

Experimental validation of a robust broadband snapshot spectropolarimeter for space applications

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ABSTRACT

The ability to measure the state of polarisation of light is of significant importance in domains such as astronomy, remote sensing, climatology, or biomedical imaging. All these applications require precise, fast and broadband measurements of any state of polarisation. In addition, minimalist and robust instrumental designs are needed to address the constraints of different working environments. To address most of these needs, we have developed over the last few years a novel method for the complete measurement of polarisation across various wavelengths. The approach is based on three birefringent prisms that serve as a modulator, with a geometry that has been thoroughly studied to ensure high precision in polarisation measurement. Followed by a linear polariser, this modulator converts information about the polarisation of light into a variation in the detected intensity along the vertical direction. A unique intensity pattern characterises each state of polarisation, and the state of polarisation can be immediately reconstructed from the snapshot measurements. We report here the experimental validation of a laboratory demonstrator of this new method. The results confirm the accurate reconstruction of the detected states of polarisation at multiple wavelengths. This paper describes the instrument concept, the calibration procedure, and the demodulation approach.

Keywords: Spectropolarimetry, polarimetry, birefringence, polarimetric modulation

1. INTRODUCTION

Polarimetric measurements provide access to the physical properties of materials and scattering media that are often inaccessible through conventional intensity measurements, supporting applications in domains such as astronomy, remote sensing, defence, and medicine.^{1–5} However, full-Stokes Spectropolarimetry over broad wavelength ranges generally requires complex instrumentation, posing significant challenges for applications that demand compactness, robustness, and high stability. To address these limitations, we developed a snapshot spectropolarimetric concept based on a modulator composed of three thin magnesium fluoride (MgF_2) prisms with differently oriented fast axes. The modulator introduces a continuous spatial variation in the phase difference between the orthogonal components of the electric field. A fixed linear analyser converts this polarisation modulation into an intensity pattern recorded by a detector array. Each state of polarisation is characterised by a unique pattern that can be used to reconstruct the Stokes vector of the incident light rapidly. Because the direction perpendicular to the modulation remains available for spectral dispersion, the architecture enables simultaneous full-Stokes measurements over an extended spectral range without moving parts or beam splitting. This work presents the first experimental demonstration of the concept and evaluates its polarimetric performance at multiple wavelengths.

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2. WORKING PRINCIPLE

The spectropolarimeter investigated in this study consists of a collimator, a spectral filter, a modulator composed of three birefringent prisms, a linear analyser, and a camera (see Fig. 1). The spectral filter was selected to simplify the experimental investigation. However, an alternative version of the instrument, incorporating a dispersive prism, was developed in collaboration with TNO. This configuration enables the simultaneous determination of the state of polarisation at multiple wavelengths.⁶

The modulator, a key component of the instrument, consists of three birefringent prisms with very small apex angles (see Fig. 2). This choice minimises the influence of Fresnel effects and ensures a negligible angular separation between the ordinary and extraordinary rays emerging from the optical assembly.

The first prism encountered by the incident light has its fast axis aligned with the x -axis. It therefore modifies any incident polarisation state except horizontal and vertical linear polarisation. The wedge geometry introduces a phase difference between the orthogonal components of the electric field, proportional to the optical path length travelled through the prism. Because the prism thickness varies continuously along the vertical direction, y , the induced phase difference also varies linearly with y .

The middle prism has its fast axis aligned with the z -axis and therefore does not alter the state of polarisation. It plays only a structural role: it maintains the modulator's compactness and robustness while preserving a continuous optical path through the assembly.

The third prism has its fast axis oriented at 45° with respect to the x -axis in the (xy) plane. Consequently, the horizontal and vertical linear polarisation states that remain unaffected by the first prism are modulated by the third prism. As in the first prism, the varying thickness of the third prism produces a continuous change in phase difference along the vertical direction, resulting in a corresponding spatial variation of the emerging polarisation state.

