

Numerical and experimental demonstration of LQG-based predictive control for point-ahead angle turbulence mitigation in GEO feeder optical links

Kyliann Robert^{a,*}, Cyril Petit^a, Perrine Lognoné^b, Yann Lucas^a, Florian Quatresooz^{a,c}, Aurélie Montmerle-Bonnefois^a, Nicolas Védrenne^a, and Thierry Fusco^a

^aDOTA, ONERA, Université Paris Saclay, 92320 Châtillon, France

^bDepartment of Physics, Durham University, South Road, Durham, DH1 3LE, UK

^cEuropean Space Agency - ESA/ESTEC, NL-2200 AG Noordwijk, Netherlands

ABSTRACT

In the field of GEO-feeder telecommunication links, we propose the use of phase and log-amplitude measurements of the downlink beam on a Shack-Hartmann wavefront sensor and the coupling to a Linear Quadratic Gaussian dynamic predictive control law, to increase the precompensation correction quality of the uplink beam. In this paper, we present the structure of this predictive controller using these combined inputs, as well as end-to-end simulation results and world-first experimental results on a laboratory bench at ONERA.

Keywords: adaptive optics, predictive control, LQG, ground-to-GEO, telecommunications, phase, log-amplitude, intensity

1. INTRODUCTION

Optical feeder telecommunication links with geostationary (GEO) satellites are challenged by the point-ahead angle (PAA), originating from the relative motion between an optical ground station (OGS) and a GEO satellite. Its value of 18.5 μrad at European latitudes implies different turbulence volumes crossed by the down- and up-links, and results in anisoplanatism phase error when correcting the uplink turbulent phase using downlink measurements with adaptive optics (AO) systems. Furthermore, no method currently allows for real-time measurement of the uplink phase disturbances, and no real-time information can be obtained from the GEO satellite to get feedback on the quality of an uplink correction. Therefore, any uplink precompensation control law is a feed-forward control law.

Currently, OGS supporting GEO feeder links^{1,2} use downlink measurements to compute the uplink precompensation. Because of anisoplanatism, this method does not allow for an efficient turbulence correction and results in recurrent signal fadings in the satellite plane. Previous works³ show that jointly using phase and log-amplitude information, from a Shack-Hartmann wavefront sensor (SHWFS) on the downlink path, allows for better static prediction of the turbulent phase in the uplink path volume. This method was numerically validated^{3,4} and first experimental results on binary star measurements have been recently reported.⁵

The next step is to include this method of static prediction of turbulent phase at PAA into a real-time control law. In this work, we consider a Linear Quadratic Gaussian (LQG) predictive control law with slopes and intensity Shack-Hartmann measurements as inputs, which represents a first to our knowledge. We call this control law $LQG_{\phi\chi}$; it will be presented in Section 2. The $LQG_{\phi\chi}$ framework allows for the use of spatio-angular covariance matrices between phase log-amplitude information which are at the core of this upgraded precompensation method. Precisions on the integration of the intensity information in the control law and how it relates to log-amplitude information are brought in Section 2.4.

A numerical end-to-end simulation tool embedding SHWFS and deformable mirror (DM) models is developed. $LQG_{\phi\chi}$ performance is compared to classic, non-predictive uplink precompensation performance (integrator) on a given turbulence profile in Section 3.

E-mail: kyliann.robert@onera.fr

The major objective is to experimentally prove the numerically-expected gain in performance. We rely on a modification of the ONERA PICOLO turbulence emulating bench⁶ and LISA AO bench⁷ to assess this gain in controlled turbulence conditions. The main aspects of the modification of the setup to meet a GEO feeder configuration are presented in Section 4. The world first experimental results of dynamic $LQG_{\phi\chi}$ control for PAA precompensation, using phase and log-amplitude measurements from the SHWFS, are presented and analyzed in Section 4.3.

Finally, the expected future works, especially the extension to on-sky validation on FEELINGS, an OGS located near Toulouse, France,² are addressed in Section 5.

2. METHODOLOGY

2.1 Adaptive optics system design

The problem is approached in a discrete time formulation. In the following, we will consider a classic AO system, with a SHWFS for the analysis of the downlink turbulence. One DM, assumed to have a perfect response, is used in the system and is common to the downlink and uplink paths. Therefore, only one path's correction can be optimized at a time. When correcting on the uplink path, the downlink correction performance will be reduced and the uplink performance increased, and vice-versa. AO control loops will run with a constant two-frames delay, typical for AO systems. The measurement of the correction performance on both paths is detailed in Section 2.6.

2.2 Control method

Different control laws can support the predictive approach. Currently, two of them are being investigated. One of them is pseudo-open loop control (POLC), and is detailed in.⁸ In this paper, we focus on the other control law considered, an LQG predictive control law. As mentioned in the introduction, the computation and the application of the uplink precompensation command will be in a feed-forward law. Therefore, we can decouple the downlink feedback law from the uplink feed-forward precompensation; but both aspects can be introduced in the same LQG control law.

