# Impact probability estimation with Partial Banana Mapping: search for virtual impactors

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## Abstract

In this work the Partial Banana Mapping (PBM) [1] method (a robust linear method for impact probability computation) has been improved. In the method we assume a Gaussian distribution of errors of orbital elements to be fulfilled on the interval from the asteroid's detection to the time of a possible collision. Thus it allows us to take into account the fact that the distribution of possible positions of the asteroid is a thin curvilinear area, stretched mostly along the nominal asteroid's orbit. The method consists of two stages. On the first stage we propagate the orbit of the asteroid with variational equations till the time of a possible collision (it is very close to the time when the distance between the Earth and asteroid's orbit is minimal). Then on the main line of the curved uncertainty area we find the virtual asteroid that is closest to the Earth. On the second stage we find the coordinates and velocities of that chosen virtual asteroid but at the epoch of observations. Then we propagate the orbit of this virtual asteroid (now it will be close to the Earth) and compute the probability. This approach takes into account all the gravitational perturbations and significantly extends the applicability of the Partial Banana Mapping method even in the cases of moderate gravitational perturbations.

## Introduction

The probability of a collision of an asteroid with the Earth arises because of the uncertainty of the position of the asteroid in space. The better we understand the behavior of the uncertainty region the more precise we can estimate the probability of a collision. In this work I am considering only linear methods of impact probability estimation. In these methods we assume that there is a linear relation between the errors of orbital parameters at the epoch of observations and at the epoch of a possible collision. The uncertainty region is determined by the covariance matrix, which is computed via integration of asteroid's equations of motion with variational equations. Thus, this kind of methods require propagation of only the nominal orbit.

The most well-known linear method is the target plane method, which was first successfully used by Paul Chodas [1] to predict a collision of comet Shoemaker-Levy 9 with Jupiter. Vavilov & Medvedev [2] introduced a concept of curvilinear coordinate system in order to take into account that the uncertainty region is curved and stretched mostly along the nominal orbit. Later Vavilov [3] extended this approach to the Partial Banana Mapping method. Here in this work I will improve the Partial Banana Mapping method, so that for a small cost of time it will produce much more reliable results.

## Target plane method

- Cartesian coordinates and velocities are linearly connected at different times, which implies a Gaussian distribution of coordinates' and velocities' errors at all times.
- The probability is assumed to happen when the asteroid comes close to the Earth.
- The uncertainty region is assumed to look like an ellipsoid.
- The uncertainty ellipsoid is projected onto the target plane.
- The probability is computed when the asteroid enters the Earth's sphere of effect as the two-dimensional integral

$$P = \frac{1}{2\pi|\det\mathbf{L}|^{\frac{1}{2}}}\int_{S_{R'_\oplus}} e^{-(\boldsymbol{u}^{\mathrm{T}}\mathbf{L}^{-1}\boldsymbol{u})/2}\mathrm{d}\boldsymbol{u}$$

where $S_{R'_\oplus}$ is the projection of the Earth (circle) with radius $R'_\oplus = R_\oplus\sqrt{1+\frac{v_s^2}{v_\infty^2}}$. $v_s$ is the escape velocity ($\approx 11.186$ km/s)

Disadvantages:

- Does not take into account the curvilinear nature of the uncertainty region.
- Requires the asteroid on the nominal orbit comes close to the Earth.
- Close approaches with major planets can ruin the linear assumption and distort impact probability values.

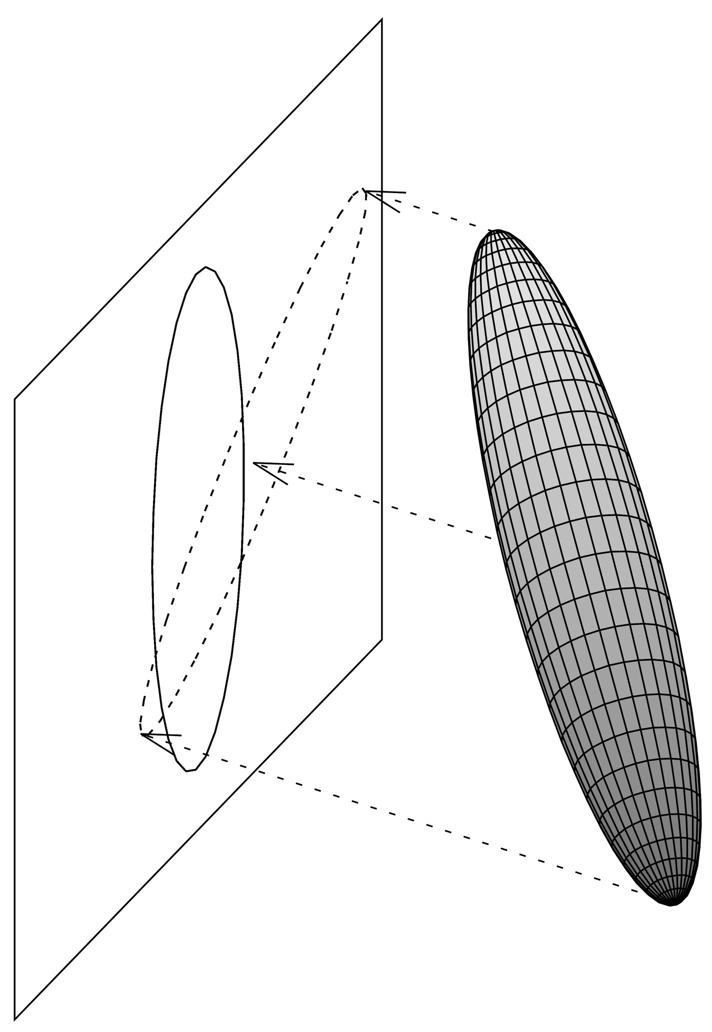

Figure 1. The schematic illustration of the target plane method.

