

Channel Estimation for Deep-Fading CO-OFDM LEO Satellite-to-Ground Optical Links Using Low-Rank Matrix Completion

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ABSTRACT

In low-Earth-orbit (LEO) satellite-to-ground optical links, prolonged atmospheric deep fading can severely corrupt pilot observations over consecutive OFDM symbols, critically degrading conventional channel estimators. To overcome this physical limitation, we propose a deep-fade-robust channel estimation framework based on low-rank matrix completion (LRMC). Grounded in the intrinsic physics of the space-optical channel—specifically, strong atmospheric temporal coherence and smooth hardware frequency response—we formulate the two-dimensional time-frequency channel as a highly concentrated low-rank matrix. We introduce an iterative LRMC algorithm to robustly reconstruct channel coefficients from severely fragmented pilot observations without requiring explicit prior statistics. The framework is comprehensively validated through a 20-Gbps CO-OFDM hardware transmission experiment. Evaluated across weak-to-strong turbulence regimes and varying pilot densities, the proposed approach significantly outperforms conventional LMMSE and Kalman filtering under severe fading conditions.

Keywords: Satellite Optical Communication (SOC), Coherent optical orthogonal frequency-division multiplexing (CO-OFDM), Channel estimation, low-rank matrix completion (LRMC), Deep fading

1. INTRODUCTION

Free-space optical (FSO) links between low-Earth-orbit (LEO) satellites and optical ground stations (OGS) are expected to support high-capacity downlinks over rapidly varying atmospheric paths. Coherent optical orthogonal frequency-division multiplexing (CO-OFDM) is a highly promising transmission format for such links, combining high spectral efficiency with robust digital signal processing (DSP) capabilities [1]. However, its practical performance depends critically on reliable channel estimation. In satellite-to-ground FSO links, atmospheric turbulence induces irradiance fluctuations and scintillation that can severely degrade the received optical signal [2]. Moreover, LEO-to-ground measurements have shown millisecond-scale temporal correlation and fade durations [3]. During such fades, the received optical power can approach the receiver sensitivity limit, severely corrupting pilot observations and producing extended regions of sparse or unreliable pilot measurements across consecutive OFDM symbols.

Conventional one-dimensional (1D) channel estimators are poorly suited to this severe space-to-ground setting. Frequency-domain interpolators such as linear minimum mean-square error (LMMSE) estimation rely on local pilot availability and accurate prior knowledge of second-order channel statistics, both of which become increasingly unreliable under time-varying atmospheric fading. Time-domain trackers, such as Kalman filtering (KF), provide adaptivity but tend to propagate prediction errors during extended pilot outages, often drifting significantly before the link recovers. Crucially, both approaches operate primarily along a single dimension—either time or frequency—and fundamentally fail to reconstruct the contiguous pilot-loss regions mathematically manifested in the two-dimensional (2D) time-frequency grid.

This physical limitation motivates a 2D structural view of the coherent FSO channel. Unlike multipath-rich terrestrial wireless channels, the FSO channel over a short local observation interval is dominated by a single line-of-sight (LoS) optical path. Atmospheric turbulence evolves comparatively slowly, resulting in strong temporal correlation over the observation interval [3]. In addition, the optical transceiver response varies smoothly across the occupied OFDM bandwidth, yielding strong frequency-domain correlation. Consequently, the channel matrix on the time-frequency grid exhibits a near-separable physical structure, motivating an effective low-rank representation analogous to low-rank OFDM

channel models [4]. This intrinsic physical property naturally connects the estimation problem to low-rank matrix completion (LRMC), enabling the reconstruction of the channel from severely fragmented observations without requiring explicit prior statistical knowledge.

In this paper, we propose a robust, physics-informed channel estimation framework for coherent FSO-OFDM systems that jointly exploits this 2D time-frequency correlation to improve channel-estimation robustness under severe pilot loss. The main contributions are as follows: First, we develop a practical, iterative LRMC-based channel recovery procedure. Unlike conventional 1D interpolators or data-driven methods requiring extensive retraining, this algorithm reconstructs channel coefficients from severely fragmented pilot observations by jointly exploiting global 2D correlations by simultaneously exploiting global 2D correlations, offering a pure DSP solution to a physical atmospheric problem. Second, moving beyond numerical simulations, we comprehensively validate the physical feasibility of the proposed framework through a 20-Gbps CO-OFDM hardware transmission experiment. Evaluated across weak-to-strong turbulence regimes and a wide range of pilot densities, the proposed recovery framework demonstrates robust performance and significantly outperforms conventional LMMSE and KF methods under practical hardware impairments.