In practice, the downlink feedback law is a temporal control law⁹ that works by using an evolution model of the downlink turbulence quantities, as well as downlink real-time measurements over the pupil. Both serve as input of a Kalman filter that computes the optimal temporal estimate (assuming linear evolution processes) of the phase on the downlink path (defined over the system pupil) one time step ahead, at each time step of the AO control loop. Then, a linear quadratic regulator (LQR) is used and serves both to compute a spatial estimate of the turbulent phase in the uplink volume and then to project the uplink phase estimate into the DM commands space.

The work is based on the work in,³ where it is shown that jointly using phase and log-amplitude information from the SHWFS on the downlink path allows for better estimation of the turbulent phase in the uplink path

volume. Therefore we can define a state vector $X_n = \begin{pmatrix} \Phi_n^0 \\ \chi_n^0 \\ \Phi_{n-1}^0 \\ \chi_{n-1}^0 \end{pmatrix}$ at time step n of the control loop, with χ_n^0 the

turbulent log-amplitude over the system pupil in the downlink turbulent volume and Φ_n^0 the turbulent phase over the system pupil in the downlink path volume (downlink quantities are denoted with an 0 exponent and uplink ones with an α_{PAA} exponent).

2.3 Evolution models

The state evolution equation is: $X_{n+1} = AX_n + v_{n+1}$ with A the state evolution matrix and v_n a noise term whose covariance matrix can straightforwardly be computed by an energy conservation formula.¹⁰ The evolution matrix A must take into account the temporal evolution for downlink quantities Φ_n^0 and χ_n^0 like in the following:

$$X_{n+1} = AX_n + v_{n+1} = \begin{pmatrix} A_\Phi & 0 & 0 & 0 \\ 0 & A_\chi & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{pmatrix} \begin{pmatrix} \Phi_n^0 \\ \chi_n^0 \\ \Phi_{n-1}^0 \\ \chi_{n-1}^0 \end{pmatrix} + v_{n+1} \quad (1)$$

with A_Φ and A_χ downlink temporal evolution matrices. Assuming the temporal evolution processes to be linear, these matrices can be optimally expressed as the product of covariance matrices of the underlying processes.⁹ Thus, we can write:

$$A_s = \Gamma_{s_{n+1}s_n} \Gamma_{s_n s_n}^{-1} \quad (2)$$

where Γ_{xy} denotes the covariance matrix between the x and y quantities, and $s_n = \{\Phi_n, \chi_n\}$ the downlink phase or log-amplitude information at time step n .

We compute analytically these matrices in the Zernike basis. Phase and log-amplitude can both be expressed in this basis¹¹ as $\Phi = \sum_{i=2}^n a_i Z_i$ and $\chi = \sum_{j=1}^m b_j Z_j$, where the Zernike polynomials Z_i are defined over the whole system pupil. In this basis, covariance matrices between Zernike modes of phase, log-amplitude, or between both, have an analytical expression^{11,12} that depends on the turbulent layers profile, strengths and associated wind speeds. Examples of these covariance matrices computed from the turbulence profile detailed in Section 3.1 are shown in Fig. 1. On these matrices, we observe dominant diagonal terms, meaning a Zernike mode is foremost coupled to itself, as well as non-zero off-diagonal terms that show terms coupling in this basis. All this information is key for a precise computation of the evolution matrices.

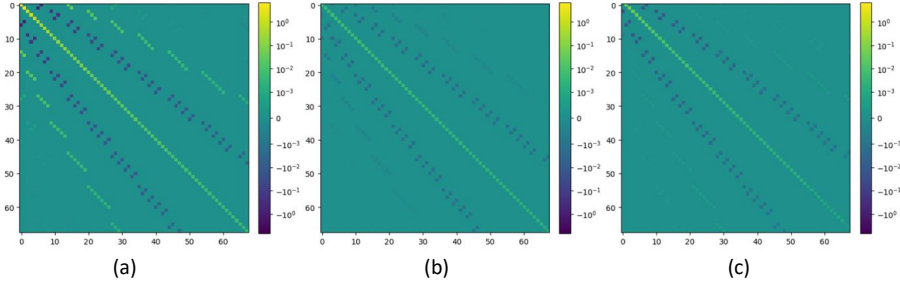


Figure 1. Auto-covariance matrices in the Zernike basis for the phase (a) and log-amplitude (b), and covariance matrix between the phase and log-amplitude (c), computed from the turbulence profile presented in Section 3.1

As for the spatial evolution model, that is used in the LQR part of the LQG control law, we can write the following evolution model:

$$\varphi_{n+1}^{\alpha PAA} = A' X_{n+1} + b_{n+1}'^v = (R_1 \quad R_2 \quad 0 \quad 0) X_{n+1} + b_{n+1}'^v \quad (3)$$

with R_1, R_2 the spatial estimation matrices that can be expressed in the same way as temporal estimation matrices as:

$$R = (R_1 \quad R_2) = (\Gamma_{\Phi PAA \Phi_0} \quad \Gamma_{\Phi PAA \chi_0}) \Gamma_{y_0 y_0}^{-1} \quad (4)$$

where $y_0 = (\Phi_0, \chi_0)$ is the downlink phase and log-amplitude information. This spatial evolution can also be computed analytically in the Zernike basis as presented above.

