

Spatio-Temporal Coherence of Atmospheric Turbulence: study towards optimal OFL pre-compensation control

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ABSTRACT

In adaptive optics systems applied to Space Situational Awareness (SSA) and to Satellite Optical Communication (SatCom) from the ground there are errors induced by anisoplanatism, when the downlink reference source and the optical feeder (up)link are angularly separated.

For example the Optical Feeder Link (OFL) from the ground has to be pointed ahead of the Satellite apparent position. If the satellite downlink signal is used to measure the atmospheric turbulence and pre-compensate the uplink OFL, the correction is affected by anisoplanatism, in most cases severely for harsh seeing conditions, low horizon elevation operation, or daytime turbulence. This substantially affects the link budget, the power losses at the OFL receiving satellite, and the data transmission rates.

The solution of using a Laser Guide Star to measure the turbulence in the direction of the OFL propagation is effective but only 13% of the wavefront errors can be corrected if the tip and tilt modes are not corrected.

So we launched a study with extensive numerical simulations, to analyse for different turbulence conditions and horizon elevations, the spatio-temporal coherence of wavefront spatial modes, to assess the effectiveness of dual-reference adaptive optics, in which the signal from the LGS and the downlink signal from the satellite can be optimally combined to improve the pre-compensation and even correct sufficiently the tip-tilt derived from the LGS.

In other words, we are studying the behaviour of the wavefront spatial modes anisoplanatism at different temporal frequencies ranges, to envisage optimal sensing schemes and also pre-compensation of tip-tilt in the OFL based on dual-reference (LGS and Satellite) wavefront sensing and control.

We present the concepts behind this investigation, its potential applications driving the research, the foreseen simulated improvements for SatCom, and the spatio-temporal studies work, which is currently in progress.

Keywords: Adaptive Optics, Propagation Delay, Atmospheric Optical Turbulence, Isoplanatic Angle, Optical Wavefront Modal Power Spectra

1. INTRODUCTION

Free-Space Optical Communications (FSOC) between optical ground stations and spacecraft offer the potential for very high data rates, high link security, and reduced spectrum-allocation constraints compared with conventional radio-frequency links. However, propagation through the Earth's turbulent atmosphere produces spatial and temporal fluctuations of the optical wavefront, resulting in beam wander, beam spreading, wavefront distortion, and scintillation. On the uplink, these effects can significantly reduce the optical power delivered to the spacecraft and increase the probability of deep fades. Adaptive Optics (AO) pre-compensation of the transmitted wavefront is therefore an important technique for improving the performance and availability of ground-to-space optical links.

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A fundamental limitation of uplink AO correction arises from the Point-Ahead Angle (PAA). Due to the relative motion of the spacecraft and the finite propagation time, the uplink beam must be transmitted ahead of the apparent position of the spacecraft. The received downlink signal and the transmitted uplink beam consequently propagate through different, although partially correlated, volumes of atmospheric turbulence. The resulting anisoplanatism limits the accuracy with which the wavefront measured from the spacecraft downlink can be used to predict and pre-compensate the turbulence encountered by the uplink. This problem becomes increasingly relevant for high-altitude and geostationary links, where the angular separation between the two propagation paths can become comparable with, or larger than, the atmospheric isoplanatic angle. Numerical studies have investigated AO-assisted ground-to-space links and the use of Laser Guide Stars (LGS) - also called Laser Beacons (LB) - positioned close to the uplink direction as a means of overcoming this limitation.^{1,2}

The use of a sodium LGS therefore provides an attractive solution for sensing the atmospheric turbulence close to the direction of the point-ahead uplink. This concept has progressed from numerical studies towards dedicated experimental facilities. The CaNaPy Laser Guide Star Adaptive Optics facility was developed as an experimental platform for LGS-AO technologies applicable to satellite communications and Optical Feeder Links (OFL).³ Installed at the ESA 1-m Optical Ground Station (OGS) at the Observatorio del Teide, Tenerife, CaNaPy was conceived to investigate technologies including high-power 589 nm sodium LGS operation, uplink wavefront pre-compensation, advanced wavefront sensing, and techniques related to the recovery of atmospheric tip and tilt from an LGS.

Building upon the CaNaPy LB-AO concept and infrastructure, the ESA ALASCA (Advanced Laser Guide Star Adaptive Optics for Satellite Communication Assessments) programme represents a further step towards an operational LGS-assisted Optical Feeder Link. ALASCA is developing a TRL 6 LGS-AO OFL demonstrator at the ESA OGS, with the objective of demonstrating reliable ground-to-satellite optical communications with GEO and LEO spacecrafts, and ultimately supporting 24/7 operation integrated systems development.⁴ The system combines sodium-LGS sensing in the uplink direction with wavefront information derived from the satellite infrared downlink, and includes uplink pre-compensation, atmospheric turbulence characterisation and operation under both night-time and challenging daytime conditions. ALASCA therefore provides an important experimental framework in which the relative information contained in the LGS and satellite wavefront measurements can be investigated and exploited. Related activities, including GEOSTAR, are also investigating simultaneous LGS and satellite wavefront sensing for mitigation of PAA-induced anisoplanatism.