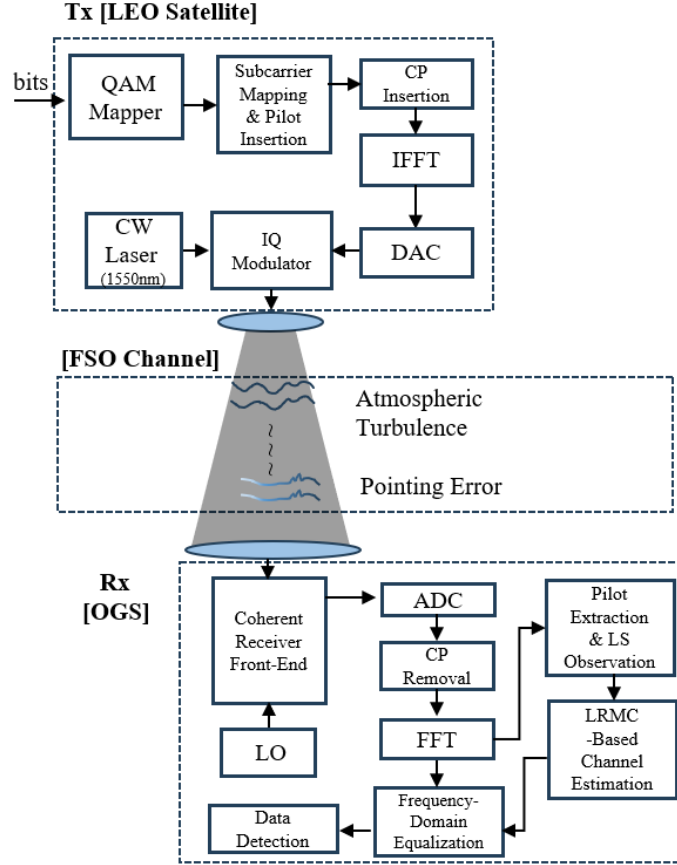


Figure 1. Block diagram of the proposed coherent FSO-OFDM system with LRMC-based channel estimation.

2. SYSTEM MODEL

Fig. 1 illustrates the considered 1550-nm CO-OFDM FSO downlink between a LEO satellite and an OGS. At the transmitter, 16-QAM data and pilot symbols are modulated over N subcarriers to form an optical signal via an IQ modulator. The signal propagates through an atmospheric slant path, where it is impaired by deterministic attenuation, turbulence-induced scintillation, and pointing errors.

Assuming perfect synchronization after coherent detection and standard OFDM demodulation, the received baseband signal on the k -th subcarrier of the n -th OFDM symbol is expressed as

$$Y[n, k] = H[n, k]X[n, k] + W[n, k], \quad (1)$$

where $X[n, k]$ is the transmitted symbol, $H[n, k]$ represents the flat-fading complex channel coefficient, and $W[n, k] \sim \mathcal{CN}(0, \sigma_w^2)$ is the additive white Gaussian noise.

Considering an observation frame of N_s consecutive OFDM symbols, the two-dimensional time-frequency channel is represented by the complex matrix

$$H \in \mathbb{C}^{N_s \times N}. \quad (2)$$

Reliable estimation of H becomes critically challenged when atmospheric deep fading severely degrades the received pilot observations. The received optical irradiance is modeled as the product of deterministic path attenuation, turbulence-induced scintillation, and pointing loss, [2],[5]:

$$I(t) = I_l I_p(t) I_a(t). \quad (3)$$

Here, I_l denotes the deterministic link attenuation for the considered link range, $I_a(t)$ follows the Gamma-Gamma irradiance model, and $I_p(t)$ denotes the pointing-loss power coefficient. Since coherent detection operates on the optical field, the corresponding complex channel coefficient is

$$h(t) = \sqrt{I(t)} e^{j\phi(t)}. \quad (4)$$

When these severe impairments simultaneously reduce the received optical signal-to-noise ratio (SNR) below a detection threshold γ_{th} , pilot symbols can no longer be detected reliably. Let Ω_p denote the set of predefined pilot locations in the OFDM time-frequency grid. The set of reliably observable pilot positions is then defined as