Together, the first and third prisms transform any incident polarisation state into a continuous spatial variation of polarisation along the y -axis. When the modulated light passes through a linear analyser with a fixed orientation, this polarisation variation is converted into a spatial intensity modulation that the camera can record. By contrast, unpolarised incident light does not produce an intensity modulation along the y -axis. No rotation or change of orientation of the analyser or the modulator is required to ensure the full determination of the Stokes vector.

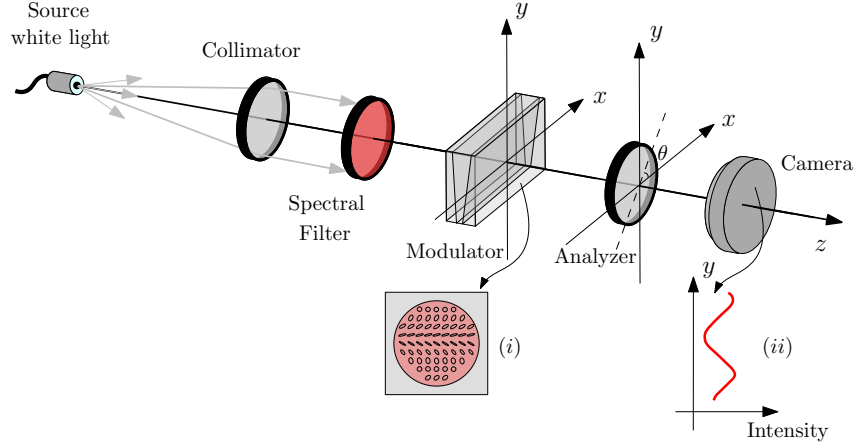


Figure 1. Representation of the operating principle: (i) The modulator converts the incoming polarisation state into a continuous variation of polarisation in the vertical direction. (ii) The analyser transforms this variation into an intensity modulation. For a given wavelength, each column of pixels will record the same pattern. From this pattern, the input polarisation state is reconstructed.

Applying the Mueller calculus and the Stokes representation of polarisation, the state of polarisation of light emerging from the compound represented by the modulator and the analyser (see Fig. 1) is:

$$S_{out} = M_A \cdot M_M \cdot S_{in}, \quad (1)$$

where M_A is the Mueller matrix of the analyzer, M_M is the Mueller matrix of the modulator and $S_{in} = (S_0, S_1, S_2, S_3)^T$, is the Stokes vector of the incoming light, with T the transposition operator. Using a modulator composed of three thin prisms with the following configuration:

- Prism 1 (entry): apex angle ξ , fast axis along x axis;
- Prism 2 (middle): apex angle $\xi + \psi$, fast axis along z axis;
- Prism 3 (exit): apex angle ψ , fast axis at 45° with respect to x axis, in the xy plane,

followed by an analyser oriented at an angle θ to the horizontal (x -axis), then the detected intensity can be expressed as:

$$I = \frac{1}{2}(S_0 + S_1 \cdot m(y, \lambda) + S_2 \cdot n(y, \lambda) + S_3 \cdot p(y, \lambda)), \quad (2)$$

where S_0, S_1, S_2, S_3 are the components of the incoming Stokes vector, y is the position along the y axis where the intensity reading takes place and λ is the wavelength of light, whereas m , n and p are the modulation functions of the polarimeter. In our previous papers,^{7,8} we showed that these functions have the following expression:

$$\begin{cases} m(y, \lambda) = \cos(2\theta) \cos(\Delta\phi_3(y, \lambda)) \\ n(y, \lambda) = \sin(2\theta) \cos(\Delta\phi_1(y, \lambda)) + \cos(2\theta) \sin(\Delta\phi_1(y, \lambda)) \sin(\Delta\phi_3(y, \lambda)) \\ p(y, \lambda) = \sin(2\theta) \sin(\Delta\phi_1(y, \lambda)) - \cos(2\theta) \cos(\Delta\phi_1(y, \lambda)) \sin(\Delta\phi_3(y, \lambda)) \end{cases}, \quad (3)$$

where $\Delta\phi_1$ and $\Delta\phi_3$ are the phase differences induced by the prisms (1) and (3):

