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**COMPUTATIONAL PROCEDURE TO INCREASE THE SHOOTING ACCURACY OF SWARMS OF SPACE-BASED LASER TRACKERS TO DEFLECT NEOS BY MEANS OF ABLATION**

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**Extended Abstract—**

Following a current trend, we introduce a computational procedure that allows increasing the shooting accuracy for lightweight spacecrafts grouped in swarms, aiming at deflecting threatening Near Earth Objects (NEOs) by means of laser ablation (see e.g. [1]).

In fact, the proper predictions of the shooting times and directions provided from our method improve the coordination of the swarm's spacecrafts, allowing them to more efficiently ablate the NEOs at designated spots.

The coordination of the spacecrafts is crucial for the multiple-action laser methods to become the most effective in deflecting NEOs - if obviously, we exclude the destructive methods. Therefore, it would make sense to check the practicability of our procedure using simulations involving known asteroids already considered for deflections, such as Apophis and 2010 KJ37. If successful, this procedure could then be easily incorporated into methods aiming at smoothly deflecting NEOs, including the above two.

Moreover, this incorporation would be advisable, since the net deviations from the shooting ranges and directions computed with the currently available methods may be non-negligible, as in scenarios considered here. In fact, at the operative distances considered in [2], the corrections to the locations of the NEOs would be decisive to precisely reach designated spots, particularly in spinning and tumbling NEOs [3]. The point is, that these corrections can amount to significant fractions of the size of many NEOs.

We stress, that according to this procedure the current predictions of the motion of the NEOs with respect to the spacecrafts in the swarms should be modified to satisfy

the post-Newtonian (p-N) equations that take into account the gravitational gradients from the spacecrafts to the NEOs. The equations read [4]

$$\frac{d^2 \bar{\mathbf{R}}_i}{ds_i^2} = \mu \int_0^1 \left[ \left( \frac{3(\bar{\mathbf{R}}_i \cdot \bar{\mathbf{r}}_i(u)) \bar{\mathbf{r}}_i(u)}{\bar{r}_i(u)^5} - \frac{\bar{\mathbf{R}}_i}{\bar{r}_i(u)^3} \right) (1 - 2u + 3u^2) - \nabla_\alpha \left( \frac{3(\bar{\mathbf{R}}_i \cdot \bar{\mathbf{r}}_i(u))^2}{\bar{r}_i(u)^5} - \frac{\bar{R}_i^2}{\bar{r}_i(u)^3} \right) (1 - u)u^2 \right] du, \quad (1)$$

where  $\mu$  is the Sun's mass,  $\bar{\mathbf{R}}_i$  is the p-N position of the NEO from the spacecraft  $\mathbf{S}_i$  in a swarm,  $\bar{\mathbf{r}}_i$  is the p-N heliocentric position of the points on the line joining  $\mathbf{S}_i$  and the NEO, and  $s_i$  is the time recorded by an atomic clock on board of  $\mathbf{S}_i$ .

To isolate the p-N effects from other perturbations, the results derived in the procedure start with the computation of the simplest heliocentric Newtonian (N) orbits of the chosen NEO, say  $D$ , derived from the orbital elements given by JPL (in this case, for Apophis as of 6/3/2021), which we consider here as final elements (Fig.1).

Of course, under real circumstances, these elements have to be considered as preliminary, so that to determine the final orbital elements of  $D$  with p-N accuracy the spacecrafts have to carry out ranging and direction measurements using the ranging formulae [5]

$$\bar{R}_i = \frac{s^{3i} - s^{1i}}{2} \left[ 1 - \frac{\mu}{2} \int_0^1 \left( 1 - u \right)^2 \left( \frac{3(\mathbf{R}_i \cdot \mathbf{r}_i(u))^2}{r_i(u)^5} - \frac{R_i^2}{r_i(u)^3} \right) du \right]. \quad (2)$$