$$\Omega = \{(n, k) \in \Omega_p : \gamma[n, k] \geq \gamma_{th}\}. \quad (5)$$

The observable portion of the ideal channel is represented by

$$H_\Omega = P_\Omega(H), \quad (6)$$

where $P_\Omega(\cdot)$ preserves only the observed locations. Crucially, because the atmospheric coherence time T_c is significantly longer than the OFDM symbol duration T_s , pilot losses do not occur as isolated random samples but manifest as contiguous missing regions (prolonged deep fades) in the time-frequency plane. This structured loss can severely degrade conventional interpolation-based estimators, necessitating an alternative approach based on the intrinsic physical dimensions of the channel.

3. PHYSICAL BASIS OF LOW-RANK FSO CHANNEL

Unlike terrestrial wireless channels characterized by rich multipath scattering, a satellite-to-ground FSO link is inherently dominated by a single line-of-sight (LoS) optical path. The physical channel impairments (turbulence and pointing errors) evolve over an atmospheric coherence time T_c that far exceeds the symbol duration ($T_s \ll T_c$), yielding strong temporal correlation. Concurrently, the negligible multipath delay spread of the clear-air channel implies that the coherence bandwidth vastly exceeds the signal bandwidth, resulting in near-perfect frequency correlation.

Because of this physical coexistence of temporal and frequency correlations, the ideal two-dimensional FSO can be modeled, under idealized conditions, as a separable process:

$$H = h_t g_f^H, \quad (7)$$

where h_t captures the slowly varying atmospheric turbulence amplitude and phase, and g_f represents the deterministic frequency response of the optical transceiver hardware. This outer product formulation directly dictates that the intrinsic channel has a rank of one ($\text{rank}(H) = 1$), directly bridging space optics physics with matrix algebra.

In practical coherent receivers, the ideal rank-one model is perturbed by hardware imperfections, including residual laser phase noise, finite analog bandwidth, synchronization jitter, and measurement noise. These practical impairments increase the numerical rank of H ; however, the channel energy remains overwhelmingly concentrated in a few dominant singular modes. Applying singular value decomposition (SVD), the practical channel matrix is accurately approximated by its truncated form:

$$H \approx \sum_{i=1}^r \sigma_i u_i v_i^H, \quad r \ll \min(N_s, N), \quad (8)$$

where only the first r principal singular values capture the essential time-frequency features of the physical link. This low-dimensional structure provides the mathematical foundation for LRMC-based reconstruction from severely fragmented pilot observations, provided that sufficient reliable samples remain available for channel recovery.

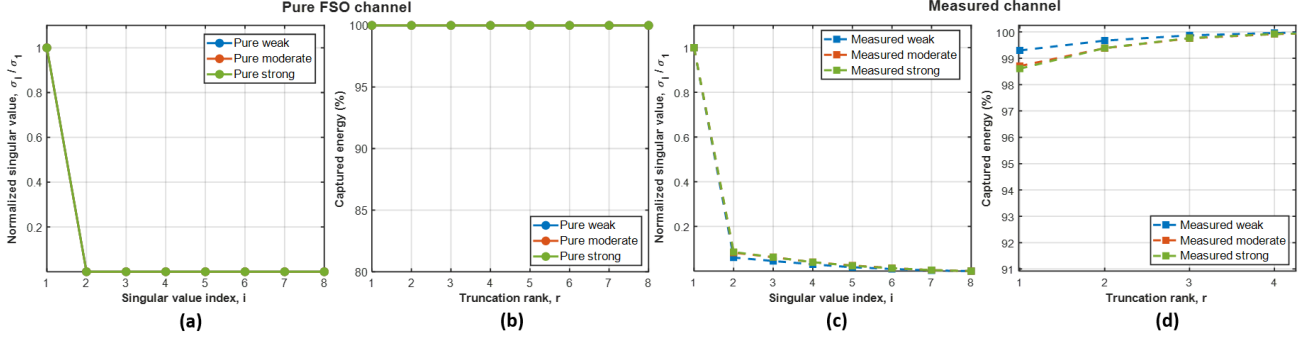


Figure 2. Normalized singular value spectra (σ_i/σ_1) and cumulative energy for (a, b) the ideal FSO channel, and (c, d) the experimentally measured channel under emulated FSO fading.