2.4 Measurement models

We use a diffractive SHWFS model for turbulence sensing. From a given turbulent field present in the system pupil, it outputs slope and intensity measurements. The phase Φ of the field can be related to slopes $\vec{s}l$ on the SHWFS via the usual matrix D :

$$\vec{s}l = D\Phi \quad (5)$$

As for the intensities management, we propose a new framework. First, we know that the integration of the signal on the SHWFS results in the intensity of the incoming field: $I(\vec{r}, t) = |\Psi|^2 = I_0(\vec{r})e^{2\chi(\vec{r}, t)}$, where I_0 corresponds to the average intensity map in the pupil. An expression of the intensity per sub-pupil can be obtained by integrating over the surface S_k of the k -th sub-pupil:

$$i_k = \iint_{S_k} I_0(\vec{r}) e^{2\chi(\vec{r}, t)} d\vec{r}$$

Here, we introduce a new quantity, the log-amplitude per subpupil: $e^{2\chi_k^{sp}} = \iint_{S_k} e^{2\chi(\vec{r},t)} d\vec{r}$. Moreover we consider that the mean intensity I_0 is averaged per sub-pupil, and can be expressed as: $I_0(\vec{r}) = \alpha_k^{sp} \mathcal{P}_k = I_0^k$, with α_k^{sp} a weighting factor for the k -th sub-pupil, corresponding to the sub-pupil spatial average of the mean intensity, and $\mathcal{P}_k$ the k -th sub-pupil indicator function (equal to 1 in the k -th sub-pupil and 0 on the rest of the pupil). Therefore the formula linking the intensity per sub-pupil i_k to the log-amplitude per sub-pupil χ_k^{sp} is $i_k = \alpha_k^{sp} \mathcal{P}_k e^{2\chi_k^{sp}} = I_0^k e^{2\chi_k^{sp}}$. It implies:

$$\chi_k^{sp} = \frac{1}{2} \ln \left(\frac{i_k}{I_0^k} \right) \quad (6)$$

In the case of small scintillation, if the turbulent regime is situated in the Rytov approximation,¹³ equation 6 can be simplified with series expansion into:

$$\chi_k^{sp} = \frac{i_k - I_0^k}{2I_0^k} \quad (7)$$

Now that we have access to the log-amplitude per sub-pupil quantity, we can create a new matrix E that links log-amplitude over the pupil χ to log-amplitude coefficients per subpupil $\vec{\chi}_{sp}$ via:

$$\vec{\chi}_{sp} = E\chi \quad (8)$$

Both D and E matrices can be numerically constructed with Zernike modes in phase or amplitude as inputs. Typical forms of these matrices are shown in Fig. 2.

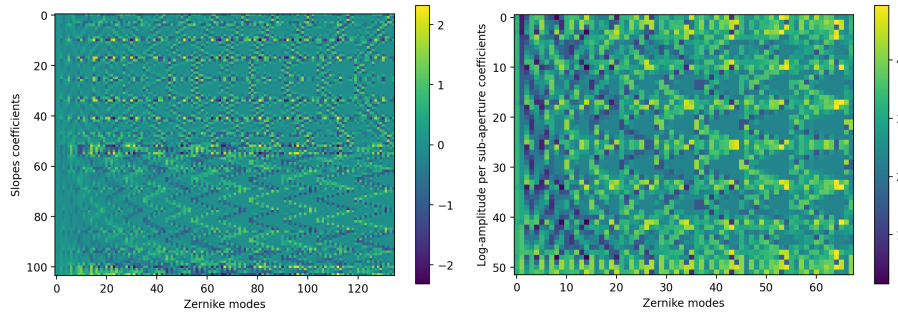


Figure 2. On the left: D matrix. On the right: E matrix.