Here,  $s^{1i}$  and  $s^{3i}$  are the emission and reception instants of the round-trips of the laser beams recorded by each spacecraft  $\mathbf{S}_i$ , the directions of the relative positions of the NEO,  $\mathbf{R}_i$ , is given by the INS on board  $\mathbf{S}_i$ ,  $\mathbf{r}_i$  is the

heliocentric position of the points on the line joining  $S_i$  and the NEO, and  $R_i$  stands for

$$R_i = \frac{s^{3i} - s^{1i}}{2}. \quad (3)$$

Next, the computation of the p-N heliocentric orbits of  $D$  and of the spacecrafts, is carried out according to the following equations [6]:

$$\frac{d^2 \vec{r}}{dt^2} = \frac{-\mu \vec{r}}{r^3} \left[ 1 - \frac{2\mu}{r} + 2 \left( \frac{d\vec{r}}{dt} \right)^2 - \left( \frac{3}{r^2} + \frac{2\mu}{r^3} \right) \left( \vec{r} \cdot \frac{d\vec{r}}{dt} \right)^2 \right] + \frac{2\mu}{r^3} \left( 1 + \frac{2\mu}{r} \right) \left( \vec{r} \cdot \frac{d\vec{r}}{dt} \right) \frac{d\vec{r}}{dt}. \quad (4)$$

The spacecrafts are grouped in two types of swarms, (i) non-coordinated ( $S1$ ,  $S2$ , and  $S3$  in the simulation below), and (ii) coordinated ( $S1c$ ,  $S2c$  and  $S3c$ ), where the spacecrafts are flying in trailing formation with  $D$ , aiming at increasing the swarm's stability. The time-span chosen (1.73 times the period of  $D$ ) allows to clearly distinguish and compare the behavior of these two types of swarms.

The next steps are as follows:

(i) computation of the p-N relative orbit of  $D$  with respect to the Earth, and of the distance from  $E$  to  $D$ , see Figs. 2 and 3,

(ii) computation of the p-N relative orbits of  $D$  with respect to  $S1$ ,  $S2$  and  $S3$  ( $S1c$ ,  $S2c$  and  $S3c$ , respectively) according to (1), see Fig. 4,

(iii) computation of the p-N distances from  $S1$ ,  $S2$  and  $S3$  ( $S1c$ ,  $S2c$  and  $S3c$ , respectively) to  $D$  according to (2), see Fig. 5,

(iv) computation of the p-N distances between the spacecrafts,  $S1$ ,  $S2$  and  $S3$  in Fig. 6, and  $S1c$ ,  $S2c$  and  $S3c$  in Fig. 7, according to (3),

(v) computation of the p-N corrections to the N positions of  $D$  with respect to  $S1$ ,  $S2$  and  $S3$  ( $S1c$ ,  $S2c$  and  $S3c$ , respectively), cf. Fig. 8,

(vi) computation of the p-N corrections to the distances to  $D$  from  $S1$ ,  $S2$  and  $S3$  ( $S1c$ ,  $S2c$  and  $S3c$ , respectively), cf. Fig. 9,

(vii) computation of the initial to final p-N elapsed coordinate times for the laser beams to reach  $D$  from  $S1$ ,  $S2$  and  $S3$  ( $S1c$ ,  $S2c$  and  $S3c$ , respectively), according to the following formula (see [5]):

$$t_{1i} = t_2 - \bar{R}_{i2} \left[ 1 + \frac{1}{6} \frac{\mu}{\bar{r}_{D2}^3} \left( \frac{3(\bar{\mathbf{R}}_{i2} \cdot \bar{\mathbf{r}}_{D2})^2}{\bar{r}_{D2}^2} - \bar{R}_{i2}^2 \right) + \frac{\mu}{\bar{r}_{D2}} + \frac{1}{2} \bar{v}_{D2}^2 \right], \quad (5)$$

where  $t_{1i}$  is the p-N emission time coordinated for the respective laser beam to reach  $D$  at the scheduled instant  $t_2$ , see Fig. 10,

(viii) computation of the differences between the p-N emission instants for  $S1$ ,  $S2$  and  $S3$  in Fig. 11, and  $S1c$ ,  $S2c$  and  $S3c$  in Fig. 12.