To quantitatively validate this low-dimensional structure, Fig. 2 evaluates the singular value distributions and cumulative captured energy for both the theoretical model and the experimentally measured channel under emulated FSO fading. As demonstrated in Fig. 2(a) and (b), the ideal pure FSO channel exhibits a perfect rank-one structure, where the principal singular value σ_1 captures 100% of the channel energy across weak, moderate, and strong turbulence regimes.

In contrast, Fig. 2(c) and (d) illustrate the singular spectra for the experimentally measured channel under emulated FSO fading, including practical hardware impairments such as residual phase noise, timing skew, and measurement noise inherent in high-speed optical transceivers. Although these practical imperfections introduce non-zero higher-order singular values, the singular spectrum still decays precipitously. Crucially, even under strong turbulence conditions, the first two singular components are sufficient to capture over 99% of the total channel energy. This strong energy concentration empirically confirms the low effective dimensionality of the measured channel and supports LRMC-based reconstruction from severely fragmented pilot observations, provided that sufficient reliable samples remain available for channel recovery.

4. LRMC-BASED CHANNEL ESTIMATION

4.1 Problem Formulation

Following the observation model introduced in Section II, channel estimation is performed only at the received pilot positions. Applying least-squares (LS) estimation to the detected pilots yields

$$\hat{H}_{LS}[n, k] = \frac{Y[n, k]}{X[n, k]}, (n, k) \in \Omega, \quad (9)$$

where Ω denotes the set of observable pilot locations.

The incomplete LS channel observation is represented as

$$\mathbf{M} = P_{\Omega}(\hat{\mathbf{H}}_{LS}), \quad (10)$$

where $P_{\Omega}(\cdot)$ is the projection operator defined in (6).

Unlike conventional interpolation methods, which estimate missing coefficients from neighboring pilots, the proposed approach seeks to recover the complete channel matrix by exploiting the intrinsic low-dimensional structure established in Section III.

Accordingly, channel reconstruction is formulated as the following rank minimization problem

$$\begin{aligned} \min_{\mathbf{H}} \quad & \text{rank}(\mathbf{H}) \\ \text{s. t.} \quad & P_{\Omega}(\mathbf{H}) = P_{\Omega}(\mathbf{M}). \end{aligned} \quad (11)$$

Equation (11) searches for the lowest-rank channel matrix that remains consistent with all available pilot observations.

4.2 Nuclear Norm Relaxation

Direct minimization of the matrix rank is a non-convex NP-hard optimization problem. Therefore, the rank function is replaced by its convex surrogate, namely the nuclear norm, defined as the sum of singular values,

$$\|\mathbf{H}\|_* = \sum_i \sigma_i(\mathbf{H}). \quad (12)$$

The reconstruction problem is consequently reformulated as

$$\begin{aligned} \min_{\mathbf{H}} \quad & \|\mathbf{H}\|_* \\ \text{s. t.} \quad & \|P_{\Omega}(\mathbf{H}) - P_{\Omega}(\mathbf{M})\|_F \leq \varepsilon, \end{aligned} \quad (13)$$

where ε accounts for receiver noise and pilot estimation errors.

The optimization seeks the minimum-complexity channel representation while preserving consistency with the observed pilot measurements.

Since the satellite-to-ground FSO channel exhibits a highly concentrated singular spectrum, as demonstrated in Section III, only a few dominant singular values are sufficient to characterize the underlying channel subspace. Consequently, the nuclear norm formulation provides an effective approximation to the original rank minimization problem.

4.3 Iterative SVT-Based Reconstruction

Motivated by the nuclear-norm formulation in (13), we employ an iterative singular-value-thresholding procedure for practical channel reconstruction[6]. Unlike conventional interpolators that rely locally on neighboring pilots, this iterative procedure reconstructs the global time-frequency subspace by executing the following five steps until convergence:

Step 1: Pilot Projection

Starting from an initial estimate, the exact observed pilot entries are enforced while the missing entries are updated. The projection is given by

$$X^{(i)} = P_{\Omega}(M) + P_{\Omega^c}(H^{(i)}), \quad (14)$$

where Ω^c denotes the set of unobserved entries, including pilot-depleted regions.

