Optimal Impulsive/Low-Thrust Trajectories for Asteroid Deflection via Kinetic Impact

Bruce A. Conway[*, #], Alessia Speziale [# *] and Ludovica Malagni [# *]

* Dept. of Aerospace Engineering, University of Illinois at Urbana-Champaign, USA
# Politecnico di Torino

Objective

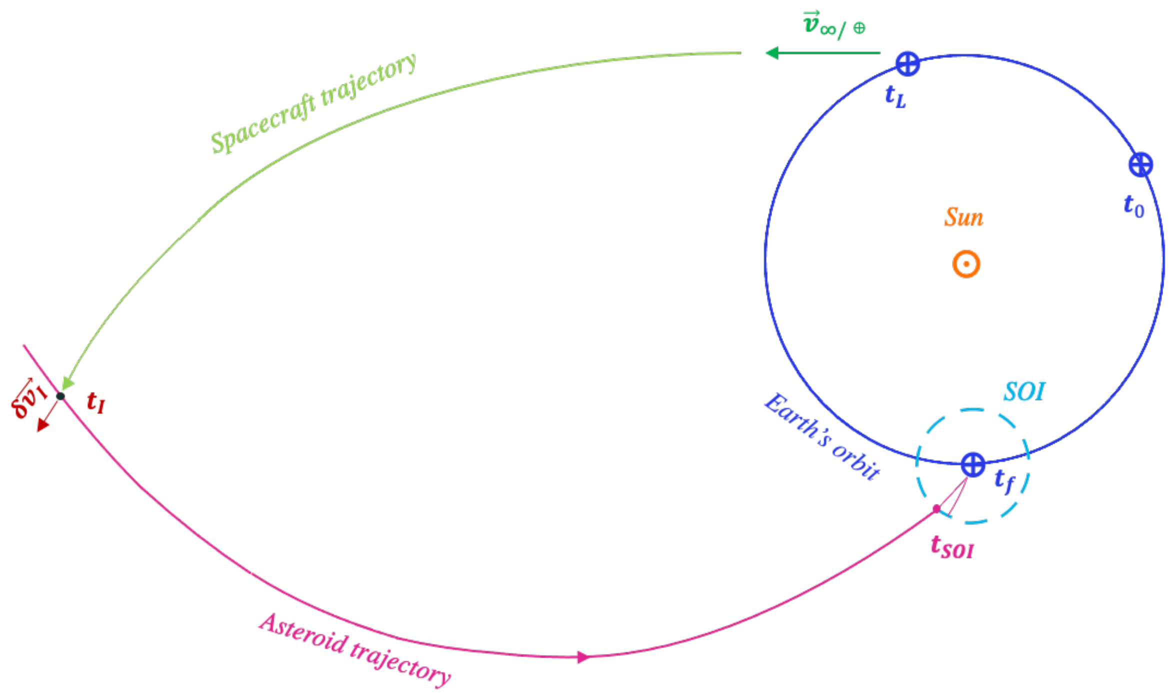
Maximize deflection of asteroid at close approach for S/C with specified mass, $V_{\infty/Earth}$, thrust magnitude, I_{sp}.

Assume flight profile similar to that of recent missions to asteroids, e.g. DART, OSIRIS-REx, Dawn.

Make result as accurate as possible; use JPL ephemeris (SPICE) for position of the asteroid target and for positions of the principal bodies causing gravitational perturbations to the flight of the spacecraft.

Method

The figure shows the simulation plan:



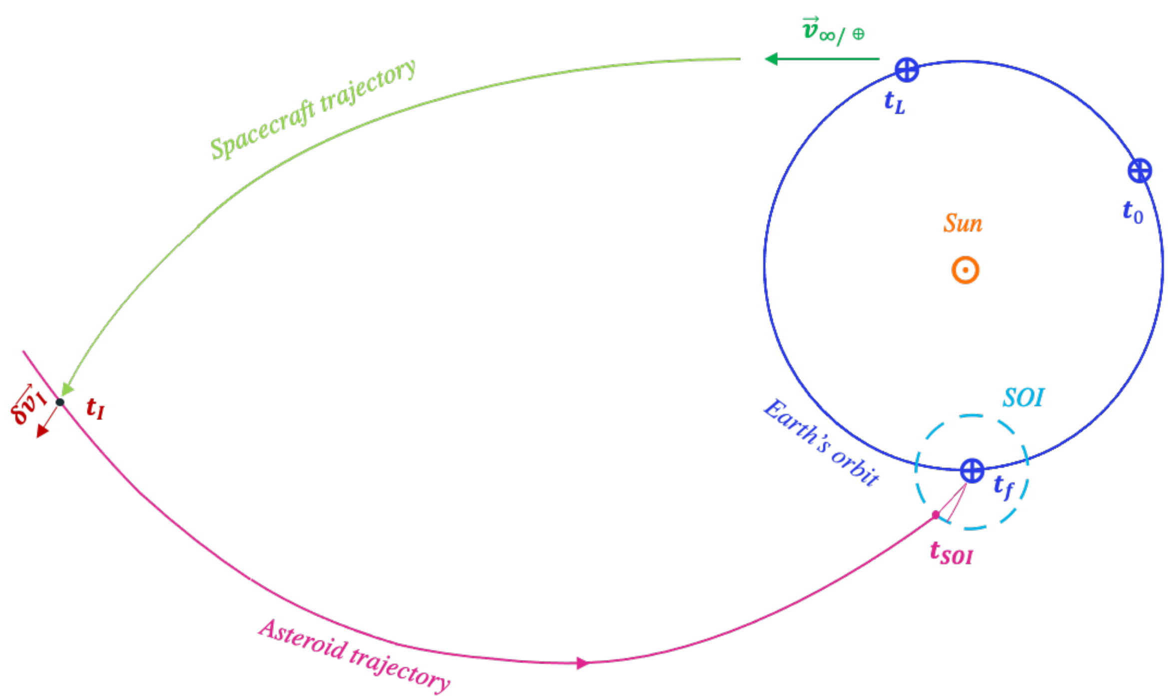
1) Earth departure; date and $V_{\infty/Earth}$ direction chosen by optimizer
2) L-T electric propulsion with thrust direction chosen by optimizer.
3) Interception/collision "constraint" satisfied on date chosen by optimizer
4) Impact causes very small δv , which depends on relative velocity, remaining mass of s/c, impact characteristics

$$\delta v_0 = \frac{m_{s/c}(v_{s/c} - v_{⊛})}{m_{⊛} + m_{s/c}}$$

5) Asteroid continues on ephemeris-generated trajectory

Method (2)

The figure shows the simulation plan:



6) At Earth SOI, s/c $\bar{r}$ and $\bar{v}$ and TOF allow determination of STM coefficients. Then

$$\begin{bmatrix} \delta\vec{r} \\ \delta\vec{v} \end{bmatrix} = \begin{bmatrix} \tilde{R} & R \\ \tilde{V} & V \end{bmatrix} \begin{bmatrix} \delta\vec{r}_0 \\ \delta\vec{v}_0 \end{bmatrix}$$

where $\delta\bar{v}_0$ is the impact-caused change in velocity.
Impact is assumed inelastic w/ no benefit from ejecta.

New

$$\bar{r} = \bar{r} + \delta\bar{r}$$
$$\bar{v} = \bar{v} + \delta\bar{v}$$

7) The asteroid motion is then integrated forward until close approach. The deflection is the increase from the nominal close approach distance.

Method (3)

Equations of Motion

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v}_x = -\frac{\mu_\odot x}{r^3} + \frac{T_x}{m} + a_{x(♀)} + a_{x(\oplus☽)} + a_{x(♂)} + a_{x(♃)} \\ \dot{v}_y = -\frac{\mu_\odot y}{r^3} + \frac{T_y}{m} + a_{y(♀)} + a_{y(\oplus☽)} + a_{y(♂)} + a_{y(♃)} \\ \dot{v}_z = -\frac{\mu_\odot z}{r^3} + \frac{T_z}{m} + a_{z(♀)} + a_{z(\oplus☽)} + a_{z(♂)} + a_{z(♃)} \\ \dot{m} = -\frac{T_{max}}{c_{exh}} \end{cases}$$

Planetary perturbations from attractions of Venus, Earth-Moon, Mars, Jupiter.

Thrust components are functions of an in-plane pointing angle β and out-of-plane pointing angle γ .

Method (4)

Optimization via two qualitatively different methods.

- PSO (particle swarm optimization)
  A heuristic method.
  Has the benefit of being initialized randomly, i.e. no initial guess needed.
  "Particles" are N-dimension potential solutions
  Particles move in N dimensional search space, to improve their cost
  Particles "communicate"; all learn best location known to the swarm
  Continuous controls need to be expressed as function of a small
  number of parameters. For this simulation, thrust pointing angles are represented by 5^{th}-degree polynomials in TOF.
  No native way to incorporate constraints; need to use penalty functions
  For this problem there are 16 PSO parameters; 12 thrust angle polynomial coefficients, 2 $V_{\infty/Earth}$ departure angles, departure date, collision date.