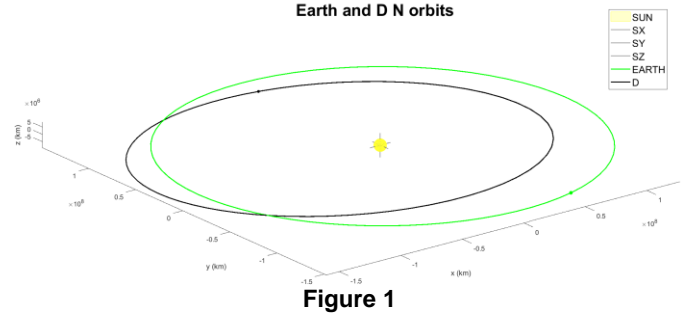


Figure 1

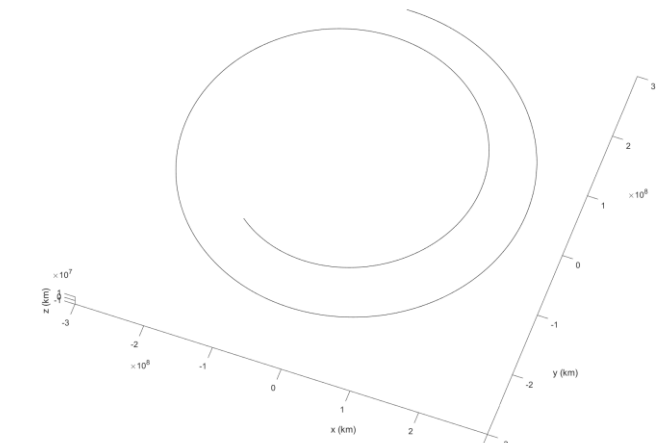


Figure 2

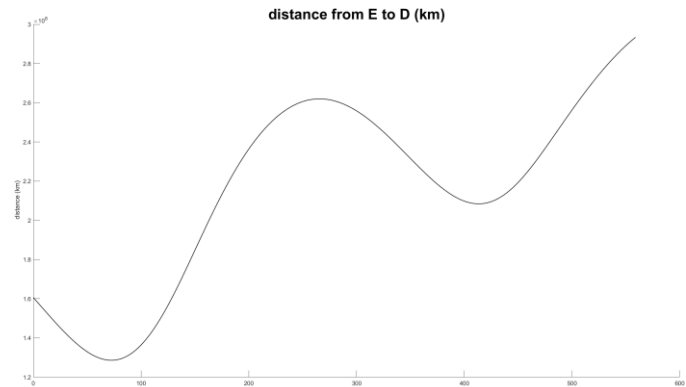


Figure 3

p-N relative orbits of D w.r.t. S1, S2 and S3, and S1c, S2c and S3c

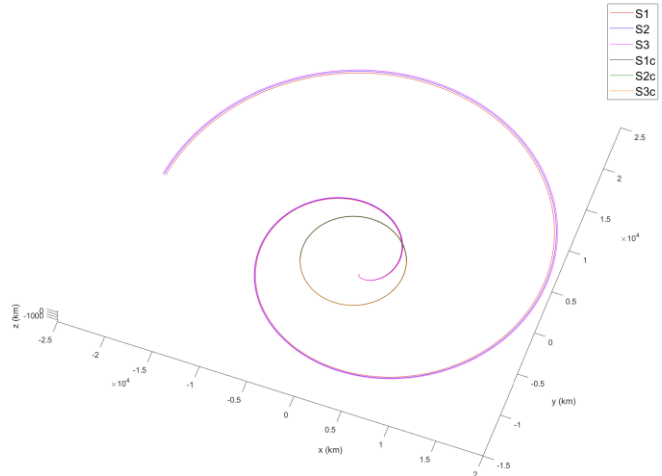


Figure 4

p-N distances S1c-S2c, S1c-S3c and S2c-S3c (km)