$$\begin{cases} \Delta\phi_1 = \frac{2\pi}{\lambda} \Delta n(\lambda)(h - y) \tan(\xi) \\ \Delta\phi_3 = \frac{2\pi}{\lambda} \Delta n(\lambda)(h - y) \tan(\psi) \end{cases}, \quad (4)$$

where $\Delta n(\lambda)$ is the birefringence of the modulator, h is the height of the modulator (in the y direction), and ξ and ψ are the apex angles of the prism (1), respectively (3). Because the middle prism has a fast axis oriented along the direction of propagation of light (z direction), no phase difference is associated with the passage through it. Equation (3) shows that the modulation of linear horizontal and vertical polarisations $(S_0, S_1, 0, 0)^T$ is governed by the function m that depends only on $\Delta\phi_3$ and the analyser orientation, θ . The absence of $\Delta\phi_1$ is a direct result of the first prism's fast axis orientation, which renders it ineffective for these states. Consequently, to ensure the determination of linear horizontal and vertical polarisations, the analyser orientation must be different from $\pi/4$.

Figure 2 shows a simulated intensity modulation pattern produced when light with the polarisation state $S = (1, 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ passes through the optical system in Fig. 1. As previously demonstrated,⁷ each incoming polarisation state generates a unique intensity pattern, whereas unpolarised light produces no modulation. This simulation assumes a modulator constructed from magnesium fluoride (MgF_2), a material widely used in polarimetry due to its ease of fabrication and handling. Furthermore, MgF_2 offers high transmission and strong birefringence across a broad waveband ($0.2\mu m$ to $8\mu m$), with particularly high birefringence in the UV domain.^{9,10}

When the detector's horizontal axis is used for spectral dispersion, each detector column captures the intensity modulation at a different wavelength, enabling snapshot full-Stokes spectropolarimetric measurements.

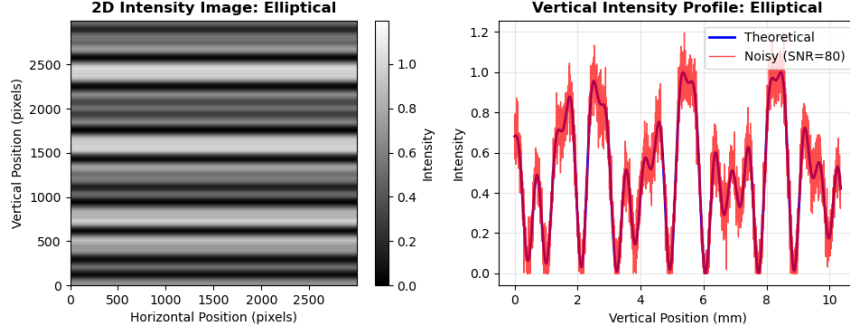


Figure 2. Simulated pattern of intensity for a hypothetical incoming state of polarisation $S_{in} = (1, 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. A noise corresponding to a signal-to-noise ratio (SNR) of 80 was also applied to this signal to illustrate real working conditions. The wavelength for this simulation is $\lambda = 0.515nm$, and the orientation of the analyser is $\theta = 73^\circ$. The left plot shows the detected image from a 3000×3000 pixel detector with a pixel size of $3.45\mu m$. In contrast, the right plot shows the variation in intensity along a column of pixels (vertical direction).

3. THE INSTRUMENTAL MATRIX

Using the Mueller calculus, the intensity measured at detector pixel (i, j) can be written as

$$I_{ij} = w_{00,ij}S_0 + w_{01,ij}S_1 + w_{02,ij}S_2 + w_{03,ij}S_3, \quad (5)$$

where $w_{00,ij}$, $w_{01,ij}$, $w_{02,ij}$, and $w_{03,ij}$ are the elements of the first row of the Mueller matrix describing the complete optical system at pixel (i, j) . Under ideal conditions, these terms approach 1, $m(y_{ij}, \lambda)$, $n(y_{ij}, \lambda)$, and $p(y_{ij}, \lambda)$, respectively.