- R-K Parallel Shooting

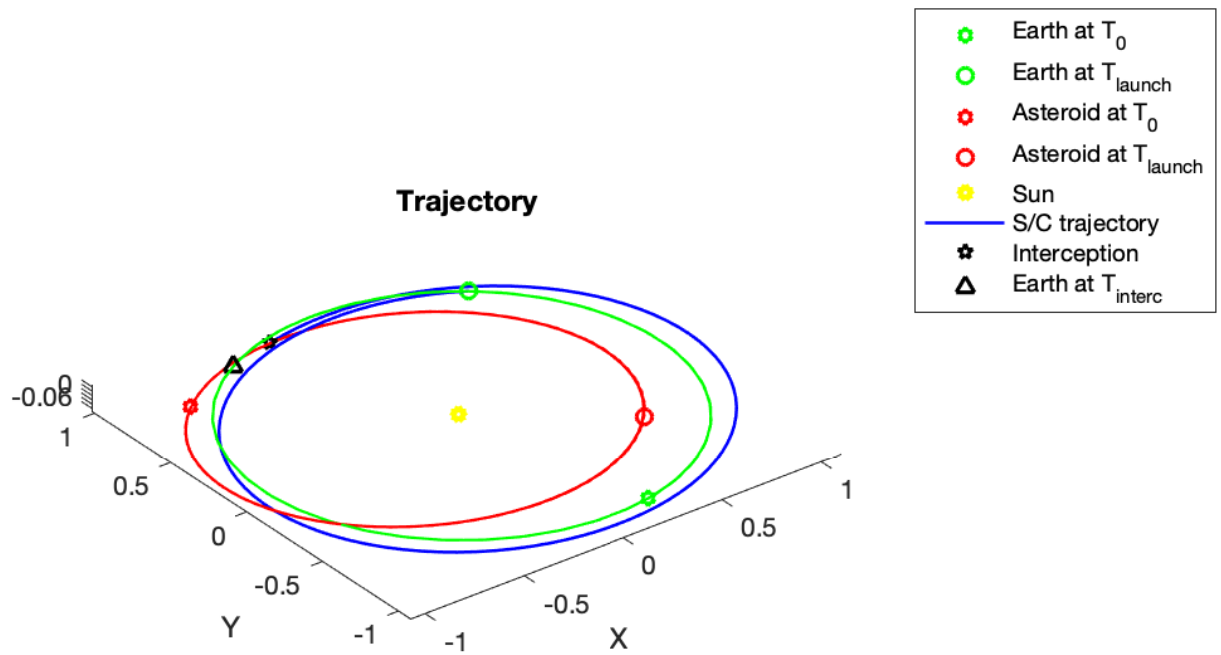
Example

Test case is deflection of Apophis. Apophis close approach is 13 April 2029.

Initial thrust accel. = 18 x 10^{-6} g
$V_{\infty/Earth}$ = 1.8 km/sec
Initial S/C mass = 10000 kg