2.5 LQG control equations

The LQG final equations are:

$$\begin{cases} \hat{X}_{k+1|k} = M_1 \hat{X}_{k|k-1} + M_1 \mathcal{H}_k(\mathbf{y}_k - (M_2 \hat{X}_{k|k-1} + M_3 \mathbf{u}_{k-1})) \\ \mathbf{u}_{k+1} = M_4 \hat{X}_{k+1|k} \end{cases} \quad (9)$$

With $\hat{X}_{k|k-1}$ being the state estimates at time step k knowing past measurements until time step $k-1$; $\mathbf{y}_k = (\vec{s}l, \vec{\chi}_{sp})$; $\mathbf{u}_k$ the DM commands; and $\mathcal{H}_k$ the Riccati gain.¹⁴

The LQG matrices M_1 to M_4 can be retrieved as:

$$M_1 = A \quad M_2 = \begin{pmatrix} 0 & 0 & D & 0 \\ 0 & 0 & 0 & E \end{pmatrix} \quad M_3 = \begin{pmatrix} M_{int} \\ 0 \end{pmatrix} \quad M_4 = \begin{pmatrix} -N^\dagger R_1 & -N^\dagger R_2 & 0 & 0 \end{pmatrix}$$

with M_{int} the AO system interaction matrix, and $N^\dagger$ the pseudo-inverse of the DM influence matrix N .

2.6 Control laws, performance metric & reciprocity principle

Different control laws will be tested in the following:

- Classic integrator control, using downlink phase measurements to compute a correction applied as the uplink precompensation;
- LQG_ϕ predictive control, using downlink phase measurements only to compute the uplink estimation and precompensation;
- $LQG_{\phi\chi}$ predictive control, using downlink phase and log-amplitude measurements for the uplink estimation and precompensation. This is the control law detailed in this paper;
- Optimal integrator control, corresponding to integrator control using uplink phase measurements to compute the uplink correction. Physically, this "optimal" control method is not reachable, as it would require to possess a beacon signal on the uplink path to measure the uplink turbulent volume. Therefore, it only serves here as an upper limit of what the performance can achieve.

To assess the performance of the different control laws, we evaluate the coupling efficiency (CE) of the signal in a single-mode fiber (SMF) onboard the GEO satellite. The objective is to obtain the highest average CE, with as few, short and shallow CE fadings to reduce significant losses of transmitted data.

To emulate this coupling both numerically and experimentally, we use a reciprocal approach: in the numerical simulations, for both downlink and uplink turbulent paths, the turbulent fields were simulated as downlink paths, with a PAA between both fields (named 'downlink on-axis' and 'downlink off-axis' respectively). We then make use of the reciprocity principle¹⁵ that applies on coupling efficiency results so that we can consider the downlink off-axis CE results as uplink results on a real system. Experimentally, the same reciprocity principle will be used; it is presented in Section 4.

3. NUMERICAL COUPLING EFFICIENCY RESULTS

3.1 Presentation

To validate and assess the LQG predictive control law performance, we use a given turbulence profile that can be reproduced both numerically and experimentally on the PICOLO bench at ONERA. For these first numerical results, we choose to use a three atmospheric layers profile with a ground level layer of $C_n^2 \cdot dz = 2.563 \times 10^{-11} \text{ m}^{-1/3}$ moving at 5 m/s, a second layer at 2.5 km altitude with $C_n^2 \cdot dz = 1.396 \times 10^{-11} \text{ m}^{-1/3}$ moving at 2 m/s, and another layer at 10 km altitude with $C_n^2 \cdot dz = 1.015 \times 10^{-12} \text{ m}^{-1/3}$ moving at 10 m/s. The profile corresponds to a Fried parameter $r_0 = 3.39 \text{ cm}$, an anisoplanatic angle $\theta_0 = 5.95 \text{ } \mu\text{rad}$, and a log-amplitude Rytov variance $\sigma_{\chi_R}^2 = 0.327$. It thus corresponds to a turbulence profile with strong scintillation, slightly out of the Rytov regime hypothesis used in Section 2.4. For this numerical validation, the complex fields are generated by the TURANDOT¹⁶ turbulent field generator.

The results are presented in two forms: CE temporal plots and cumulated density functions (CDF), that are cumulated histograms of the temporal plots. The latter visually represent the probability of the signal to dip lower than a given threshold; this allows for a visualization of the reduction of CE fadings depending on the control law used.

3.2 Numerical results

The end-to-end simulation tool uses SHWFS and DM direct models. The measurement noise sources are still neglected in this first result. As input, the simulation takes the turbulent field at each time step of the control loop and computes a correction according to the different control laws.

The temporal CE results for all four control laws are plotted on Fig. 3. Visually, it can be observed that many signal fadings from the classic integrator (in blue line) are erased by $LQG_{\phi\chi}$ control (in green), thanks to the addition of the intensity information in the control loop inputs. $LQG_{\phi\chi}$ is also able to increase the signal average value compared to classic integrator control. It can also be noticed that LQG_ϕ , predictive control using

only downlink phase information, barely manages to do better than integrator control. Some fadings are erased but others tend to appear; and the mean CE is slightly increased, but the gain is minor compared to the gain when adding log-amplitude information in the control law.