## References

[1] P. W. Chodas. Estimating the Impact Probability of a Minor Planet with the Earth. In *BAAS*, volume 25, page 1236, Jun 1993.

[2] D. E. Vavilov and Yu. D. Medvedev. A fast method for estimation of the impact probability of near-Earth objects. *MNRAS*, 446(1):705–709, Jan 2015.

[3] Dmitrii E. Vavilov. The partial banana mapping: a robust linear method for impact probability estimation. *MNRAS*, 492(3):4546–4552, March 2020.

## Partial Banana Mapping method

- The uncertainty region is thin, curved and stretched mostly along the nominal asteroid's orbit.
- The collision happens when the Earth comes close to the nominal asteroid's orbit.
- The covariance matrix in orbital elements much better represent the actual shape of the uncertainty region (see Fig.1) in the two-body formalism (assume Gaussian distribution in orbital elements).
- Find the point on the main axis of curvilinear uncertainty region, which is closest to the Earth (point be in Fig. 2).
- Compute the probability by a target plane method but for point B.

Advantages:

- The uncertainty region at possible collision time is well represented

Disadvantages:

- Close approaches with major planets can ruin the linear assumption and distort impact probability values (general problem for linear methods)

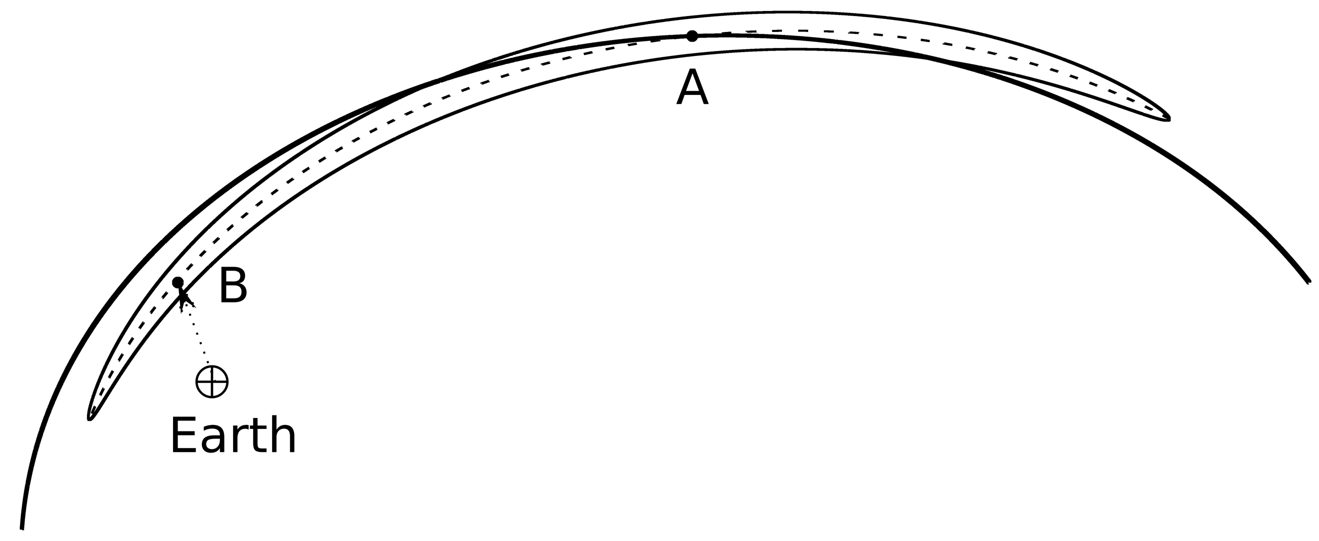


Figure 2. The scheme of the banana shaped uncertainty region of the asteroid. Point A is the nominal position of the asteroid, point B — the virtual asteroid of the main axis of the uncertainty region, which is closest to the Earth. The bold line is the nominal asteroid's orbit.

## PBM: search for virtual impactors

In order to suppress the drawback of close approaches the following we can find in the original uncertainty region the orbital elements of the virtual asteroid that lead to point B:

$$\mathrm{V}^{-1}[E^{min} - E] = [x_0^{min} - x_0] \tag{1}$$

where $x_0$ — state vector at epoch of observations, $x$ — state vector at time of possible collision, $E$ — Keplerian elements at time of possible collision, $E^{min}$ — Keplerian elements of point B, $x_0^{min}$ — state vector at epoch of observations that should lead to point B, and matrix V is a multiplication of two partial derivative matrices:

$$\mathrm{V} = \left[\frac{\partial E}{\partial x}\right]\left[\frac{\partial x}{\partial x_0}\right].$$

Propagate the orbit of $x_0^{min}$ and compute probability to it.