Polarisation retrieval is based on the intensity modulation along the vertical direction of the detector. The procedure described below is applied independently to each detector column because the modulation properties may vary across the field. Although four pixels are theoretically sufficient for direct demodulation, using $N > 4$ pixels improves robustness to noise and reduces the effects of weak intensity variations or linear dependence among measurements. For fitting-based demodulation, the number of pixels must also satisfy the Nyquist criterion. A detailed analysis of the required number and size of pixels as a function of wavelength was performed in a previous research.⁸

Because each detector pixel samples a distinct position within the spatially varying modulator, its modulation coefficients must be determined experimentally rather than assumed to follow their theoretical values.

The instrumental matrix is calibrated by recording the response to $s > 4$ known, linearly independent input Stokes vectors. For a set of N pixels within column j , the calibration measurements can be expressed compactly as

$$I = WS, \quad (6)$$

where $I \in \mathbb{R}^{N \times s}$ contains the measured intensities, $S \in \mathbb{R}^{4 \times s}$ contains the known input Stokes vectors, and $W \in \mathbb{R}^{N \times 4}$ is the instrumental matrix. Its i th row is

$$W_{ij} = (w_{00,ij} \quad w_{01,ij} \quad w_{02,ij} \quad w_{03,ij}). \quad (7)$$

Using the normal-equation approach,^{11,12} the right pseudoinverse of S is

$$S^+ = S^T (SS^T)^{-1}, \quad (8)$$

and the instrumental matrix is obtained from

$$W = IS^+. \quad (9)$$

This solution requires S to have rank 4. The condition number of SS^T should also be kept as low as possible because a poorly conditioned calibration matrix amplifies errors in the estimated input Stokes vectors.

Once W has been calibrated, an unknown input Stokes vector can be retrieved from the measured intensity vector using the left pseudoinverse of W :

$$W^+ = (W^T W)^{-1} W^T, \quad (10)$$

$$S = W^+ I. \quad (11)$$

The demodulation matrix W^+ is specific to the optical configuration and depends on parameters such as analyser orientation, modulator position, wavelength, and temperature. The conditioning of $W^T W$ therefore determines the sensitivity of the retrieved Stokes vector to measurement errors.

4. EXPERIMENTAL SETUP

The primary objectives of the experimental work were to determine the instrumental matrix W and to evaluate the instrument's demodulation performance.

After the alignment of the optical setup, we devised the activities in four major steps:

1. Characterisation of the optical components
2. Characterisation of the S matrix
3. Instrumental matrix (W) retrieval
4. Test of demodulation

The complete experimental setup, used for the demodulation of various states of polarisation, is presented in Fig.3. From left to right, it comprises the following elements:

- Source of white light;
- Diaphragm;
- Collimator;
- Filter wheel;
- Ultra-broadband wire grid linear polariser;
- Diaphragm
- Achromatic quarter-wave plate;
- Modulator in MgF_2 ;
- Ultra-broadband wire grid linear polariser;
- Camera.

The modulator was fabricated in MgF_2 . Just like in the blueprints, it consists of three wedges with tiny apex angles and fast axes oriented differently. The main geometrical and optical features are:

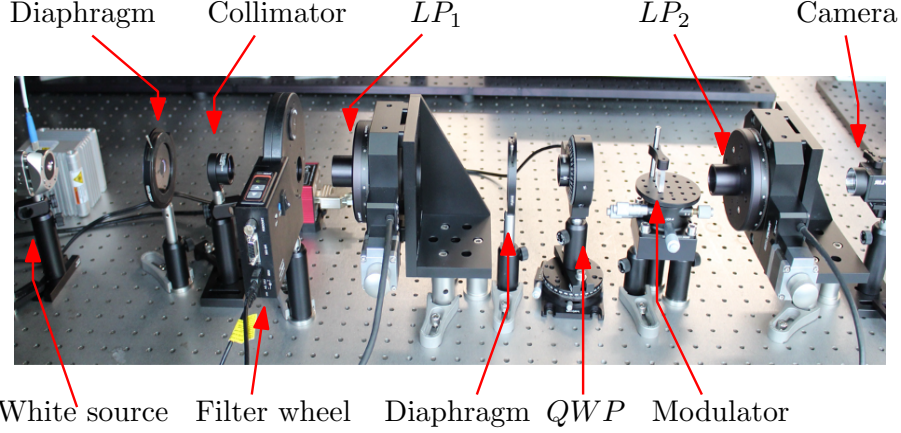


Figure 3. Experimental setup for validating the novel spectropolarimeter. Light enters from the left, passing sequentially through a diaphragm, collimator, and filter wheel. A polarisation state generator (*PSG*), comprising a rotating linear polariser (LP_1) and a rotating quarter-wave plate (*QWP*), prepares the input beam. This light then passes through the modulator, which is mounted on a triple-axis rotation stage. Finally, the light passes through a fixed analyser (LP_2) before reaching the camera. During measurements, LP_1 and the *QWP* rotate to generate the required polarisation states, while the analyser remains stationary.

- Prism 1: apex angle $\xi = 2.6^\circ$, fast axis along x axis;
- Prism 2: apex angle $\xi + \psi$, fast axis along z axis;
- Prism 3: apex angle $\psi = 1.8^\circ$, fast axis at 45° to x axis, in the (xy) plane.
- width=height = $2cm$;
- Additional thickness: $d_0 = 1mm$.

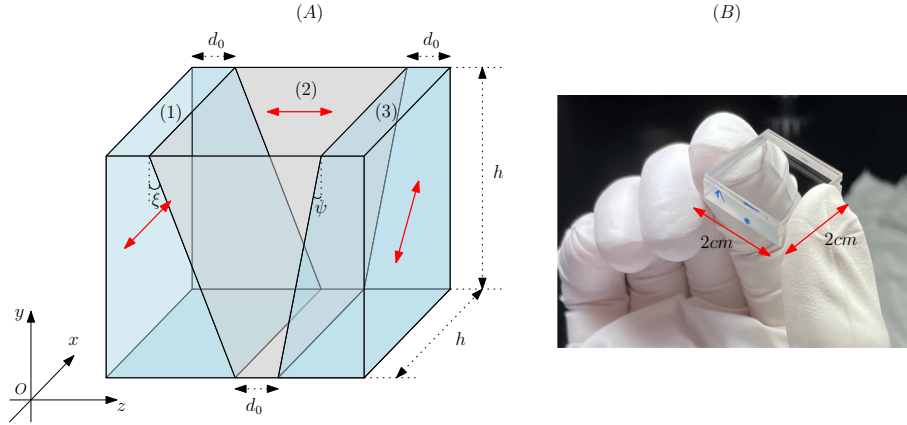


Figure 4. (A) Geometrical design of the modulator. The fast axes orientation for each prism is indicated in red. The additional thickness d_0 was considered to facilitate manipulation and reduce the risk of damage to the compound. On the right (B), the manufactured modulator is shown. A slight misalignment of three prisms can be observed at careful inspection.

5. INSTRUMENTAL MATRIX RETRIEVAL

According to Eq.9, the determination of the W matrix requires the usage of a series of well-known states of polarisation and of the corresponding recorded modulations of the intensity. The states of polarisation were

generated using a polarisation state generator (*PSG*), composed of LP_1 and the *QWP*. Because no reference spectropolarimeter was available, these states were estimated from the calibrated properties of both elements (LP_1 and *QWP*). The necessary states of polarisation were generated by choosing two orientations of the *QWP*, 0° and 45° . For each of these orientations, the linear polariser was rotated from 0° to 180° , with a step of 1° . For each position, two images were recorded. Throughout the process, the position of LP_2 was held fixed.

6. TEST ON VARIOUS POLARIZATIONS

After the retrieval of the instrumental matrix W , another series of polarisation states was generated by rotating LP_1 from 0° to 180° , while keeping the *QWP* oriented at 30° . This series of polarisation was used as a test.

An example of the results for $\lambda = 515nm$ is presented in Fig.(5). As it was expected, the test and the demodulated states are slightly falling inside the Poincaré sphere because of the retardance of the *QWP*, which is below the ideal value.