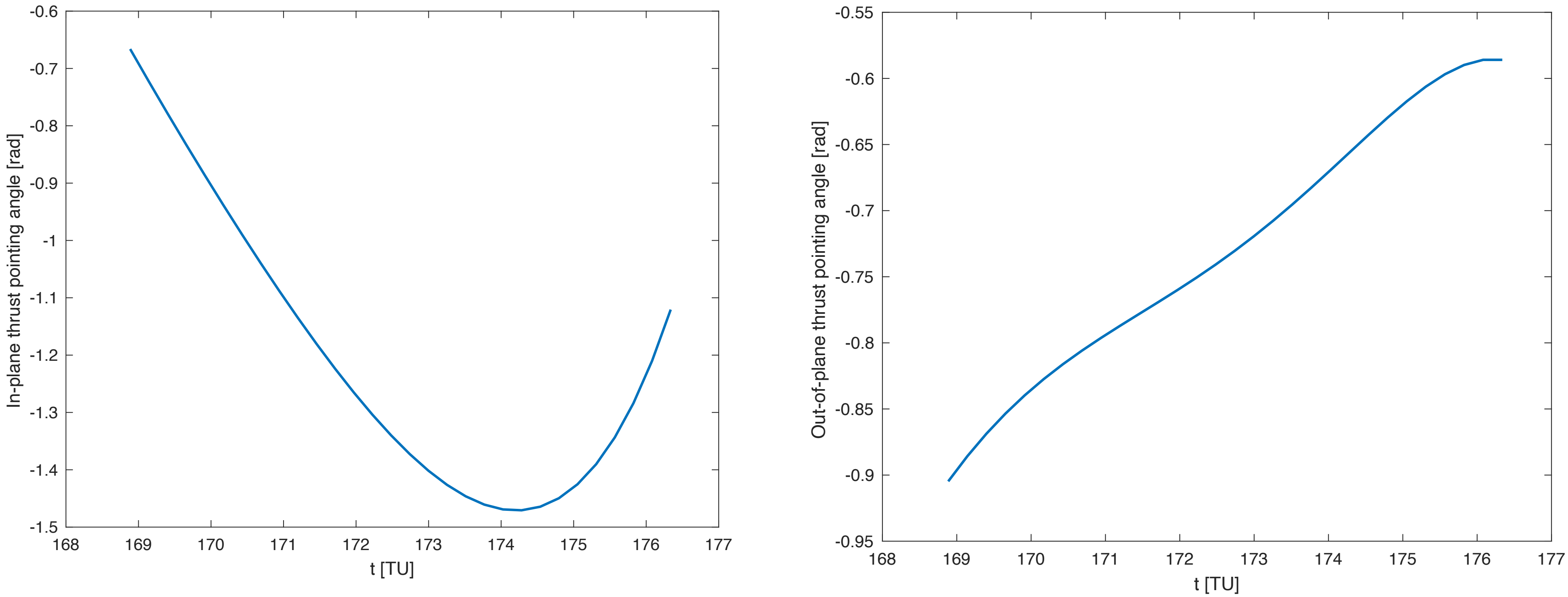
Epoch date is 1/1/2026.
Optimizer chooses departure date of 11/13/2026 and impact date of 1/19/2028
S/C mass at impact = 7764 kg

Impact results in deflection of 1267 km



Example (2)

Thrust pointing angles during powered flight, parametrized by 5th degree polynomials in TOF



Results - Variation with Departure $V_{\infty/Earth}$ and Thrust Magnitude

T_{max}/m_0 (10^{-6} g)	$V_{\infty/Earth}$ (km/s)	Defl (km)	Interception (AU)	Departure*	Impact
30	1.80	-1371	7.50E-07	12/30/2026	1/19/2028
24	1.80	-1361	7.40E-12	12/11/2026	1/19/2028
18	1.80	-1267	3.30E-11	11/13/2026	1/19/2028
18	1.65	-1217	2.90E-11	11/10/2026	1/19/2028
18	1.50	-1147	7.80E-12	11/7/2026	1/19/2028
12	1.50	-846	2.80E-10	10/11/2026	1/19/2028
12	1.35	-828	4.00E-11	10/14/2026	1/19/2028
12	1.20	-851	9.00E-12	10/22/2026	1/19/2028

* Earth departure is possible any day after 1/1/2026

Confirmation of PSO (heuristic) Result with R-K (NLP-based) Result

Same deflection of Apophis prior to April 2029 close approach
S/C Initial thrust accel. = 30 x 10^{-6} g
Exhaust velocity = 29.78 km/sec (I_{sp} = 3035 sec)
$V\infty/Earth$ = 1.8 km/sec
Initial S/C mass = 10000 kg
Epoch date is 1/1/2026

R-K result

Departure date of 12/30/2026
Impact date of 1/19/2028
S/C mass remaining at impact = 6674 kg
Interception (collision) error = 5.3E-8 AU
Impact results in deflection of 1376 km

PSO result

Departure date of 12/30/2026
Impact date of 1/19/2028
S/C mass remaining at impact = 6674kg
Interception (collision) error = 7.5E-7 AU
Impact results in deflection of 1371 km

Conclusions

- A heuristic (PSO) optimizer has successfully found optimal strategies for asteroid deflection missions.
- This solution method is straightforward and benefits from not needing to require an initial guess, which can prejudice convergence to a local minimum.
- A qualitatively different optimization method, similar to collocation, in which the problem is converted to a (large) NLP problem, has confirmed the solution obtained by PSO.
- The use of the system STM is simplifying and also adds to accuracy, since forward integration of the EOM post-collision is numerically difficult because the delta-V caused by the impact is only a fraction of 1 m/sec.
- Interestingly, for the case of Apophis, the optimizer chooses a lengthy wait time before departure, in order to improve the relative geometry of Earth and Apophis.