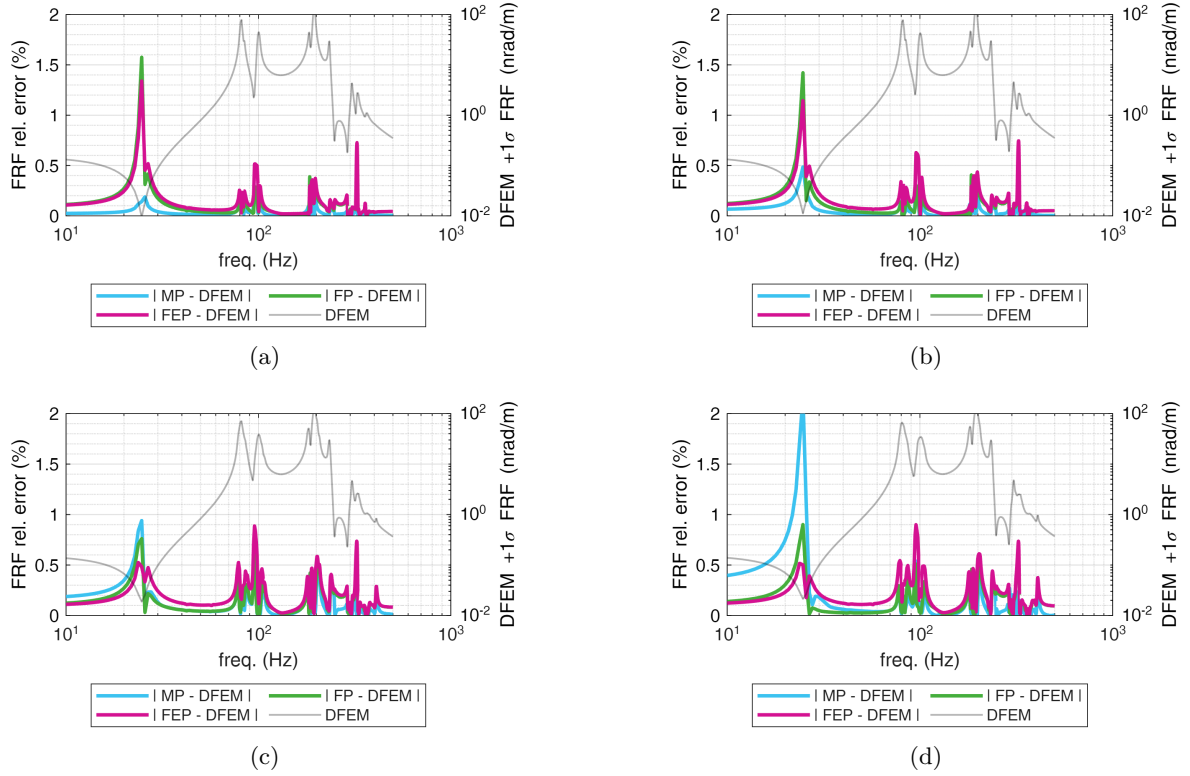


Figure 5: Absolute value of the relative error of the $+1\sigma$ FRF obtained by each method (FP, MP, and FEP) with respect to the one obtained by DFEM, used as a benchmark, varying all the parameters at the same time. Each parameter is assumed to be normally distributed around its nominal value, with standard deviation respectively equal to 2%, 3%, 5%, and 7%, for (a), (b), (c), and (d). The DFEM curve is also reported as a reference.

5. CONCLUSIONS

This work extends the technique proposed in Ref. 27 by incorporating the participation factors of each body (or material) into the computation of the FEP method. Furthermore, it demonstrates that all the proposed techniques can be applied to the case in which multiple parameters are affected by uncertainty, enabling the prediction of the FRF uncertainty envelope, defined as the $+1\sigma$ FRF.

Moreover, this work highlights the challenges associated with computing the Jacobians (for both the natural frequencies and the mode shapes), which are required by both the MP and FP methods. Their accurate evaluation requires careful calibration of the finite-difference step size according to the sensitivity of the FRF to each uncertain parameter. Furthermore, different FRFs may exhibit different sensitivities to the same parameter, implying that, in principle, a different finite-difference step size should be adopted for each FRF to accurately compute the corresponding Jacobians. This significantly increases the computational effort, as additional simulations are required to reduce the error introduced by the finite-difference approximation. In contrast, the FEP method is not affected by this issue and always requires only a single simulation to estimate the $+1\sigma$ uncertainty envelope of the FRF.