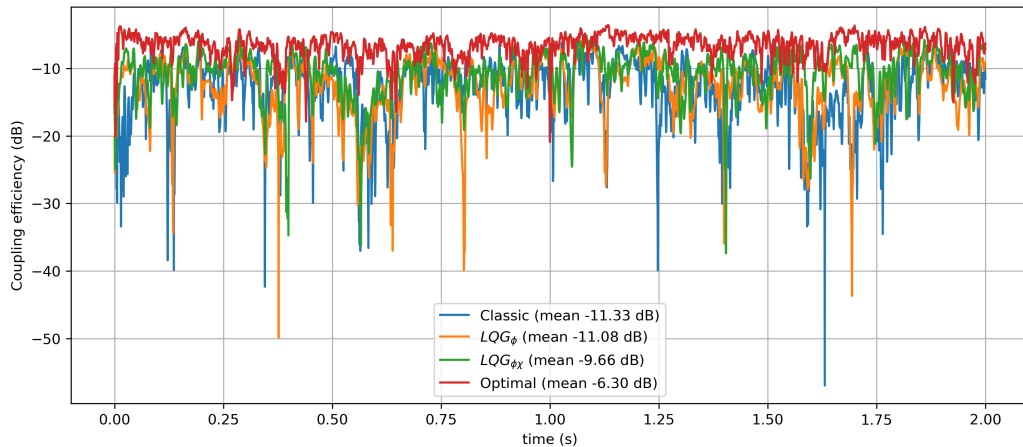


Figure 3. Temporal CE plots, for classic and optimal integrator control, LQG_ϕ and $LQG_{\phi\chi}$ predictive control.

A deeper analysis of signal fadings is led with the CE CDF plot on Fig. 4. On this plot, the diminution of signal fadings at all probability thresholds for $LQG_{\phi\chi}$ control compared to integrator or LQG_ϕ control can be clearly seen. Notably, $LQG_{\phi\chi}$ predictive control improves by an average of 5 dB the performance at the 10^{-3} to 10^{-1} thresholds. Yet on this CDF plot, we notice that the statistical convergence is not yet reached with the 2 second numerical simulation; larger datasets are expected to be computed to address this lack of convergence and enhance the analysis.

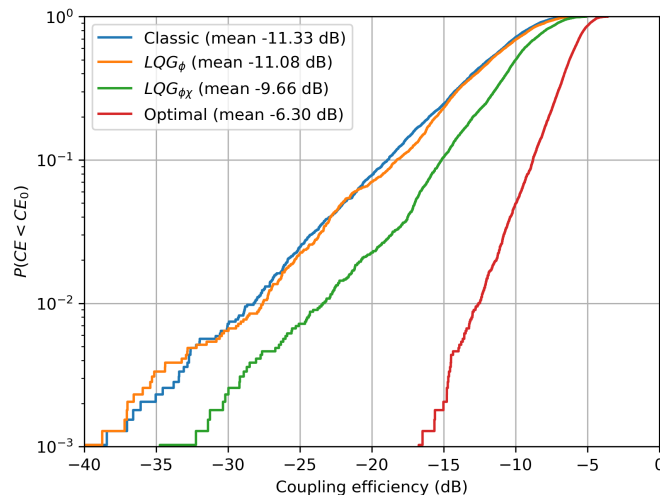


Figure 4. CDF plots, for classic and optimal integrator control, LQG_ϕ and $LQG_{\phi\chi}$ predictive control.

These first numerical results prove that using $LQG_{\phi\chi}$ is beneficial to the final precompensation performance, and comforts us in testing this method experimentally. However, these first results were obtained in simplified conditions where no noise sources were accounted for on the SHWFS. Scintillation in particular can have a detrimental effect on the phase measurement process. In reality, on the experimental bench at ONERA, these noise terms might modify the performance. In the following, we present the experimental validation.

4. EXPERIMENTAL VALIDATION OF $LQG_{\phi\chi}$ ON PICOLO+LISA

The experimental validation takes place on the PICOLO+LISA laboratory bench at ONERA, with PICOLO the turbulence emulator bench⁶ and LISA the AO system.⁷ We here introduce their composing elements as well as the recent design modifications to experimentally emulate the uplink precompensation scheme.

4.1 PICOLO, the turbulence emulator bench

PICOLO is a turbulence emulator bench at ONERA. It models a whole turbulent channel, from emission sources that represent laser-beam-emitting satellites in space, then phase screens that are statistically representative of the emulated turbulent channel, to an aperture materializing a 40 cm diameter OGS on Earth. Different on-sky turbulence scenarios can be reproduced on PICOLO using turbulence compression.⁶ The phase screens can be moved along the optical path to be placed at corresponding atmospheric layers' altitudes, and can be rotated to give them a translation speed.

