ASTEROID SIZE ESTIMATION WITH DATA FROM THE RUBIN OBSERVATORY LEGACY SURVEY OF SPACE AND TIME Željko Ivezić[1], Vedrana Ivezić[2], [1]University of Washington, 3910 15th Avenue NE, Seattle, WA 98195, USA; ivezic@uw.edu; [2]Princeton University, 35 Olden St, Princeton, NJ 08540, USA

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Abstract: The accuracy of optical color-based size estimates is only a factor of 1.3 to 1.4 worse than for IR-based estimates when sufficiently accurate multi-band optical photometry and IR-based training sample are available. The Rubin Observatory Legacy Survey of Space and Time (LSST) will obtain size estimates more accurate than 25% for several million asteroids.

Introduction:

The knowledge of asteroid size is important in the context of planetary defense because the potential damage caused by an impactor scales with its size [1]. Size are also important in the context of studying the formation and evolution of the asteroid belt as encoded in its size distribution [2].

Despite this importance, there are fewer than a thousand asteroids with direct size measurements [3], and most size estimates are derived indirectly from flux measurements. The currently largest asteroid sample with size estimates is based on thermal flux modeling and infrared fluxes measured by the WISE survey [4]. A series of papers that produced size estimates accurate to about 15-20% for about 164,000 asteroids, as well as constraints on asteroid emissivity properties, was reviewed and summarized by [5].

Traditionally, optical size estimates were inferior to infrared-based size estimates because asteroid surfaces are not very shiny: their reflectivity (albedo) is low, which implies high emissivity via Kirchhoff's law [6]. Therefore, dynamic range for optical albedo is much larger than dynamic range for infrared emissivity, and this difference propagates to a difference in resulting size uncertainties. For example, an uncertainty range in reflectivity of a factor of 2 around a fiducial value of 0.1 corresponds to an emissivity uncertainty range of less than 10%, and an implied ratio of size uncertainties of almost 10.

However, the observed strong correlation between optical colors and optical albedo of asteroids can be used to obtain much more accurate asteroid size estimates when multi-band optical data are available [7]. It turns out that without color information the observed asteroid albedo distribution would result in a scatter of optical size estimates of about 50-60%, and demonstrably non-Gaussian uncertainty distribution. This level of accuracy is a factor of 3 to 4 worse than the accuracy of infrared-based sizes. Instead, [7] showed that size estimates based on multi-band SDSS data, which delivered precise optical color measurements for over 100,000 asteroids, can be tied to WISE-based estimates with an uncertainty of only 17%. This small scatter demonstrates substantial improvement compared to single-band estimates.

We recently revisited analysis reported in [7] and explored the correlation between SDSS colors and WISE-based albedo using sophisticated data-driven models [8]. Here we briefly highlight the main results from that study (referred to as IvIv21 hereafter).

Correlation between asteroid albedo and optical colors:

Given an estimate of asteroid size based on WISE data, D, its visual albedo p_V is computed as

$$p_V = \left(\frac{1329\,\mathrm{km}}{D}\right)^2 10^{-0.4H}, \qquad (1)$$

where H is optical absolute magnitude based on SDSS measurements. With an intrinsic scatter of about 5% for D values, and 0.05 mag for H, the resulting p_V uncertainty is about 11% (relative error, or precision) and dominated by D uncertainties.

The variation of WISE-based estimates of p_V with optical SDSS colors is shown in Figure 1. Given these WISE-based estimates of p_V and optical SDSS colors, various statistical and machine learning models can be used to map the variation of p_V with SDSS colors and estimate the best-fit model p_V^{model}, which is only a function of SDSS colors. Using such a model and SDSS data, SDSS-based size can be then estimated by transforming Eq. 1 into

$$D = 1329\,\mathrm{km}\,\frac{10^{-0.2H}}{\sqrt{p_V^{model}}}. \qquad (2)$$

Such mapping of multi-dimensional color space to a scalar quantity, such as albedo, is not new to astronomy (e.g., photometric redshifts for galaxies and photometric metallicity for stars).

Asteroid size estimates with optical colors:

A Gaussian Mixture model that accounts for Gaussian measurement errors is known in astronomy as the Extreme Deconvolution (XD) method[1] The XD method was applied to the 4-dimensional