REFERENCES

- [1] McElwain, M. W., Feinberg, L. D., Perrin, M. D., Clampin, M., and et al., “The James Webb Space Telescope Mission: Optical Telescope Element Design, Development, and Performance,” *Publications of the Astronomical Society of the Pacific* **135**, 058001 (May 2023). Publisher: The Astronomical Society of the Pacific.

- [2] Ramsay, S., Amico, P., Bezawada, N., Cirasuolo, M., and et al., “The ESO Extremely Large Telescope instrumentation programme,” in [*Advances in Optical Astronomical Instrumentation 2019*], Ellis, S. C. and d’Orgeville, C., eds., *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series* **11203**, 1120303 (Jan. 2020).
- [3] Danzmann, K., Amaro-Seoane, P., Audley, H., Babak, S., and others, “LISA - Laser Interferometer Space Antenna,” tech. rep. (Jan. 2017).
- [4] Straume-Lindner, A. G. and Ingmann, P., “ADM-Aeolus: The first space-based high spectral resolution Doppler Wind Lidar,” in [*2007 IEEE International Geoscience and Remote Sensing Symposium*], 4969–4974 (2007).
- [5] Kompella, M. S. and Bernhard, R. J., “Measurement of the statistical variation of structural-acoustic characteristics of automotive vehicles,” in [*Noise Vibration Conference Exposition*], SAE International (may 1993).
- [6] Lyon, R. H., DeJong, R. G., and Lyon, R. H., [*Theory and application of statistical energy analysis*], Butterworth-Heinemann, Boston, 2nd ed ed. (1995).
- [7] Lu, J., Zhan, Z., Apley, D. W., and Chen, W., “Uncertainty propagation of frequency response functions using a multi-output gaussian process model,” *Computers Structures* **217**, 1–17 (2019).
- [8] Holzlöhner, R., Kellerer, A., Lampater, U., Lewis, S., and Zaroni, C., “Structural, thermal, and optical performance analysis applied to subsystems of the European Extremely Large Telescope,” *Journal of Astronomical Telescopes, Instruments, and Systems* **8**(2), 021504–021504 (2022).
- [9] Gilmozzi, R. and Spyromilio, J., “The European extremely large telescope (E-ELT),” *The Messenger* **127**(11), 3 (2007).
- [10] Craig Jr, R. R. and Bampton, M. C., “Coupling of substructures for dynamic analyses.,” *AIAA journal* **6**(7), 1313–1319 (1968).
- [11] Yotov, V., Remedias, M., Aglietti, G., and Richardson, G., “Improved Craig–Bampton stochastic method for spacecraft vibroacoustic analysis,” *Acta Astronautica* **178**, 556–570 (Jan. 2021).
- [12] De Lellis, S., Stabile, A., Aglietti, G. S., and Richardson, G., “Structural uncertainty estimation through a Craig-Bampton Stochastic Method optimisation in satellites structures,” *Journal of Sound and Vibration* **469**, 115123 (Mar. 2020).
- [13] Remedias, M., Aglietti, G. S., Richardson, G., and Sweeting, M., “Integrated Semiempirical Methodology for Microvibration Prediction,” *AIAA Journal* **53**, 1236–1250 (May 2015).
- [14] Remedias, M., Aglietti, G., and Richardson, G., “A stochastic methodology for predictions of the environment created by multiple microvibration sources,” *Journal of Sound and Vibration* **344**, 138–157 (May 2015).
- [15] Abolfathi, A., O’Boy, D. J., Walsh, S. J., and Fisher, S. A., “Investigating the sources of variability in the dynamic response of built-up structures through a linear analytical model,” *Journal of Sound and Vibration* **387**, 163–176 (Jan. 2017).
- [16] Calvi, A., “Uncertainty-based loads analysis for spacecraft: Finite element model validation and dynamic responses,” *Computers & Structures* **83**, 1103–1112 (May 2005).
- [17] Stewart, E. C., Belloch, P. A., and Coppolino, R. N., “Improving Accuracy of Structural Dynamic Modification with Augmented Residual Vectors,” in [*AIAA Scitech 2019 Forum*], American Institute of Aeronautics and Astronautics, San Diego, California (Jan. 2019).
- [18] Laborde, S. and Calvi, A., “Spacecraft base-sine vibration test data uncertainties investigation based on stochastic scatter approach,” *Mechanical Systems and Signal Processing* **32**, 69–78 (2012).
- [19] Cataldo, G., Qian, E., and Auclair, J., “Multifidelity uncertainty quantification and model validation of large-scale multidisciplinary systems,” *Journal of Astronomical Telescopes, Instruments, and Systems* **8**, 038001 (July 2022).
- [20] Uebelhart, S., Miller, D., and Blaurock, C., [*Uncertainty Characterization in Integrated Modeling*] (2005).
- [21] Zhou, K., Wang, Z., Gao, Q., Yuan, S., and Tang, J., “Recent advances in uncertainty quantification in structural response characterization and system identification,” *Probabilistic Engineering Mechanics* **74**, 103507 (2023).
- [22] DiazDelaO, F., Adhikari, S., Saavedra Flores, E., and Friswell, M., “Stochastic structural dynamic analysis using bayesian emulators,” *Computers Structures* **120**, 24–32 (2013).