Step 2: Subspace Decomposition

The projected matrix is then factorized into its singular components via SVD to expose the underlying low-rank structure of the FSO channel:

$$X^{(i)} = U\Sigma V^H. \quad (15)$$

Step 3: Soft-Thresholding

To suppress noise and eliminate artifacts caused by missing observations, soft-thresholding is applied to each singular value:

$$\tilde{\sigma}_j = \max(\sigma_j - \tau, 0), \quad (16)$$

where τ is the threshold parameter controlling the effective matrix rank.

Step 4: Subspace Reconstruction

The updated channel estimate is geometrically reconstructed using the thresholded singular values:

$$H^{(i+1)} = U\tilde{\Sigma}V^H, \quad (17)$$

where $\tilde{\Sigma} = \text{diag}(\tilde{\sigma}_1, \tilde{\sigma}_2, \dots)$.

Step 5: Convergence Check

The iterative process (Steps 1–4) continues until the residual difference between consecutive estimates falls below a predefined tolerance δ :

$$\frac{\|H^{(i+1)} - H^{(i)}\|_F}{\|H^{(i)}\|_F} < \delta. \quad (18)$$

By jointly exploiting correlations across both time and frequency dimensions through this five-step logic, the algorithm reconstructs channel coefficients within severely pilot-depleted intervals by exploiting the remaining reliable observations and the low-rank time-frequency structure.

4.4 Channel Equalization

After convergence, the reconstructed channel matrix $\hat{\mathbf{H}}$ is employed for one-tap frequency-domain equalization.

The transmitted symbol on each subcarrier is estimated by

$$\hat{X}[n, k] = \frac{Y[n, k]}{\hat{H}[n, k]}. \quad (19)$$

The equalized symbols are subsequently forwarded to the 16-QAM demapper and the corresponding bit sequence is recovered.

The estimation accuracy directly determines the effective post-equalization SNR. Assuming the reconstructed channel is expressed as

$$\hat{H} = H + e, \quad (20)$$

where e denotes the channel estimation error, the received signal after equalization becomes

$$\hat{X} = X + \frac{W - eX}{\hat{H}}. \quad (21)$$

Accordingly, the estimation error contributes an additional interference term, reducing the effective SNR,

$$\text{SNR}_{\text{eff}} \approx \frac{E\{|HX|^2\}}{E\{|eX|^2\} + \sigma_w^2}. \quad (22)$$

Therefore, improving channel reconstruction accuracy directly enhances the post-equalization SNR and consequently reduces the bit-error rate (BER). The following section experimentally evaluates the proposed LRMC estimator under various atmospheric turbulence conditions.

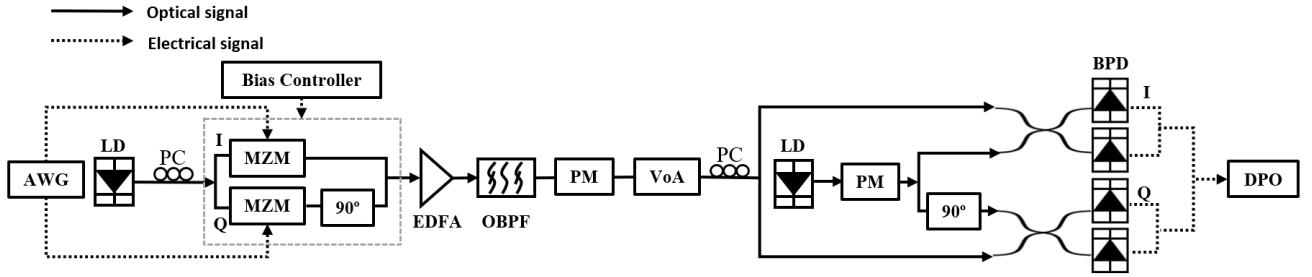


Figure 3. Experimental setup of the 20-Gbps CO-OFDM optical transmission system. (AWG: arbitrary waveform generator; PC: polarization controller; MZM: Mach-Zehnder modulator; EDFA: erbium-doped fiber amplifier; OBPF: optical bandpass filter; VOA: variable optical attenuator; PM: power monitor; DPO: digital phosphor oscilloscope.)

Table 1. Experimental and channel-emulation parameters of the 20-Gbps CO-OFDM system.