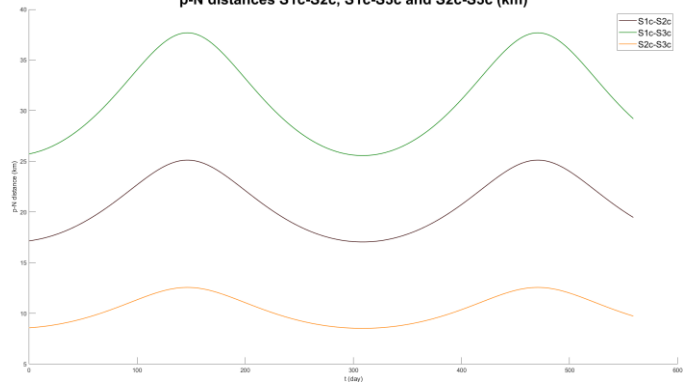


Figure 7

p-N distances S1-D, S2-D and S3-D, and S1c-D, S2c-D and S3c-D (km)

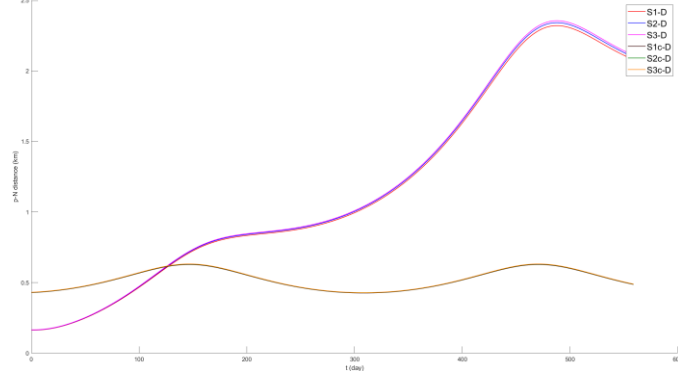


Figure 5

p-N corrections to relative positions of D w.r.t. S1, S2 and S3, and S1c, S2c and S3c (cm)

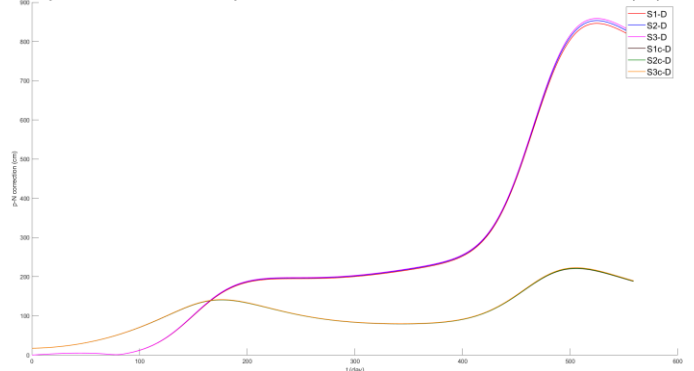


Figure 8

p-N distances S1-S2, S1-S3 and S2-S3 (km)

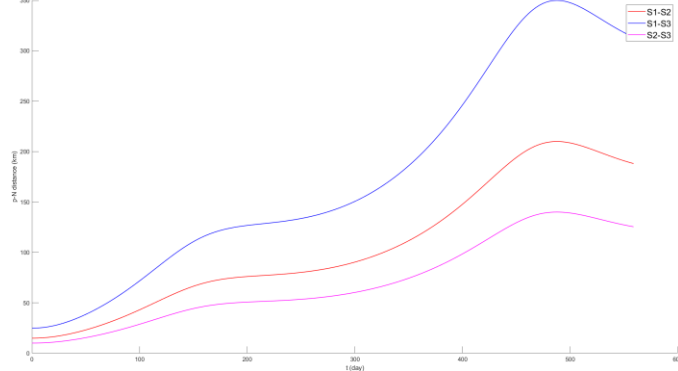


Figure 6

S1-D, S2-D and S3-D, and S1c-D, S2c-D and S3c-D p-N ranging corrections (cm)