A fundamental limitation nevertheless remains when an LGS is used as the AO reference. An LGS cannot directly provide an absolute measurement of atmospheric tip and tilt, i.e. the turbulence-induced image motion, because the laser beacon propagates through the atmosphere on both the uplink and the downlink. The resulting reciprocity prevents a conventional LGS wavefront sensor from distinguishing the absolute atmospheric tilt affecting the outgoing beam. This limitation is particularly important because approximately 87% of the atmospheric phase variance is contained in the tip and tilt modes, hence the impact of LB-AO will be significant, in harsh atmospheric conditions, only if the uplink tip-tilt can be sensed from the laser beacon, i.e. the laser guide star (LGS).⁵

This is the holy-grail of LB-AO technologies for space applications. Several methods have been proposed to estimate or pre-compensate the uplink tip and tilt. For a monostatic LGS launch configuration, the *Propagation Delay* method⁶ estimates the uplink tip and tilt from their temporal derivatives. Detector and measurement noise, however, can produce low-frequency drift in the reconstructed tip/tilt signal,⁷ requiring an independent low-frequency reference on timescales of approximately 0.5–1 s. The spacecraft downlink signal can provide such a reference. Its line of sight, however, is angularly separated from the LGS/uplink direction by approximately the PAA, and the accuracy with which the two measurements can be combined is therefore determined by their atmospheric spatio-temporal correlation.

This observation motivates a description of anisoplanatism that goes beyond the conventional, purely spatial isoplanatic angle. Atmospheric turbulence is transported by altitude-dependent wind fields and simultaneously it evolves with time. Consequently, two angularly separated lines of sight that are only partially correlated at a given instant may exhibit a stronger correlation at a finite time delay when the same turbulent structures are transported from one line of sight towards the other. Conversely, intrinsic evolution of the turbulence progressively destroys this correlation. The information shared between the satellite and LGS measurements

is therefore expected to depend jointly on angular separation, atmospheric-layer altitude, wind velocity and direction, temporal frequency, and spatial or modal order.

Under Taylor’s Frozen Turbulence Hypothesis (TFTH), turbulent structures are assumed to evolve slowly compared with their translation by the wind. This approximation provides a direct relationship between spatial and temporal statistics and is widely used in AO simulations, where atmospheric layers are represented by translating phase screens. More general numerical descriptions have introduced autoregressive phase-screen models containing both frozen-flow and stochastic, or “boiling”, components.⁸ Such models have demonstrated that even relatively small departures from pure frozen flow can modify the temporal statistics relevant to closed-loop AO performance. Recent experimental investigations of optical turbulence have further examined the validity of TFTH and reported evidence for additional temporal decorrelation mechanisms, including behaviour consistent with random-sweeping models.⁹ These studies indicate that the temporal evolution of optical turbulence cannot, in general, be accurately described solely by rigid translation of frozen turbulent layers.

For ground-to-space optical communications, this suggests that the conventional isoplanatic angle alone is not sufficient to quantify the information that can be transferred between the satellite and LGS lines of sight. Instead, the relevant quantity is a joint spatio-temporal covariance, or equivalently a frequency-dependent angular coherence, of the atmospheric wavefront. Furthermore, this coherence should be considered on a modal basis: tip/tilt and other low-order Zernike modes are dominated by different spatial scales and may exhibit different angular and temporal correlation properties from higher-order aberrations. A characterisation of the modal cross-correlation or cross-spectral density as a function of angular separation and temporal frequency could therefore provide a more appropriate metric for the design of multi-reference AO control systems.

Knowledge of these spatio-temporal statistics may enable predictive AO controllers to optimally combine measurements from an LGS located close to the uplink direction with measurements obtained from the spacecraft itself. In such an architecture, the LGS can provide high-bandwidth information on the turbulence encountered close to the uplink path, while the spacecraft signal provides an absolute reference for modes that cannot be directly recovered from the LGS and can constrain slowly varying estimation errors. The relative weighting of the two measurements need not be fixed, but may instead depend on temporal frequency, angular separation, atmospheric conditions, and wavefront mode.

We hypothesise that considerably improved uplink pre-compensation can be achieved using predictive control methods that explicitly exploit the measured spatio-temporal covariance of the atmospheric wavefront. Artificial Intelligence (AI) and Machine Learning (ML) techniques represent one possible approach for learning or adapting such relationships from atmospheric telemetry, particularly where the covariance evolves with atmospheric conditions. More generally, the measured statistics can provide the basis for model-based predictive estimators and optimal control techniques. In either case, the objective is to combine LGS and spacecraft wavefront information according to their measured spatio-temporal coherence rather than treating them as independent reference signals.

We present here the preliminary results of STACS—*Spatio-Temporal Atmospheric Coherence Studies*—a programme aimed at numerically characterising the modal spatio-temporal statistics of atmospheric optical turbulence for ground-to-space FSOC, anchoring and validating the simulations to real measurements data. The project addresses two primary questions:

1. How does the angular correlation of atmospheric wavefront modes vary as a function of temporal frequency when comparing a sodium laser guide star with a nearby reference source, such as a satellite or natural guide star, and can this behaviour be represented by an effective frequency-dependent isoplanatic angle?
2. How can predictive, potentially AI-assisted, optimisation of AO control loops exploit this spatio-temporal atmospheric coherence to optimally combine wavefront information from multiple reference sources?