Parameter	Value
Wavelength (λ)	1550 nm
AWG / DPO Sampling Rate	25 GS/s
Modulation Format	16-QAM CO-OFDM
Raw Data Rate	20 Gbps (5 Gbaud)
OFDM FFT Size (N)	256
Cyclic Prefix (N_{cp})	44
Frame Size (N_{symbol})	5000
Link Range	600 km
Rytov Variance (σ_R^2)	0.2, 0.5, 0.8
Experimental capture duration	240 μ s
Oversampling factor	4
Pointing Error	10 μ rad

5. RESULTS AND DISCUSSION

To validate the proposed framework, we conducted a 20-Gbps (5-Gbaud) laboratory experiment using a coherent OFDM transmission system. The experimental configuration is shown in Fig. 3, and the principal transmission and channel-emulation parameters are summarized in Table 1. A 1550-nm 16-QAM CO-OFDM signal was generated using a 256-point FFT with a cyclic-prefix length of 44 samples and sampled at 25 GS/s. The emulated channel conditions correspond to a representative 600-km LEO-to-ground link, with Rytov variances of 0.2, 0.5, and 0.8 used for the three investigated turbulence regimes. Rather than relying on simple hardware attenuation, time-varying FSO fading was numerically synthesized to emulate representative LEO-to-ground channel conditions, including Gamma-Gamma turbulence and pointing errors. These composite fading time series were digitally applied to the CO-OFDM waveform before being loaded into the AWG.

The resulting electrical waveform drove an optical IQ modulator at 1550 nm, ensuring that the critical contiguous deep-fade events were precisely reproduced within a physical optical setup. After propagation, the signal was coherently detected by a 25-GS/s DPO and processed offline for channel reconstruction and equalization.

The advantage of the proposed 2D LRMC estimator becomes evident under severe pilot depletion. As shown in Fig. 4, conventional one-dimensional estimators exhibit substantial performance degradation during prolonged deep fades. LMMSE exhibits an error floor due to ill-posed interpolation across pilot-depleted regions, while KF suffers increased estimation error as the fade duration exceeds its effective temporal tracking range. In contrast, LRMC reconstructs the missing channel coefficients by exploiting the global low-rank subspace across both weak and strong turbulence conditions.

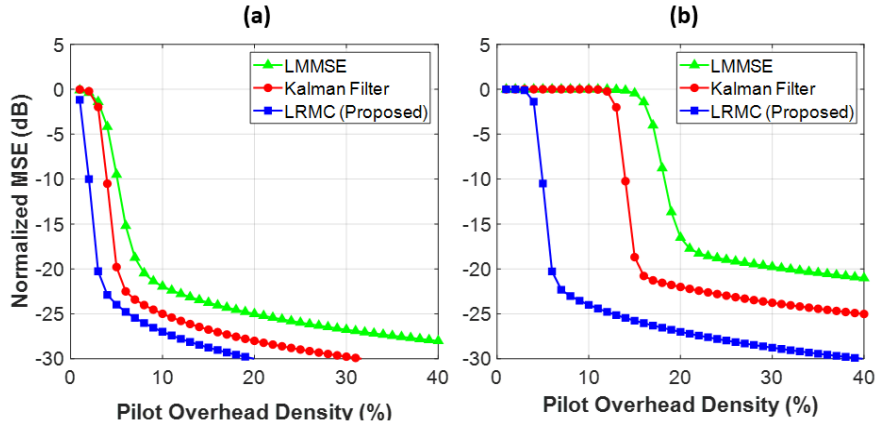


Figure 4. NMSE performance of the channel estimators versus pilot density under (a) weak and (b) strong turbulence conditions. The proposed LRMC avoids the severe error floors and tracking failures experienced by conventional 1D interpolators during contiguous pilot outages.

The improved channel reconstruction directly translates into enhanced post-equalization BER performance. Figure 5 compares the experimental BER with theoretical predictions obtained using an effective-SNR model, in which the channel-estimation error of each estimator is represented as an estimator-dependent SNR penalty. The resulting effective SNR is then mapped to the analytical BER of coherent 16-QAM over an AWGN channel.

The experimental results exhibit trends consistent with the theoretical predictions, while the performance gap between the conventional estimators and LRMC becomes more pronounced as the turbulence strength increases. Under strong turbulence, the pre-FEC BER target of 3.8×10^{-3} is reached at E_b/N_0 values of 23.0 dB, 17.5 dB, and 12.5 dB for LMMSE, KF, and LRMC, respectively. Accordingly, the proposed LRMC reduces the required E_b/N_0 by 10.5 dB relative to LMMSE and by 5.0 dB relative to KF. These results indicate that exploiting the low-rank time-frequency channel structure effectively suppresses the channel-estimation penalty under severe fading conditions.