- [23] [A Comparative Study of Uncertainty Propagation Methods for Black-Box Type Functions], *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference Volume 6: 33rd Design Automation Conference, Parts A and B* (09 2007).
- [24] Ng, L. W. T. and Willcox, K. E., “Multifidelity approaches for optimization under uncertainty,” *International Journal for Numerical Methods in Engineering* **100**(10), 746–772 (2014).
- [25] Gupta, S. and Ghosh, D., [Uncertainty Quantification in Structural Engineering: Current Status and Computational Challenges], ch. Chapter 5, 119–149.
- [26] Mace, B. and Shorter, P., “A local modal/perturbational method for estimating frequency response statistics of built-up structures with uncertain properties,” *Journal of Sound and Vibration* **242**, 793–811 (May 2001).
- [27] Dalla Ricca, E., Zaroni, C., and Redaelli, E. M. A., “Accounting for material property uncertainty in the preliminary vibration analysis of opto-mechanical systems,” *Journal of Astronomical Telescopes, Instruments, and Systems* **10**(4), 044002–044002 (2024).
- [28] Meirovitch, L., [Fundamentals of vibrations], Waveland Press (2010).
- [29] Rao, S. S., [Vibration of continuous systems], John Wiley & Sons (2019).
- [30] Redaelli, E. M. A., Aliverti, M., Saggin, B., and Farinato, J., “MAORY/MORFEO at ELT: optomechanical preliminary design,” in [Advances in Optical and Mechanical Technologies for Telescopes and Instrumentation V], Navarro, R. and Geyl, R., eds., **12188**, 121883V, International Society for Optics and Photonics, SPIE (2022).
- [31] Liemohn, M. W., Shane, A. D., Azari, A. R., Petersen, A. K., Swiger, B. M., and Mukhopadhyay, A., “Rmse is not enough: Guidelines to robust data-model comparisons for magnetospheric physics,” *Journal of Atmospheric and Solar-Terrestrial Physics* **218**, 105624 (2021).
- [32] Ren, J., Zhang, W., Yang, J., Shen, Z., Zhao, J., Zhou, Y., and Wang, Z., “A comparison of numerical and lu modeling of water flow and heat transport with laboratory experiments,” *Environmental Earth Sciences* **78**, 1–16 (2019).
- [33] Obilor, E. I. and Amadi, E. C., “Test for significance of Pearson’s correlation coefficient,” *International Journal of Innovative Mathematics, Statistics & Energy Policies* **6**(1), 11–23 (2018).
- [34] Montgomery, D. C. and Runger, G. C., [Applied statistics and probability for engineers], John wiley & sons (2010).
- [35] Papoulis, A., [Probability and statistics], Prentice-Hall, Inc. (1990).