[1] See https://github.com/jobovy/extreme-deconvolution

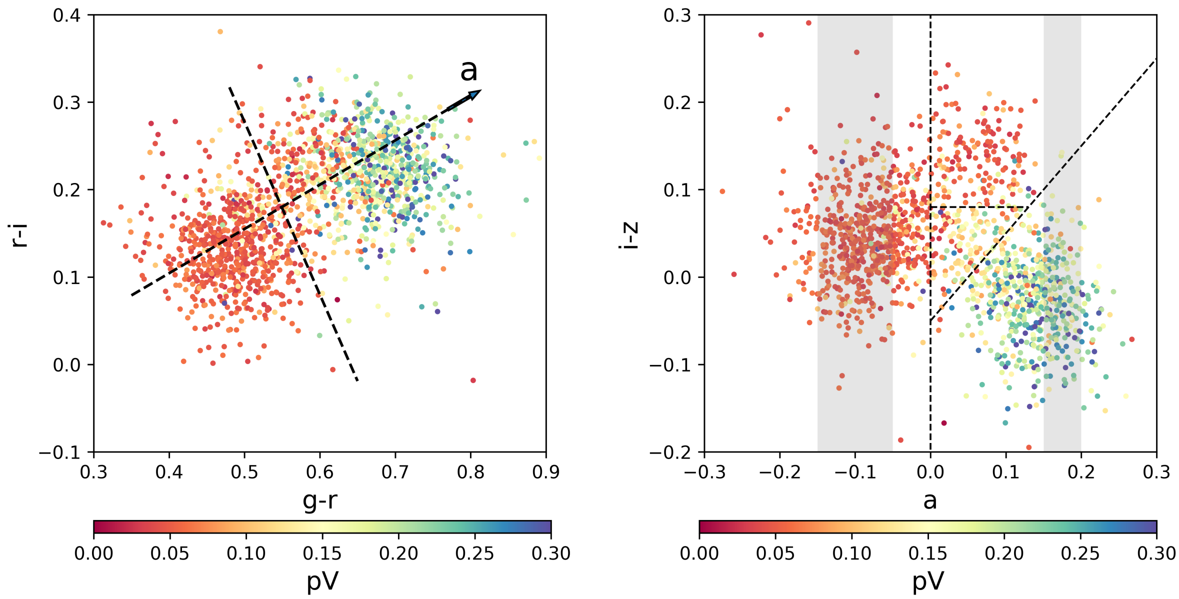

Figure 1: The SDSS color-color distributions for 1,557 asteroids that also have WISE-based albedo estimates. The symbols are color-coded by the values of albedo p_V, as marked at the bottom. The dashed lines in the left panel show the principal axes that define the a color, used in the right panel. The dashed lines in the right panel outline color regions with nearly uniform albedo distributions. The two gray rectangles mark the a color range that can be used to select clean subsamples of C type (left) and S type (right) asteroids. This figure was generated using *this python notebook*.

space spanned by the SDSS $g-r$, $r-i$ and $i-z$ colors, and the WISE-based albedo p_V. The number of Gaussian components was set to ten, which was found to be sufficient to capture details in the data; given the sample size, there is no danger of overfitting with that many components.

The performance of size estimates based on XD mapping is illustrated in Figure 2. The SDSS-based diameters obtained with the XD method match WISE-based values with a scatter of 15% and a reasonably Gaussian distribution. Compared to the accuracy of WISE-based size estimates of 15-20%, the implied accuracy of optical size estimates is thus only a factor of 1.3 to 1.4 worse.

Findings and Implications for Rubin Observatory LSST:

In IvIv21, we revisited a correlation between SDSS optical colors and optical albedo derived using WISE-based size estimates and developed several improved methods to estimate asteroid sizes with optical data alone. The best-performing approach uses the so-called Extreme Deconvolution method, a Gaussian mixture model that accounts for measurement errors, to clone a large sample statistically consistent with the data, and then assigns the best-fit albedo and its uncertainty using nearest neighbors. Optical color-based size estimates, calibrated to agree with WISE-based size estimates with a precision of 15%, deliver size accuracy in the range 21-25% after addition of WISE-based accuracy, 15-20%, in quadrature. Therefore, **the accuracy of optical color-based size estimates is only a factor of 1.3 to 1.4 worse than for IR-based estimates when sufficiently accurate multi-band optical photometry and an IR-based training sample are available**.

This size estimation accuracy is significantly better than commonly assumed for optical data. For example, the recent National Academy of Sciences report “Finding Hazardous Asteroids Using Infrared and Visible Wavelength Telescopes” [1], conservatively assumed that optical size estimates are as much as a factor of 4 worse than infrared-based size estimates, which may impact some of the conclusions presented there. The significantly better optical size estimation accuracy demonstrated in IvIv21 is due to accurate and homogeneous multi-band photometry delivered by SDSS, the large accurate calibration sample delivered by the WISE survey, and adequate data-