The reciprocity principle introduced in Section 2.6 is also applied on the experimental bench. Therefore, instead of using one downlink beam to precompensate another uplink beam, we use two downlink sources presented on Figure 5 in the red and purple boxes, that are angularly separated on the bench by an angle that corresponds to the PAA on-sky. We also separate the two sources in wavelength, with the on-axis source at $\lambda_{on} = 1550$ nm and the off-axis source at $\lambda_{off} = 1600$ nm. The two collimated beams go through several phase screens (two in green in the figure), that possess different perturbation statistics,¹⁷ and are collected in a telescope-like aperture highlighted in gold. The screens can be placed and rotated accordingly to emulate atmospheric layers altitudes and wind speeds.

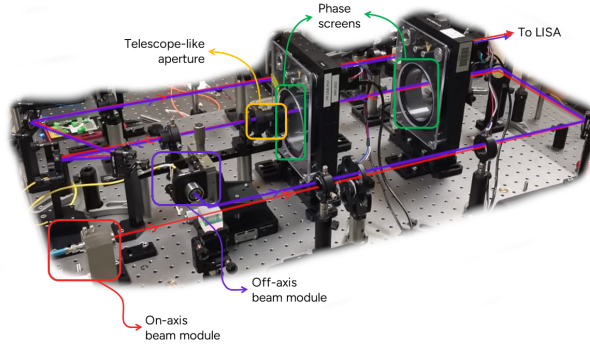


Figure 5. Picture of the PICOLO turbulence emulator bench, with annotations.

4.2 LISA, the AO system

The turbulent beams from the on-axis and off-axis modules of PICOLO are then coupled into LISA, presented in Figure 6. It is a typical AO system with a 97-actuators DM, an 8×8 sub-apertures SHWFS with 52 valid ones, and scientific paths to experimentally acquire coupling efficiency data. A filter centered around λ_{on} is put in front of the SHWFS so that only downlink information can be seen by the AO system. The DM can then compensate either the on- or off-axis beam, depending on the control solution. The on-axis injection modules are in the green boxes, and aligned with the downlink beam; the off-axis injection modules are in the blue boxes and are aligned with the off-axis ('uplink') beam. Each injection path has been calibrated to account for differential aberrations, and normalize properly each CE.

4.3 Experimental validation

We want to experimentally demonstrate the gain in performance brought by the $LQG_{\phi\chi}$ control method. However, as mentioned in Section 3, the three-layered turbulence profile presents strong scintillation. There is yet no scintillation management method available on LISA, meaning that it is potentially sensitive to this effect. Therefore, we choose to modify the profile used for the experimental validation to a two-layers profile, corresponding to an on-sky layer at ground level with $C_n^2 \cdot dz = 2.563 \times 10^{-11} \text{ m}^{-1/3}$ moving at 5 m/s, and another layer at

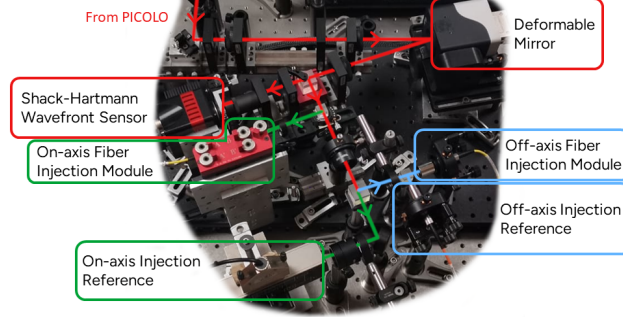


Figure 6. Picture of the LISA AO system bench, with annotations.

10 km altitude with $C_n^2 \cdot dz = 1.015 \times 10^{-12} \text{ m}^{-1/3}$ moving at 10 m/s. This profile exhibits a Fried parameter $r_0 = 4.36 \text{ cm}$, an anisoplanatic angle $\theta_0 = 9.74 \text{ } \mu\text{rad}$, and a log-amplitude Rytov variance $\sigma_{\chi_R}^2 = 0.064$, much more acceptable for first experimental tests. This two-layered profile is reproduced on PICOLO, and we acquire experimental coupling efficiency data with the different control laws.

We present experimental CE CDF plots of the four control laws on Figure 7. Visually, we can easily confirm that the use of $LQG_{\phi\chi}$ allows for an improvement in the CE on the uplink path: fading at 10^{-3} probability are decreased by a factor of 8.4 dB compared to classic integrator control; and the average CE value increases from -3.24 dB with integrator control to -2.55 dB with $LQG_{\phi\chi}$ control. Moreover, this plots allows for the comparison of results between LQG_{ϕ} control (in green), using only downlink phase information for the computation of the uplink precompensation, and $LQG_{\phi\chi}$ control. It is also evident based on the plot that adding the log-amplitude information brings significant improvement to the estimation quality and the final performance: it is this aspect that is capital to the performance gain.