Advantages:

- Takes into account gravitational perturbations.

## Results

Table 1. Impact probabilities computed by different methods.

| Designation | $P_{MC} \pm 3\sigma_{MC}$ | $P_{TP}$ | $P_{PBM}$ | $P_{IPBM}$ |
|---|---|---|---|---|
| 2006 JY26 | $(5.6 \pm 1.7)\cdot 10^{-5}$ | $\mathbf{1.0\cdot 10^{-4}}$ | $\mathbf{1.0\cdot 10^{-4}}$ | $6.0\cdot 10^{-5}$ |
| 2006 QV89 | $(1.8 \pm 0.1)\cdot 10^{-3}$ | $\mathbf{2.0\cdot 10^{-3}}$ | $1.8\cdot 10^{-3}$ | $1.8\cdot 10^{-3}$ |
| 2010 UK | $(3.1 \pm 0.7)\cdot 10^{-3}$ | $\mathbf{1.8\cdot 10^{-3}}$ | $\mathbf{1.8\cdot 10^{-3}}$ | $2.9\cdot 10^{-3}$ |
| 2011 AG5 | $(5.3 \pm 1.3)\cdot 10^{-4}$ | $4.2\cdot 10^{-4}$ | $5.7\cdot 10^{-4}$ | $5.5\cdot 10^{-4}$ |
| 2007 VK184 | $(6.2 \pm 2.0)\cdot 10^{-6}$ | $\mathbf{2.7\cdot 10^{-5}}$ | $\mathbf{2.6\cdot 10^{-5}}$ | $8.8\cdot 10^{-6}$ |
| 2007 VE191 | $(6.4 \pm 1.0)\cdot 10^{-4}$ | $\mathbf{0.0}$ | $6.8\cdot 10^{-4}$ | $7.0\cdot 10^{-4}$ |
| 2008 JL3 | $(3.0 \pm 0.4)\cdot 10^{-4}$ | $\mathbf{7.5\cdot 10^{-4}}$ | $\mathbf{4.0\cdot 10^{-4}}$ | $3.3\cdot 10^{-4}$ |
| 2014 WA | $(3.5 \pm 2.4)\cdot 10^{-7}$ | $\mathbf{0.0}$ | $5.4\cdot 10^{-7}$ | $5.3\cdot 10^{-7}$ |
| 2009 JF1 | $(7.4 \pm 1.2)\cdot 10^{-4}$ | $7.3\cdot 10^{-4}$ | $8.0\cdot 10^{-4}$ | $7.6\cdot 10^{-4}$ |
| 2012 MF7 | $(3.1 \pm 0.8)\cdot 10^{-4}$ | $\mathbf{0.0}$ | $\mathbf{4.8\cdot 10^{-4}}$ | $3.2\cdot 10^{-4}$ |
| 2008 CK70 | $(6.4 \pm 1.0)\cdot 10^{-4}$ | $5.8\cdot 10^{-4}$ | $6.9\cdot 10^{-4}$ | $6.5\cdot 10^{-4}$ |
| 2005 BS1 | $(1.4 \pm 0.2)\cdot 10^{-4}$ | $\mathbf{0.0}$ | $1.4\cdot 10^{-4}$ | $1.4\cdot 10^{-4}$ |
| 2005 QK76 | $(4.3 \pm 0.9)\cdot 10^{-5}$ | $\mathbf{0.0}$ | $4.1\cdot 10^{-5}$ | $4.3\cdot 10^{-5}$ |
| 2007 KO4 | $(7.3 \pm 4.0)\cdot 10^{-7}$ | $\mathbf{0.0}$ | $\mathbf{2.2\cdot 10^{-6}}$ | $9.7\cdot 10^{-7}$ |
| Apophis 2036 | $(1.4 \pm 0.8)\cdot 10^{-5}$ | $\mathbf{0.0}$ | $\mathbf{0.0}$ | $1.2\cdot 10^{-5}$ |
| 2010 RF12 | $(0.0 \pm 1.7)\cdot 10^{-6}$ | $\mathbf{5.1\cdot 10^{-2}}$ | $\mathbf{4.9\cdot 10^{-2}}$ | $0.0$ |

*Notes:* 'Designation' is the asteroid's designation, $P_{MC}$ is the probability computed by the Monte-Carlo method, $\sigma_{MC}$ is a standard deviation of $P_{MC}$, $P_{TP}$ and $P_{PBM}$ are probabilities calculated by the target plane method and the partial banana mapping method correspondingly. $P_{IPBM}$ is the probability computed by the improved version of PBM method.

## Conclusions

The Partial Banana Mapping method was significantly improved so that now it doesn't suffer from gravitational perturbations from major planets, and produces accurate results of impact probability values. The cost of the improvement is that now instead of propagating only the nominal orbit of the asteroid, it requires a couple of more propagations.

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