The STACS approach combines atmospheric measurements with numerical modelling of the propagation through altitude-dependent turbulent layers. The objective is to determine the modal spatial and temporal covariance between separated lines of sight and to investigate its dependence on atmospheric turbulence strengths, types and wind profiles. The results are intended both to the physical description of frequency-dependent angular

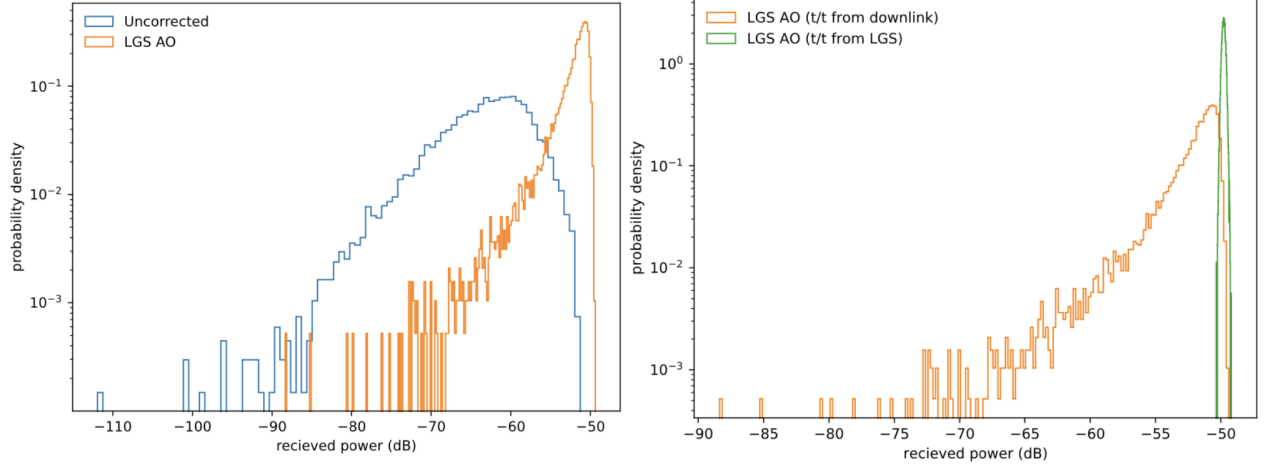


Figure 1. Distribution of received power (or Power Loss compared to the ground OFL laser Emission) at GEO for a 1 m launch aperture. Left: uncorrected propagation (blue) and LGS-based AO (LB-AO), assuming perfect high-order correction but retaining the residual optical feeder-link tilt produced by atmospheric anisokinetism over a 4'' point-ahead angle (orange). Right: LB-AO (orange) and LB-AO including correction of the LGS tip/tilt (green).

coherence in atmospheric optical propagation and to provide the statistical basis required for the development and validation of predictive multi-reference AO control strategies for current or future optical ground stations and Optical Feeder Links.

2. THEORY

Ground-to-space Free-Space Optical Communications (FSOC) are significantly affected by the Point-Ahead Angle (PAA) required to compensate for spacecraft motion during the propagation time. A sodium Laser Guide Star (LGS) located close to the uplink direction can provide a measurement of the high-order atmospheric aberrations encountered by the transmitted beam. However, absolute atmospheric tip and tilt cannot be directly recovered from the LGS because the beacon propagates through the turbulent atmosphere on both its uplink and downlink paths.

The importance of recovering this information is illustrated in Fig. 1. For the simulated GEO link with Alpha-Sat, correcting the residual tilt in addition to the high-order LGS-AO correction produces a substantial increase and a much smaller variability in the received OFL optical power. Accurate estimation and prediction of the tilt affecting the uplink is therefore a key requirement for high-performance LGS-assisted optical feeder links.

The atmospheric wavefront phase over the telescope aperture can be represented by a Zernike expansion

$$\phi(\mathbf{r}, t) = \sum_j a_j(t) Z_j(\mathbf{r}), \quad (1)$$

where $a_j(t)$ is the time-dependent coefficient of the j -th Zernike mode. Classical atmospheric turbulence theory provides the spatial covariance of these coefficients for one or more lines of sight.^{10,11} For two angularly separated sources, however, the wavefronts are both spatially separated and temporally evolving. The appropriate statistical quantity is therefore a spatio-temporal modal cross-covariance

$$C_{jk}(\boldsymbol{\theta}, \tau) = \left\langle a_j^{(1)}(t) a_k^{(2)}(t + \tau) \right\rangle, \quad (2)$$

where $\boldsymbol{\theta}$ is the angular separation vector between the two lines of sight and τ is their relative time delay. The corresponding temporal cross-power spectral density is obtained from the Fourier transform

$$S_{jk}(\boldsymbol{\theta}, f) = \int_{-\infty}^{+\infty} C_{jk}(\boldsymbol{\theta}, \tau) e^{-i2\pi f\tau} d\tau. \quad (3)$$

Analytical expressions for the spatio-temporal covariance and temporal cross-power spectral density of turbulent phase and Zernike coefficients have been developed for two distinct optical beams.¹² This formulation is particularly relevant here because it demonstrates that anisoplanatism is intrinsically frequency dependent when the temporal evolution of the atmosphere and the filtering of an AO control loop are considered. Earlier work also derived spatial-temporal Zernike correlation functions for the stochastic modelling and control of AO systems.⁷