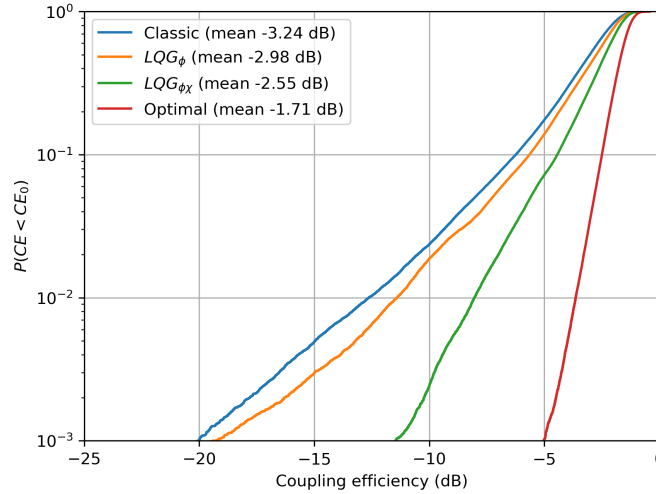


Figure 7. Experimental CDF plots, for classic and optimal integrator control, LQG_{ϕ} and $LQG_{\phi\chi}$ predictive control.

5. CONCLUSION & PERSPECTIVES

In this paper, we propose, to the best of our knowledge, the first control solution to perform optimized uplink precompensation based on downlink measurements, validated both numerically and experimentally. It embeds optimal estimation of uplink phase at PAA, based on both phase and log-amplitude fluctuations on the downlink path, and rely on an LQG control solution. We addressed the theoretical bases of incorporating turbulent phase and log-amplitude evolution models into $LQG_{\phi\chi}$ as well as the management of intensity and log-amplitude information. The first end-to-end dynamical simulations present the interest of this control method on uplink

signal precompensation in terms of average CE but most importantly of signal fadings reduction. Experimental results on PICOLO+LISA, the first of their kind using slope and intensity measurements, are very encouraging as they prove that this enhanced control solution can be ported to experimental setups with great results.

The next step is to run numerical validation on the two-layered profile to compare the results. Then, we intend to study the behavior of the SHWFS to increasing scintillation, allowing us to quantify if tests with profiles presenting stronger scintillation can be done on LISA. If so, more experimental results can be acquired and the performance robustness to scintillation can be assessed. Comparisons between numerical results with the end-to-end simulation tool, and experimental results, on similar turbulence profiles, would also prove useful for a deeper understanding of the conditions in which $LQG_{\phi\chi}$ presents an interest.

Moreover, with this successful experimental validation, we now want to go towards on-sky validation on FEELINGS,² an OGS located near Toulouse, France. This OGS supports GEO-feeder links as well as LQG control. Several points remain to be addressed for this extension to FEELINGS, such as the creation of numerical AO models adapted to FEELINGS' AO elements, or end-to-end simulations on various profiles encountered on-sky. Another practical aspect when going on-sky is the turbulence profile live estimation. That can be done with profilometry techniques;¹⁸ using profilometry is necessary to compute turbulence-adjusted covariance matrices for the different evolution models. The update frequency required for the LQG matrices (due to evolving turbulence conditions) would have to be assessed to make sure $LQG_{\phi\chi}$ remains optimal during the whole exchange duration.

ACKNOWLEDGMENTS

We acknowledge support from project PEPR Origins, reference ANR-22-EXOR-0017, supported by the France 2030 plan managed by Agence Nationale de la Recherche.