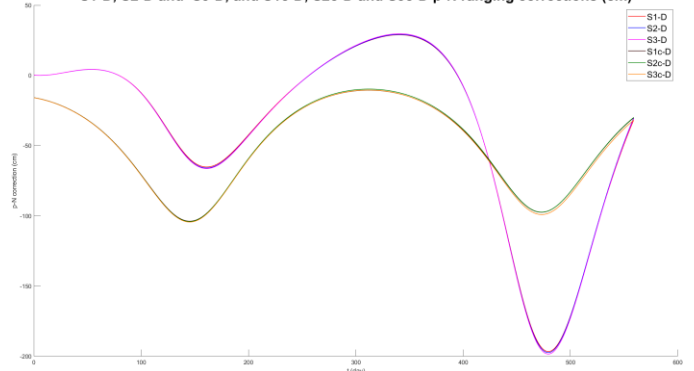


Figure 9

Initial to final instants p-N elapsed time for S1, S2 and S3, and S1c, S2c and S3c (sec)

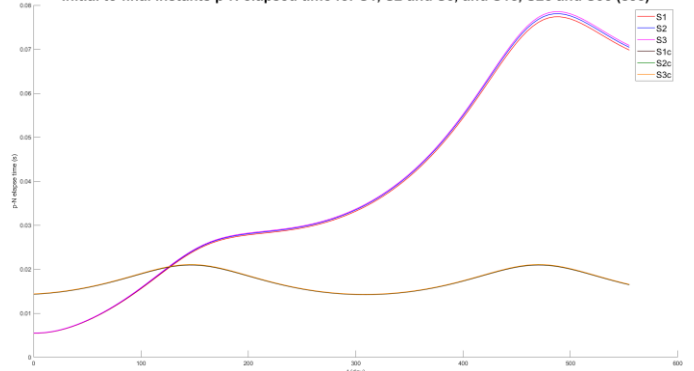
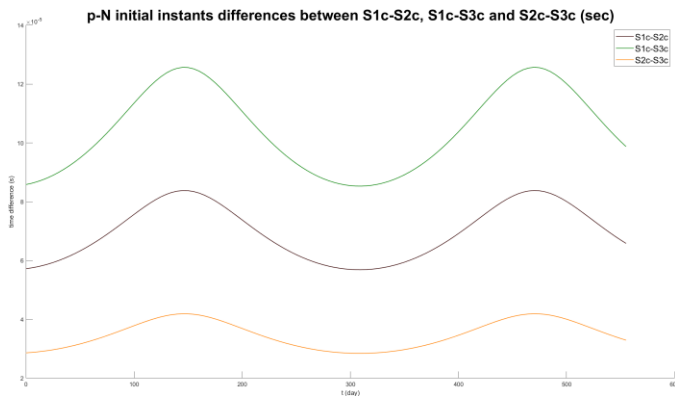
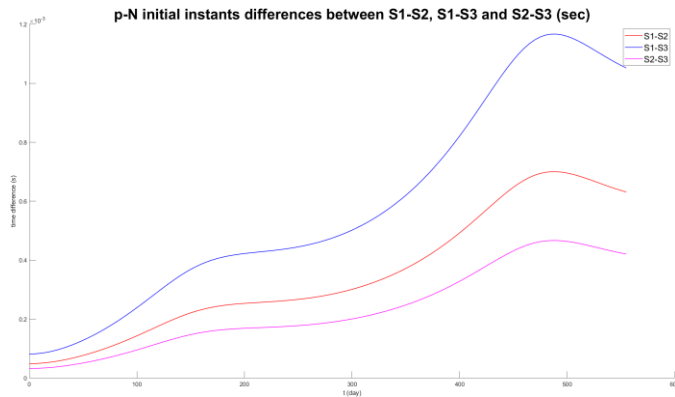


Figure 10



[x1Bud\\_Mc0Y8y8UBVN3rhSvmTX6C5o3xzJbGuA?e=icHZhM](https://doi.org/10.1117/12.2197261)

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The above sequence of figures is shown in the following video:

<https://iesalpajes-my.sharepoint.com/:v:/p/lgarciadelpino/EXuAWMDa->