Under Taylor's Frozen Flow Hypothesis (TFH), each turbulent layer is assumed to retain its spatial structure while being transported by the layer wind velocity $\mathbf{v}_l$. For a layer at altitude h_l , two lines of sight separated by $\boldsymbol{\theta}$ sample points separated approximately by

$$\boldsymbol{\rho}_l(\tau) = h_l \boldsymbol{\theta} - \mathbf{v}_l \tau. \quad (4)$$

The spatial and temporal components of anisoplanatism are therefore coupled. Maximum correlation is expected when wind advection partially compensates the geometrical separation of the two footprints, i.e. when $\mathbf{v}_l \tau$ approaches $h_l \boldsymbol{\theta}$. Conversely, when the separation is perpendicular to the wind direction, temporal translation does not bring the same turbulent structures successively through the two lines of sight. This immediately predicts the dependence of the measured coherence on wind speed, wind direction and relative source orientation investigated in this work.

For the STACS application we are particularly interested in the same Zernike mode measured along the LGS and satellite lines of sight. A useful normalized modal spatio-temporal coherence can therefore be defined as

$$\gamma_j(\boldsymbol{\theta}, f) = \frac{|S_{jj}^{(12)}(\boldsymbol{\theta}, f)|}{\sqrt{S_{jj}^{(11)}(f)S_{jj}^{(22)}(f)}}, \quad 0 \leq \gamma_j \leq 1, \quad (5)$$

where $S_{jj}^{(12)}$ is the cross-spectrum between the two lines of sight and $S_{jj}^{(11)}$ and $S_{jj}^{(22)}$ are their corresponding auto-spectra. Equation (5) provides a direct measure of how much information about mode j observed in one direction remains available in the second direction at a given temporal frequency. $\gamma_j = 1$ corresponds to full normalized coherence between the two lines of sight.

This suggests a generalisation of the conventional isoplanatic or isokinetic description for our intended application to OFL and ground to spacecraft LB-AO. Rather than assigning a single angular coherence scale, an effective modal coherence angle $\theta_{c,j}(f)$ may be defined from a chosen threshold of $\gamma_j(\boldsymbol{\theta}, f)$. Such a quantity provides a compact description of the angular separation over which useful information about a given atmospheric mode remains available as a function of temporal frequency. In particular, the low-frequency coherence of tip and tilt is directly relevant to combining the LGS estimate with the absolute tilt information available from the satellite signal.

The expected modal spatio-temporal statistics consequently depend on:

- the $C_n^2(h)$ turbulence profile and layer altitudes;
- wind speed and direction as a function of altitude;
- angular separation and orientation of the two lines of sight;
- turbulence outer scale L_0 and Fried parameter r_0 ;
- telescope aperture and the Zernike mode considered; and
- measurement frame rate and exposure time.

One strategic objective of the STACS simulations is to quantify these dependencies for the LGS-satellite geometry and, in particular, to determine the frequency range over which the LGS measurement retains predictive information about the atmospheric modes affecting the point-ahead uplink.

These statistics can subsequently provide the covariance analytical model or training information required by predictive and machine-learning-based AO controllers.

Parameter	Value
L_0	20 m
Pupil Diameter	0.5 m
Orientations	Parallel, Perpendicular and 45°
Frame rate	100 Hz
Exposure Time	instantaneous
Separations	0 – 30"
Zenith Angle	0°
Wavelength	500 nm
Propagation method	Physical
Noise Assumptions	Noiseless

Table 1. A table of the simulation parameters used.

3. SIMULATION

A Monte Carlo simulation tool is under development to investigate the modal spatio-temporal statistics of optical turbulence in the atmosphere. Full details of this simulation workflow is given in 2. The SPECULA¹³ and AoTools¹⁴ python packages are used to produce atmospheric phase screens and to generate Zernike modes. An advantage of using the SPECULA package is its capability to work with GPUs, enabling significantly faster computing time.

SCIDAR data from La Palma (Canary Islands) Observatorio del Roque de los Muchachos has been used to model the turbulent atmosphere.¹⁵ Fifteen typical turbulence profiles have been generated using a hierarchical clustering method.¹⁶ These fifteen C_n^2 profiles are given in Fig. 3. Distinct wind profiles were also generated from this method. The profiles were binned into 10 layers using the optimal grouping method.¹⁷

The parameters given in Table. 1 have been used in the simulation. The two lines of sight have been calculated as point sources and Taylor’s Frozen Flow Hypothesis has been assumed. Hence, these results provide an upper limit on the expected coherence distance that could be measured on-sky for the parameters given in Table. 1.

Three relative line of sight orientations were used to investigate how the spatio-temporal statistics vary with relative position. The orientations are shown in Fig. 4. For each orientation, a range of line of sight angular separations were used to investigate the spatial statistics.

4. PRELIMINARY RESULTS

In this section, we present the preliminary results investigating the spatio-temporal statistics for fifteen turbulence profiles given in Fig. 3. Previous results for a single turbulent layer have been presented.¹⁸ For multiple layers, a revised preliminary metric for the coherence distance has been defined using the correlation coefficient between modal time series measured along the two lines of sight with a threshold of 0.9 defined as where the coherence distance occurs. This revision is needed since for a multi-layer turbulence profile, the correlation coefficient for the ground layer, typically the strongest layer, is always one, thus keeping the coherence relatively high even at large angular separations. Hence, a higher threshold is required for the coherence distance measurement.