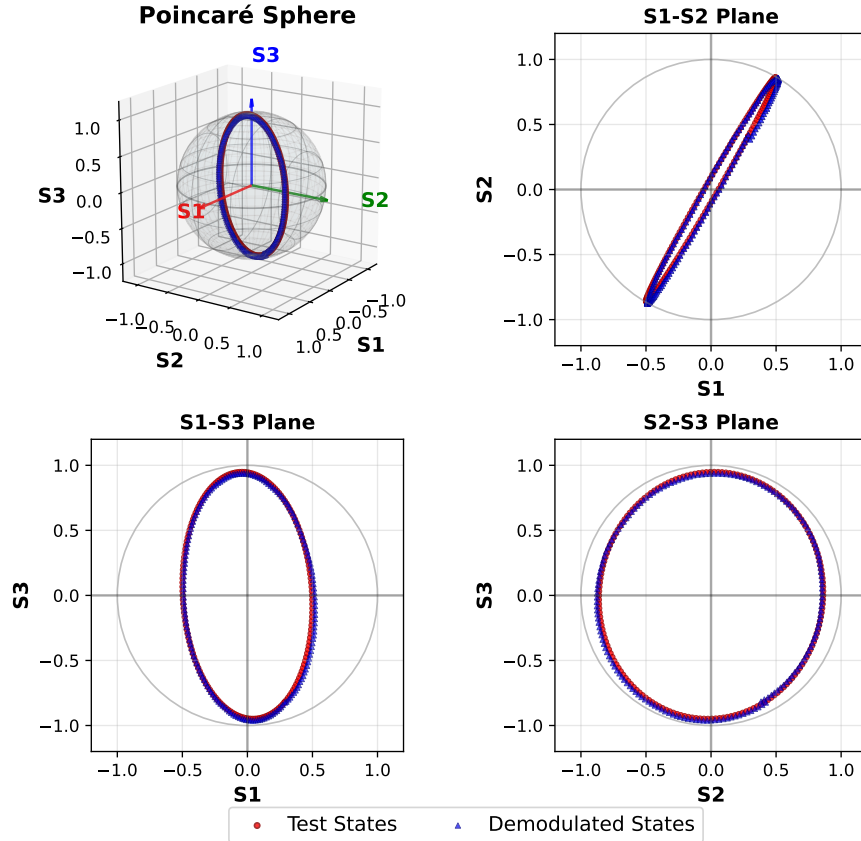


Figure 5. Test of the demodulation algorithm with 181 states generated by turning LP_1 in front of the *QWP* oriented at 30° . The red dots represent the approximated incoming states, and the blue dots represent the demodulated states. The analyser of the system was oriented in this case at 10° , and the wavelength was $\lambda = 515nm$.

Figure 5 demonstrates the system's capacity to retrieve polarisation states for the given *PSG* configuration and the previously described modulator. However, interpreting the quality of these results is challenging due to the uncertainty in the generated Stokes vectors. To address this, the demodulated Stokes vectors can be compared against the region of possible input vectors defined by the uncertainties of the *PSG*.

Figure 6 provides a general overview of the input and demodulated polarisation states for the *PSG* configured with the *QWP* at 30° and the analyser at 105° . Rather than quantifying the error with respect to a single

nominal input value, we evaluate it as the distance from the demodulated parameter to the confidence region of the possible corresponding Stokes parameter. Demodulated results falling outside this confidence region are marked with a black $\times$. Overall, the demodulated parameters remain very close to the input states, largely within the confidence region (green zone). Corresponding results for the other two wavelengths are presented in Figures 7 and 8.