REFERENCES

- [1] Artaud, G., Eymar, L., Forriere, H., Coret, L., Baranger, H., Benammar, B., Lacoste, F., Battaglini, D., Geneslay, P., O’Kane, F., Bondoux, E., Drouilly, R., Garcia, L., Jans, B., Lanz, T., Samain, E., Vincenti, J., Schimpf, A., Poulenard, S., and Richard, J.-C., “Final architecture of FrOGS, validation, and first results of GEO optical links with TELEO,” in *[International Conference on Space Optics — ICSO 2024]*, Bernard, F., Karafolas, N., Kubik, P., and Minoglou, K., eds., **13699**, 136993D, International Society for Optics and Photonics, SPIE (2025).
- [2] Védrenne, N., Montmerle-Bonnefois, A., Petit, C., Chalali, E., Lai-Tim, Y., Krafft, L., Henrion, J., Houy, J., Gustave, F., Caillault, K., Bedoya, A., Barbon-Dubosc, D., Vincent-Randonnier, A., Quatresooz, F., Kermarrec, G., Richard, J.-C., Poulenard, S., Coret, L., Chouteau, J.-F., and Anfray, T., “First results of ground to GEO optical links channel monitoring with FEELINGS,” in *[International Conference on Space Optics — ICSO 2024]*, Bernard, F., Karafolas, N., Kubik, P., and Minoglou, K., eds., **13699**, 136990P, International Society for Optics and Photonics, SPIE (2025).
- [3] Lognoné, P., Conan, J.-M., Rekaya, G., and Védrenne, N., “Phase estimation at the point-ahead angle for AO pre-compensated ground to GEO satellite telecoms,” *Opt. Express* **31**, 3441–3458 (Jan 2023).
- [4] Lognoné, P., *Optimization of High Data Rate Ground to Satellite Links Pre-compensated by Adaptive Optics*, theses, Institut Polytechnique de Paris (Dec. 2023).
- [5] Lognoné, P., Wilson, R., and Osborn, J., “The first on-sky demonstration of intensity-based angular phase estimation,” *Opt. Express* **34**, 26309–26329 (Jul 2026).
- [6] Robles, P., Petit, C., Velluet, M.-T., Leuch, L. L., Montmerle-Bonnefois, A., Paillier, L., Conan, J.-M., Cassaing, F., Montri, J., Neichel, B., and Védrenne, N., “Emulating and characterizing strong turbulence conditions for space-to-ground optical links: the PICOLO bench,” *Journal of Astronomical Telescopes, Instruments, and Systems* **9**(4), 049002 (2023).
- [7] Lim, C. B., Petit, C., Saab, K., Fleury, B., Mendez, F., Montri, J., Védrenne, N., and Michau, V., “Single-mode fiber coupling for satellite-to-ground telecommunication links corrected by adaptive optics,” in *[SF2A-2018: Proceedings of the Annual meeting of the French Society of Astronomy and Astrophysics]*, Di Matteo, P., Billebaud, F., Herpin, F., Lagarde, N., Marquette, J.-B., Robin, A., and Venot, O., eds., Di (Dec. 2018).

- [8] Lognoné, P., Robert, K., Lucas, Y., Montmerle-Bonnefois, A., Védrenne, N., Obsorn, J., and Petit, C., “Closing the Loop with Intensity-Based Estimation for Pre-Compensated Optical Ground-to-Satellite Links : Simulation and First Experimental Results,” this conference.
- [9] Robert, K., Petit, C., Maulion, G., Robles, P., and Fusco, T., “Experimental performance evaluation of VAR LQG predictive control in adaptive optics for LEO satellites,” *Opt. Express* **34**, 28341–28364 (Jul 2026).
- [10] Correia, C., Jackson, K., Vérant, J.-P., Andersen, D., Lardière, O., and Bradley, C., “Static and predictive tomographic reconstruction for wide-field multi-object adaptive optics systems,” *J. Opt. Soc. Am. A* **31**, 101–113 (Jan 2014).
- [11] Mahé, F., Michau, V., Rousset, G., and Conan, J.-M., “Scintillation effects on wavefront sensing in the Rytov regime,” in [*Propagation and Imaging through the Atmosphere IV*], Roggemann, M. C., ed., **4125**, 77 – 86, International Society for Optics and Photonics, SPIE (2000).
- [12] Chassat, F., *Propagation optique à travers la turbulence atmosphérique : étude modale de l’anisoplanétisme et application à l’optique adaptative*, PhD thesis, Université Paris-XI (1992).
- [13] Tatarski, V. I., [*Wave Propagation in a Turbulent Medium*], Courier Dover Publications (1961).
- [14] Petit, C., *Etude de la commande optimale en OA et OAMC, validation numérique et expérimentale*, PhD thesis, Université Paris-XIII (2006). Thèse de doctorat dirigée par Viaris de Lesegno, Patrick.
- [15] Shapiro, J. H. and Puryear, A. L., “Reciprocity-enhanced optical communication through atmospheric turbulence — Part I: Reciprocity proofs and far-field power transfer optimization,” *Journal of Optical Communications and Networking* **4**(12), 947–954 (2012).
- [16] Védrenne, N., Velluet, M. T., Séchaud, M., Conan, J.-M., and Toyoshima, M., “Turbulence effects on bi-directional ground-to-satellite laser communication systems,” (2012).
- [17] Robles, P., Petit, C., Velluet, M.-T., Leuch, L. L., Montmerle-Bonnefois, A., Paillier, L., Conan, J.-M., Cassaing, F., Montri, J., Neichel, B., and Védrenne, N., “Emulating and characterizing strong turbulence conditions for space-to-ground optical links: the PICOLO bench,” *Journal of Astronomical Telescopes, Instruments, and Systems* **9**(4), 049002 (2023).
- [18] Quatresooz, F., Robert, K., Petit, C., Bonnefois, A., Henrion, J., Kermarrec, G., Alaluf, D., and Védrenne, N., “Line-of-sight optical turbulence and wind profile monitoring during free-space optical communication links,” *Opt. Express* **34**, 15074–15099 (Apr 2026).