The mean results for the fifteen profiles are given in Fig. 5. In all orientations, filtering the modal time series to retain temporal frequencies below 1 Hz produces a clear increase in coherence distance compared with the unfiltered data. This increase is most pronounced for the lowest-order modes, particularly tip and tilt, for which the coherence distance can increase by approximately a factor of two. In contrast, the coherence distances measured from the >1 Hz component are generally lower, indicating that the higher-frequency temporal variations decorrelate over shorter angular separations.

The effect of temporal filtering becomes less pronounced for the higher-order modes, for which the coherence distances are generally smaller and the differences between the filtering cases are reduced. Nevertheless, the consistent increase observed for the <1 Hz component suggests that a significant fraction of the spatially coherent modal behaviour is contained within the low-frequency temporal variations. This has potentially important implications for AO control strategies, as optimising control loops to account for these longer-timescale correlated variations could increase the angular range over which useful correction can be achieved.

NUMERICAL SIMULATION FLOW-CHART

Ground-to-Space FSO Link through Atmospheric Turbulence

Multi-Phase-Screen + Physical Optics Propagation

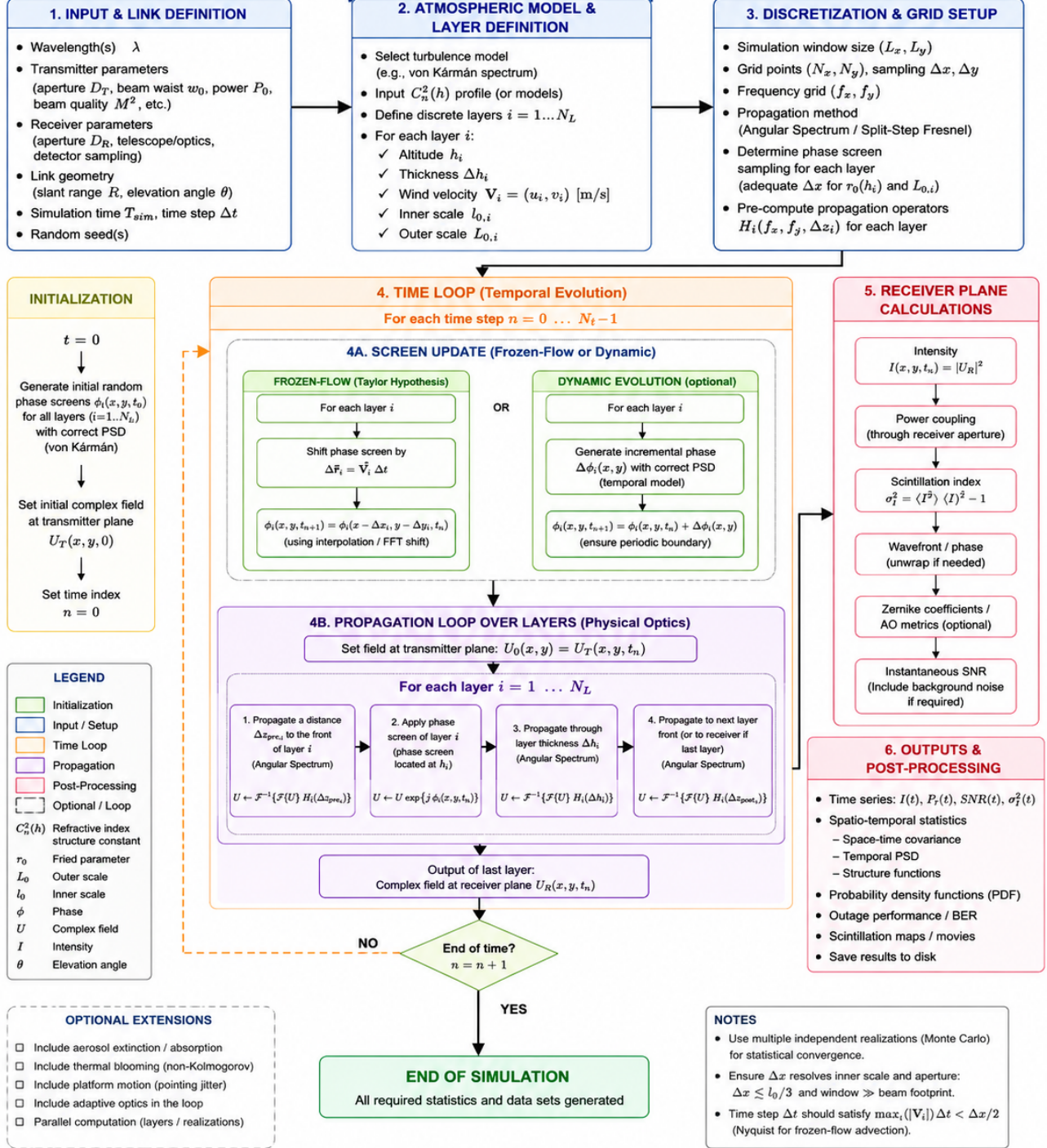


Figure 2. The designed simulation workflow.

