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OPTIMAL IMPULSIVE/LOW-THRUST TRAJECTORIES FOR ASTEROID DEFLECTION VIA KINETIC IMPACT

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Extended Abstract—

Introduction

This work considers the trajectory optimization of a kinetic impactor spacecraft, which is sent to collide with a threatening near-Earth asteroid. As a result of the impact, the subsequent path of the asteroid is very modestly changed. A modest body of work on this subject is now growing as it appears to becoming perhaps the most feasible method of hazard mitigation, especially after the success of the DART mission[1-6].

The goal, in those works and in this, is to maximize the perigee radius of the deflected asteroid (in this instance Apophis) at its closest approach to Earth. Here the important variables such as the date of Earth departure, the direction of the departure, the thrust program for the low-thrust motor, and the date of the collision are all optimization parameters. The mission is assumed to be qualitatively similar to that of the DART mission; it departs Earth on a local hyperbolic trajectory and then uses low-thrust electric propulsion for the heliocentric phase until impact. High fidelity is achieved by using the SPICE ephemeris for the motion of the asteroid target, the motion of the Earth, and the positions of the planets as needed to determine their perturbing effects on the spacecraft trajectory. To avoid a loss in accuracy of the amount of deflection obtained, at the time of close approach to Earth the deflection is obtained by using the system state transition matrix and the small, known change in the velocity of the asteroid as a result of the earlier impact.

The problem is solved using Particle Swarm Optimization (PSO), a swarm intelligence method that requires that the optimization be transcribed into a parameter optimization problem with a modest number (i.e. 10's) of free parameters[7-9] . This is accomplished in part by assuming *a priori* that the programs for the history of the in-plane and out-of-plane thrust pointing angles can be represented by 5th degree polynomials in (flight) time. Since the PSO has no native method for incorporating equality constraints the constraint that enforces interception, i.e. impact, is included as a penalty function in the objective. The PSO solution, which is necessarily sub-optimal because of the assumed form for the thrust pointing histories, has in a few cases been confirmed using a separate numerical optimization approach. In this "direct" solution the continuous optimal control problem is transcribed into a nonlinear programming (NLP) problem, now using many 100's of NLP parameters, and the equations of motion become nonlinear equality constraints.

This transcription required the development of a Runge-Kutta (RK) parallel-shooting code, implemented in MATLAB for the first time[10-11] . When the PSO solution is used as the required initial guess for the NLP problem the results are virtually the same, showing that the "true" optimal thrust pointing is in fact well approximated by the smooth 5th degree polynomials assumed.

Method

The structure of the simulation is shown in Figure 1.

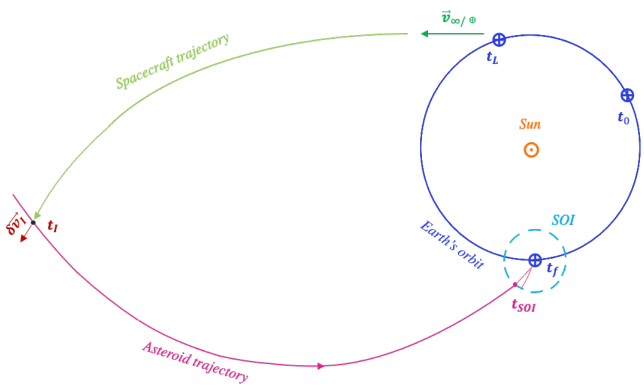

Figure 1. Cartoon showing the simulation plan.

The steps in the numerical simulation/optimization are:

1) Earth departure; with date and direction chosen by optimizer,
2) L-T electric propulsion with thrust direction program chosen by optimizer,
3) Interception/collision "constraint" satisfied on date chosen by optimizer,
4) Impact causes very small δv, which depends on relative velocity, remaining mass of s/c, and impact characteristics,

$$\delta v_0 = \frac{m_{s/c}(v_{s/c} - v_{\circledast})}{m_{\circledast} + m_{s/c}}$$

The impact is assumed inelastic with no benefit from ejecta. Thus, the resulting deflection is likely a lower bound for what would actually occur.

5) Asteroid continues on ephemeris-generated trajectory
6) At Earth SOI, s/c heliocentric position and velocity vectors and the TOF (time of flight since launch) allow determination of STM coefficients. The method and details are in Battin[12] . Then

$$\begin{bmatrix} \delta \vec{r} \\ \delta \vec{v} \end{bmatrix} = \begin{bmatrix} \tilde{R} & R \\ \tilde{V} & V \end{bmatrix} \begin{bmatrix} \delta \vec{r}_0 \\ \delta \vec{v}_0 \end{bmatrix}$$

where δv_0 is the impact-caused change in velocity.