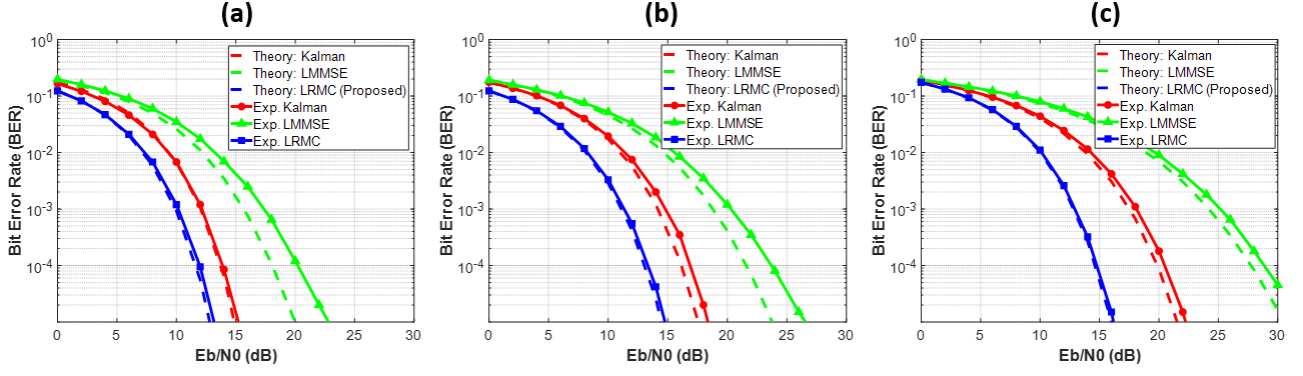


Figure 5. BER versus E_b/N_0 for the coherent 16-QAM system under weak, moderate, and strong turbulence conditions. Solid and dashed curves denote experimental results and theoretical predictions based on the effective-SNR model, respectively

6. CONCLUSION

We proposed a physics-informed channel estimation framework for coherent FSO-OFDM, leveraging the intrinsic near-separable time-frequency structure of the space-optical channel. The developed five-step iterative SVT algorithm reconstructs channel coefficients within severely pilot-depleted intervals by exploiting the remaining reliable observations and the low-rank time-frequency structure. The framework was validated through a 20-Gbps laboratory experiment emulating dynamic LEO-to-ground fading environments. Experimental results demonstrate that the proposed method consistently outperforms conventional estimators, achieving up to a 10.5-dB reduction in the required (E_b/N_0) relative to LMMSE and a 5.0-dB reduction relative to KF at the pre-FEC BER target under strong turbulence. By reducing channel-estimation penalties under severe fading, the proposed approach improves link robustness and relaxes the required SNR margin for LEO optical downlinks.

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REFERENCES

- [1] Shieh, W., Bao, H., and Tang, Y., "Coherent optical OFDM: theory and design," *Opt. Express* 16, 841–859 (2008). DOI: 10.1364/OE.16.000841.
- [2] Andrews, L. C., and Phillips, R. L., *Laser Beam Propagation through Random Media*, 2nd ed., SPIE Press, Bellingham, WA (2005). DOI: 10.1117/3.626196.
- [3] Moll, F., "Experimental analysis of channel coherence time and fading behavior in the LEO-ground link," International Conference on Space Optical Systems and Applications (ICSOS 2014), Kobe, Japan, May 7–9, 2014.
- [4] T. Liu, K. F. Masood, J. Tong, J. He, and J. Xi, "Two-Fold Sampling-Based Super-Resolution Estimation of Low-Rank MIMO-OFDM Channels," *IEEE Open Journal of the Communications Society*, vol. 5, pp. 7434–7446, 2024, doi: 10.1109/OJCOMS.2024.3500782.
- [5] Farid, A. A., and Hranilovic, S., "Outage capacity optimization for free-space optical links with pointing errors," *J. Lightwave Technol.* 25, 1702–1710 (2007). DOI: 10.1109/JLT.2007.899174.
- [6] J.-F. Cai, E. J. Candès, and Z. Shen, "A singular value thresholding algorithm for matrix completion," *SIAM J. Optim.*, vol. 20, no. 4, pp. 1956–1982, 2010. DOI: 10.1137/080738970.