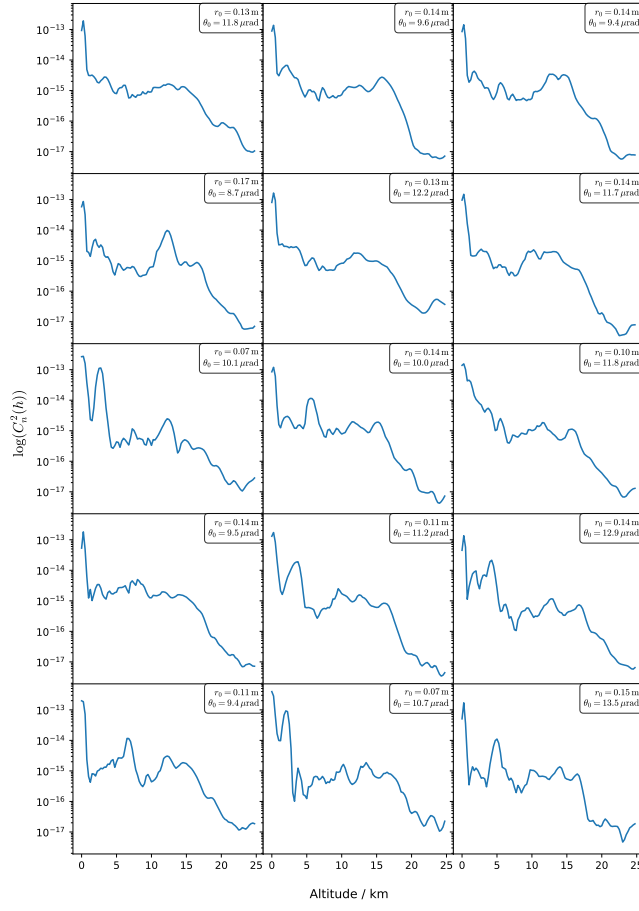


Figure 3. The fifteen typical La Palma profiles generated using a hierarchical clustering method¹⁶

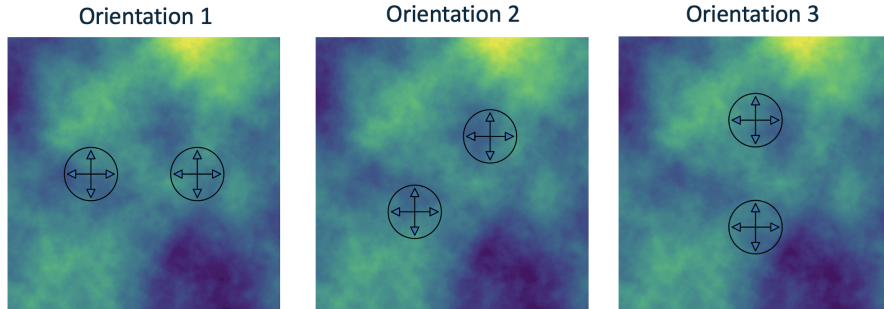


Figure 4. The three relative line of sight orientations used in the simulation.

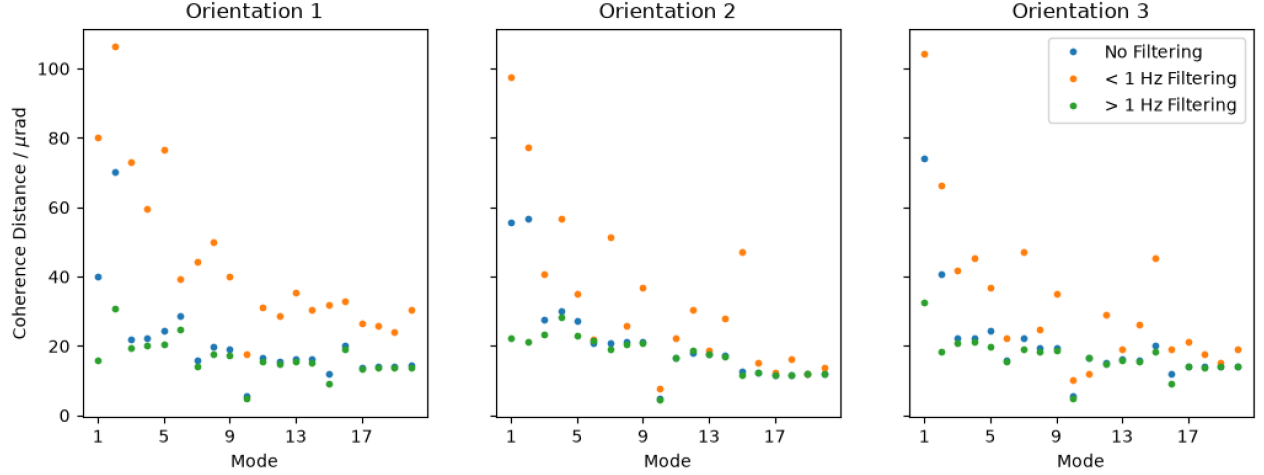


Figure 5. The mean coherence distance for the 15 profiles for the three different line of sight orientations for the first 20 modes. For each, the coherence distance without any temporal filtering and with a > 1 Hz filtering and < 1 Hz filtering are plotted.