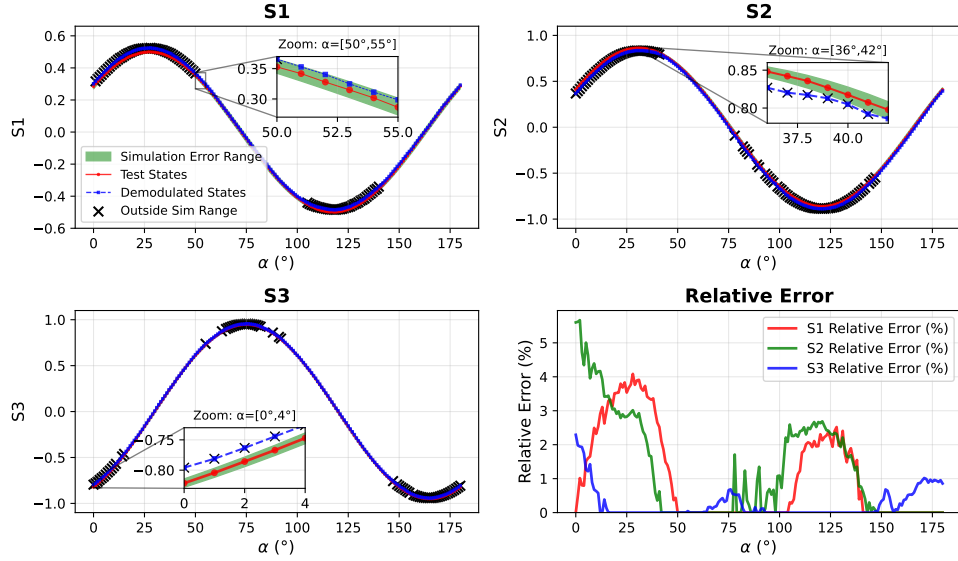


Figure 6. Detailed representation of the incoming Stokes vectors and demodulated values for $\lambda = 515nm$. S_1 , S_2 and S_3 present the values of the corresponding Stokes parameters when LP_1 is rotated from 0° to 180° . The green region highlighted in the zoom window is the confidence interval within which the parameter can take values. The test states are located randomly in this interval. The demodulated states (in blue) stay very close to or within the interval. The regions where the demodulation falls outside the green zone are underlined with the red $\times$. The relative error plot shows the relative error of the demodulation with respect to the nearest point within the confidence interval for cases where the demodulation falls outside the confidence interval. As long as the demodulation falls within the confidence interval, we set the relative error to zero.

7. CONCLUSION

This research demonstrates that the spectropolarimeter concept, which has been thoroughly investigated in recent years, can be successfully realised as a functional instrument. The system can measure arbitrary polarisation states across multiple wavelengths, and the demodulation algorithm is straightforward to implement. We have experimentally validated the instrument's performance by successfully demodulating polarisation at three distinct wavelengths ($515nm$, $632.8nm$, and $488nm$) using a large series of Stokes vectors.

An important step in evaluating demodulation quality was estimating the input polarisation states generated by the *PSG*. We identified multiple factors affecting these Stokes vectors and accounted for uncertainties in the linear polariser's transmission axis, the *QWP* orientation, and its retardance. This allowed us to define a probability region for the generated Stokes vectors, which was used as the reference for assessing demodulation error.

Consequently, the error of each demodulated parameter was defined as its distance to the nearest edge of the input probability region, rather than to a single nominal vector. This approach directly accounts for the spatial inhomogeneity of the generated Stokes vectors across the pixel column used for demodulation.

Using this definition of the error, a relative error below 2% was generally observed for all the parameters, for most of the orientations of the LP_2 . The detailed analysis of the consequences of different analyser orientations provided important information for the next practical implementation of this spectropolarimetry method. It shows the orientations to use to enhance the precision of the Stokes parameters.