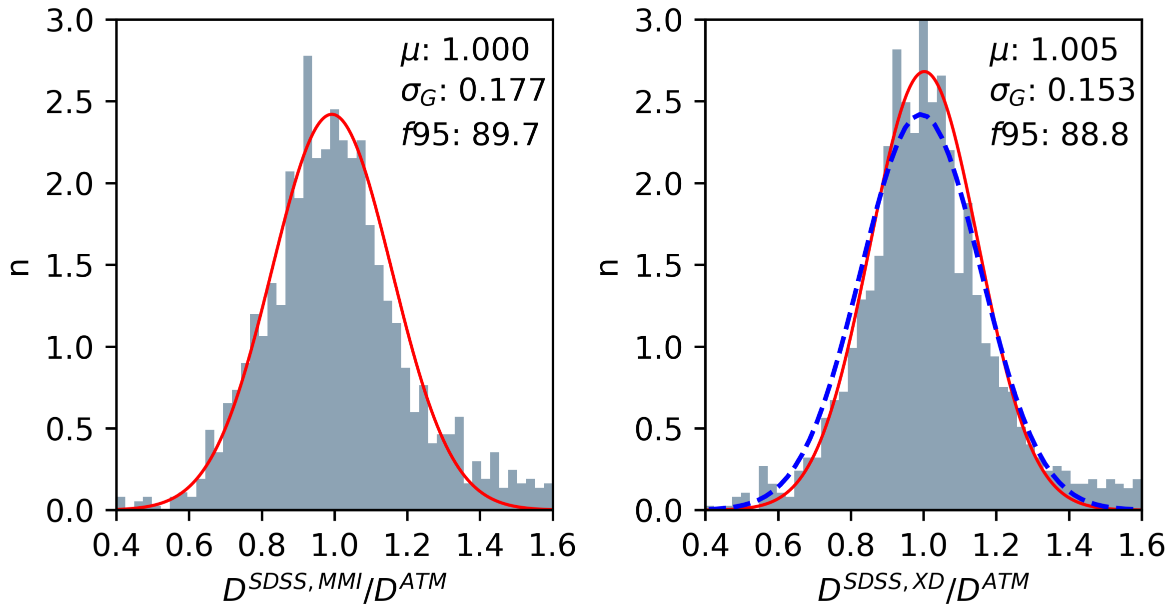

Figure 2: A comparison of size estimate uncertainties for a simple mapping method (left panel) and the XD method (right panel). The insets list the mean (μ), robust standard deviation (σ_G) and the fraction of sample within 2σ_G from the median (f_{95}, 95% for normal distribution). The solid lines show the corresponding normal (Gaussian) distributions, $N(\mu, \sigma_G)$. The dashed line in the right panel is the same as the solid line in the left panel and illustrates the improved performance of the XD method. The SDSS-based diameters obtained with the XD method match WISE-based values with a scatter of 15% and a reasonably Gaussian distribution. This figure was generated using ***this python notebook*****.**

driven machine learning methods for mapping colors to albedo discussed there.

These findings bode well for future asteroid studies. For example, the Rubin Observatory Legacy Survey of Space and Time (LSST) will obtain time-resolved astrometric and photometric data for about 6 million asteroids [9]. Detailed simulations show that LSST will obtain about 200-300 photometric measurements per asteroid during its 10 year survey (see Section 5 in [10] and [11]). Given the small color variability, all bands can be combined together in order to estimate the rotational period and fit the light curve, yielding a color accuracy of the order 0.01 mag for sufficiently bright asteroids, and not worse than 0.05 mag for faint objects. This color accuracy will be sufficient to obtain size estimates better than 25% for the majority of objects.

Lastly, motivated by a desire to enable transparency and reproducibility of results, we publicly release all the data and Jupyter Notebook files with supporting Python code used to perform analysis presented here. They are available from this GitHub[2] site.

References: [1] National Academies of Sciences, Engineering, and Medicine (2019) *Finding Hazardous Asteroids Using Infrared and Visible Wavelength Telescopes* The National Academies Press, Washington, DC ISBN 978-0-309-49398-7 `doi`. [2] A. Parker, et al. (2008) *Icarus* 198:138 `doi`. `arXiv:0807.3762`. [3] N. Myhrvold (2018) *Icarus* 314:64 `doi`. [4] E. L. Wright, et al. (2010) *Astronomical Journal* 140:1868 `doi`. `arXiv:1008.0031`. [5] A. Mainzer, et al. (2015) in *Asteroids IV* (Edited by P. Michel, et al.) 89–106 University of Arizona Press, Tucson, AZ `doi`. [6] N. Myhrvold (2018) *Icarus* 303:91 `doi`. [7] J. Moeyens, et al. (2020) *Icarus* 341:113575 `doi`. [8] V. Ivezić, et al. (2021) *Icarus* 357:114262 `doi.arXiv:2007.05600`. [9] Ž. Ivezić, et al. (2019) *Astrophysical Journal* 873:111 `doi.arXiv:0805.2366`. [10] LSST Science Collaboration, et al. (2009) *arXiv e-prints* arXiv:0912.0201.`arXiv:0912.0201`. [11] R. L. Jones, et al. (2018) *Icarus* 303:181 `doi.arXiv:1711.10621`.

[2]See `https://github.com/ivezicV/2share/tree/master/AsteroidPaper`