A dependence on relative orientation is also apparent in all three panels of Fig. 5. Although the results are averaged over fifteen multi-layer turbulence profiles, the coherence distances remain systematically different between orientations. This is likely to arise from prevailing wind directions at the observing site, which introduce anisotropy into the spatio-temporal evolution of the turbulence. The persistence of this orientation dependence after averaging over multiple turbulence profiles demonstrates that site-specific atmospheric characteristics may be important when designing and optimising wide-field AO systems. In particular, knowledge of the local wind distribution and turbulence profile could potentially be used to inform AO system geometry and control strategies.

Figure 6 shows the effective modal coherence angle, $\theta_{c,j}(f)$ defined in eq. 5, for tip and tilt as a function of temporal frequency for the three orientations investigated. A threshold value of 0.9 has been used as the ground layer which dominates in strength results in very high coherence. In all cases, the coherence angle is largest at low temporal frequencies and decreases rapidly with increasing frequency, indicating that the angular range over which modal information remains coherent is substantially larger for slowly varying atmospheric aberrations. The coherence angle subsequently approaches a lower, approximately constant value at higher frequencies. The detailed behaviour differs between orientations, particularly at low frequencies, reflecting the directional dependence of the spatio-temporal atmospheric statistics. While tip and tilt exhibit similar overall trends, their low-frequency coherence angles can differ substantially, with the relative magnitude depending on orientation. This is consistent with the directional nature of frozen-flow turbulence, for which the temporal relationship between the two lines of sight depends on their separation relative to the wind direction.

5. DISCUSSION

We have performed simulations on fifteen 10 layer SCIDAR profiles measured in La Palma. The results confirm that the isoplanatic angle is larger for low-order modes at lower temporal frequencies. However, this work currently assumes frozen flow and noiseless conditions and hence gives an upper boundary on the typical spatio-temporal coherence that could be measured on-sky.

The model will be further refined and results will be compared with analytical theory where available, as well as to measure on-sky data using the ESA Optical Ground Station in Tenerife. With this data we will demonstrate the increase in the coherence between two lines of sight at lower temporal frequencies, and the improved performances with machine learning controls taking into account both, the beacon wavefront sensor data and the satellite downlink data.

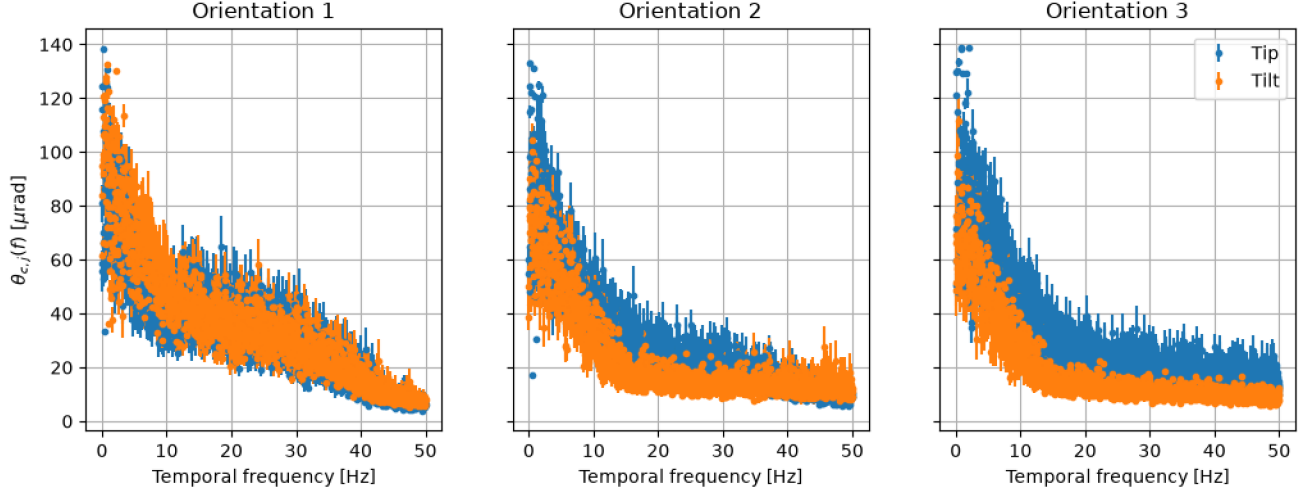


Figure 6. The spatio-temporal coherence defined in eq. 5 for the tip and tilt modes. A threshold value of 0.9 has been used to define the modal coherence angle.

The results presented in this conference paper are preliminary as the work is still in progress. Work to determine the optimal metrics to use for this investigation is underway. Future work will involve as well simulation of atmospheres with no TFFH and with high Rytov variance, giving high scintillation.

6. CONCLUSIONS

We have framed an R&D oriented to detailed numerical simulations of spatio-temporal coherences in realistic atmospheric conditions, including daytime and low-altitude laser propagations. We aim this strategic research to the applications of space optical communications, space situation awareness, space debris laser ranging.

Our initial results encourage our hypothesis to use in the future for LB-AO control different values of modal anisoplanatism, depending on the temporal frequency ranges used in sensing and control. Further R&D work is ongoing and will be reported. The aim is to be able to not only optimize the LB-AO performance in ground to space LB-AO applications, but also to optimally precompensate uplink laser beams, such as e.g. Optical Feeder Links, including the detection and correction of tip-tilt at the PAA using a laser beacon.