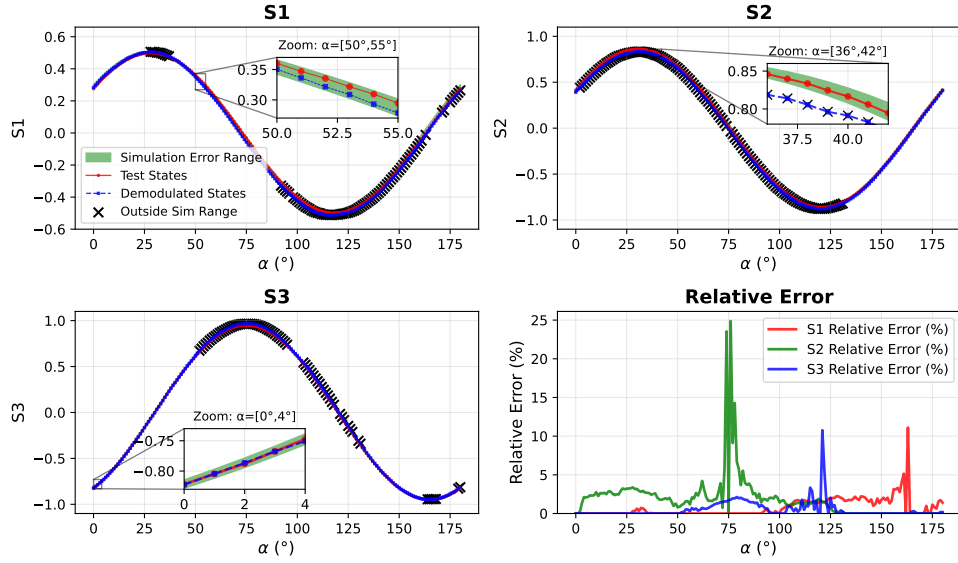


Figure 7. Results obtained for $\lambda = 488nm$. Changing the wavelength results in a small variation in the Stokes vectors for the same LP_1 orientations as before. At the same time, as underlined in chapter 2, different wavelengths correspond to different patterns of intensity. This variation can be the origin of a different recorded error value.

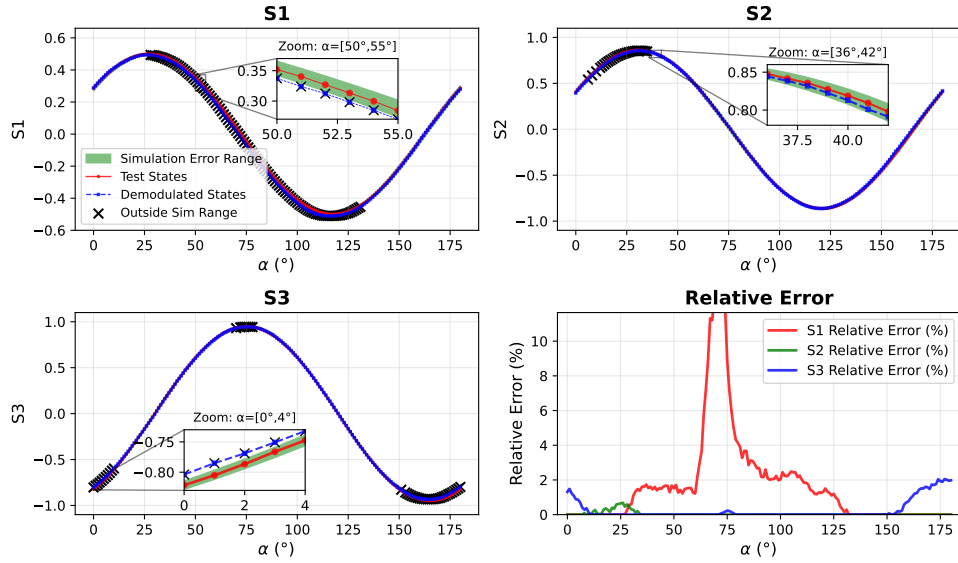


Figure 8. Results obtained for $\lambda = 632.8nm$

The method presented and validated here offers significant advantages in speed, compactness, and versatility. Once the system's instrumental matrix is determined, polarisation demodulation is performed almost instantly, as it requires only a single matrix multiplication.

Furthermore, the modulator's small size and the absence of moving parts enable a highly compact and robust implementation, compatible with most optical platforms. This characteristic is crucial for adapting the technology to a wide range of applications.

Beyond the filter-based operating mode demonstrated here, the instrument is inherently capable of hyper-spectral polarimetry due to its horizontal redundancy. By incorporating a dispersive element and a cylindrical lens into the optical layout, the polarisation of light can be measured simultaneously at multiple wavelengths.

Finally, because a certain level of redundancy also exists in the vertical direction and the demodulation is based on an overdetermined system, the number of pixels used for modulation and demodulation could be reduced. This would allow the polarimeter to be fitted into even more compact instruments or would free up the vertical dimension to achieve imaging spectropolarimetry.

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