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REFERENCES

- [1] Martínez, N., Rodríguez-Ramos, L. F., and Sodnik, Z., "Toward the uplink correction: application of adaptive optics techniques on free-space optical communications through the atmosphere," *Optical Engineering* **57**(7), 076106 (2018).
- [2] Osborn, J., Townson, M. J., Farley, O. J. D., Reeves, A. P., and Mata Calvo, R., "Adaptive optics pre-compensated laser uplink to LEO and GEO," *Optics Express* **29**(4), 6113–6132 (2021).
- [3] Bonaccini Calia, D., Centrone, M., Pinna, E., Alaluf, D., Martinez, N., Osborn, J., Hackenberg, W., Townson, M., Faccini, M., Di Paola, A., Rodriguez Ramos, L. F., Speziali, R., Reyes, M., Janout, P., Jenkins, D., Wei, D. P., Fischer, T., Perdigues, J., and Clements, W. R. L., "Canapy: Satcomm lgs-ao experimental platform with laser uplink pre-compensation," in *[International Conference on Space Optics – ICSO 2020], Proceedings of SPIE* **11852**, 118521A (2021).
- [4] Biasi, R. et al., "ALASCA: the ESA laser guide star adaptive optics optical feeder link demonstrator," in *[International Conference on Space Optics – ICSO 2022], Proceedings of SPIE* (2023).

- [5] Pfrommer, T. and Hickson, P., “High-resolution mesospheric sodium observations for extremely large telescopes,” in [*Adaptive Optics Systems II*], Ellerbroek, B. L., Hart, M., Hubin, N., and Wizinowich, P. L., eds., **7736**, 773620, International Society for Optics and Photonics, SPIE (2010).
- [6] Ragazzoni, R., “Propagation Delay of a Laser Beacon as a Tool to Retrieve Absolute Tilt Measurements,” **465**, L73 (July 1996).
- [7] Niero, T., Ragazzoni, R., and Farinato, J., “Propagation delay as a tool to retrieve high-frequency absolute tilt,” in [*Atmospheric Propagation, Adaptive Systems, and Lidar Techniques for Remote Sensing II*], Devir, A. D., Kohle, A., Schreiber, U., and Werner, C., eds., **3494**, 168 – 174, International Society for Optics and Photonics, SPIE (1998).
- [8] Srinath, S., Poyneer, L. A., Rudy, A. R., and Ammons, S. M., “Computationally efficient autoregressive method for generating phase screens with frozen flow and turbulence in optical simulations,” *Optics Express* **23**(26), 33335–33349 (2015).
- [9] Pérez, D. G., Farfán-Bachilloglu, H., and Sepúlveda, M., “Spatiotemporal statistics of optical turbulence beyond Taylor’s frozen turbulence hypothesis,” *Journal of the Optical Society of America A* **41**(6), B135–B143 (2024).
- [10] Noll, R. J., “Zernike polynomials and atmospheric turbulence,” *Journal of the Optical Society of America* **66**(3), 207–211 (1976).
- [11] Hu, P. H., Stone, J., and Stanley, T., “Application of Zernike polynomials to atmospheric propagation problems,” *Journal of the Optical Society of America A* **6**(10), 1595–1608 (1989).
- [12] Plantet, C., Carlà, G., Agapito, G., and Busoni, L., “Spatiotemporal statistics of the turbulent piston-removed phase and Zernike coefficients for two distinct beams,” *Journal of the Optical Society of America A* **39**(1), 17–27 (2022).
- [13] Rossi, F., Puglisi, A., and Agapito, G., “Introducing a new generation adaptive optics simulation framework: from PASSATA to SPECULA,” *Journal of Astronomical Telescopes, Instruments, and Systems* **12**(1), 019001 (2026).
- [14] Townson, M. J., Farley, O. J. D., de Xivry, G. O., Osborn, J., and Reeves, A. P., “Aotools: a python package for adaptive optics modelling and analysis,” *Opt. Express* **27**, 31316–31329 (Oct 2019).
- [15] Shepherd, H. W., Osborn, J., Wilson, R. W., Butterley, T., Avila, R., Dhillon, V. S., and Morris, T. J., “Stereo-SCIDAR: optical turbulence profiling with high sensitivity using a modified SCIDAR instrument,” *Monthly Notices of the Royal Astronomical Society* **437**, 3568–3577 (11 2013).
- [16] Farley, O. J. D., Osborn, J., Morris, T., Sarazin, M., Butterley, T., Townson, M. J., Jia, P., and Wilson, R. W., “Representative optical turbulence profiles for eso paranal by hierarchical clustering,” *Monthly Notices of the Royal Astronomical Society* **481**, 4030–4037 (12 2018).
- [17] Saxenhuber, D., Auzinger, G., Louarn, M. L., and Helin, T., “Comparison of methods for the reduction of reconstructed layers in atmospheric tomography,” *Appl. Opt.* **56**, 2621–2629 (Apr 2017).
- [18] Hartley, K., Calia, D. B., Osborn, J., and Jenkins, D., “Wavefront modal spatio-temporal coherence in turbulent atmospheres,” in [*Adaptive Optics Systems X*], Jackson, K. J., Schmidt, D., and Vernet, E., eds., **14150**, 141507K, International Society for Optics and Photonics, SPIE (2026).