New

$r = r + \delta r$
$v = v + \delta v$

7) The asteroid motion is then integrated forward until close approach. The deflection is the increase from the nominal close approach distance.

Governing Equations

The system spacecraft equations of motion are written in Cartesian coordinates to simplify the many instances in which the SPICE ephemeris is used, e.g. for the determination of the perturbing planetary attractions, which depend on instantaneous planetary position, and for the formulation of the asteroid impact constraint, which requires the position of the target asteroid.

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v}_x = -\frac{\mu_\odot x}{r^3} + \frac{T_x}{m} + a_{x(♀)} + a_{x(⊕☽)} + a_{x(♂)} + a_{x(♃)} \\ \dot{v}_y = -\frac{\mu_\odot y}{r^3} + \frac{T_y}{m} + a_{y(♀)} + a_{y(⊕☽)} + a_{y(♂)} + a_{y(♃)} \\ \dot{v}_z = -\frac{\mu_\odot z}{r^3} + \frac{T_z}{m} + a_{z(♀)} + a_{z(⊕☽)} + a_{z(♂)} + a_{z(♃)} \\ \dot{m} = -\frac{T_{max}}{c_{exh}} \end{cases}$$

Planetary perturbations are from attractions of Venus, Earth-Moon, Mars, Jupiter.

The thrust components T_x , T_y , T_z are functions of an in-plane pointing angle β and out-of-plane pointing angle γ.

Optimization

The optimization is accomplished with two qualitatively different methods.

- PSO (particle swarm optimization)[7-9]

  A heuristic method.
  Has the benefit of being initialized randomly, i.e. no initial guess needed.
  "Particles" are N-dimension potential solutions
  Particles move in N dimensional search space, to improve their cost
  Particles "communicate"; all learn best location known to the swarm.

Continuous controls need to be expressed as a function of a small number of parameters. For this simulation, the thrust pointing angles are represented by 5^{th}-degree polynomials in TOF.
No native way to incorporate constraints; need to use penalty functions
For this problem there are 16 PSO parameters; 12 thrust angle polynomial coefficients, two V∞/*Earth* departure angles, departure date, collision date.

- R-K (Runge-Kutta) Parallel Shooting[10,11]

In this method the optimization problem is transcribed into a NLP problem.
The TOF is divided by a large number of equally spaced "nodes". The state and control variables at each node become NLP variables; typically there are several hundred such parameters.
There are a small number of additional NLP parameters such as departure date, two V∞/*Earth* departure angles, and date of impact at the asteroid.
The system EOM are enforced by stepping forward from one node to a subsequent node by using the explicit 4-step R-K procedure. If the resulting states do not agree with the current values of the corresponding states that becomes a nonlinear constraint that the solver needs to force to zero.
In addition, candidate trajectories must satisfy a nonlinear interception "constraint", i.e. that when the spacecraft crosses the asteroid path the asteroid is precisely at that point. Thus, unlike PSO, this method does not require the use of a penalty function to enforce the collision and does not require the parameterization of the control history.

<u>Example and Results</u>

Test case is deflection of 99942 Apophis. Apophis close approach is 13 April 2029.

S/C Initial thrust accel. = 18 x 10^{-6} g
Exhaust velocity = 29.78 km/sec (Isp = 3035 sec
V∞/Earth = 1.8 km/sec
Initial S/C mass = 10000 kg
Epoch date is 1/1/2026
Optimizer chooses:

Departure date of 11/13/2026 (i.e. it waits 317 days from Epoch for geometry to improve)
Impact date of 1/19/2028
S/C mass remaining at impact = 7764 kg
Impact results in deflection of 1267 km

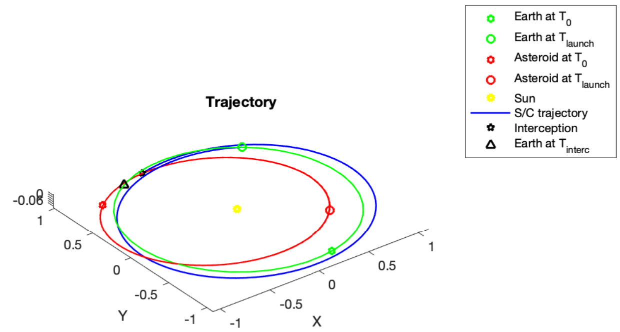

Figure 2. Asteroid (Apophis) and spacecraft trajectories

Figure 3 shows the optimal thrust pointing angle time histories for this example. Both are represented by 5^{th} degree polynomials in TOF. Thus the PSO has chosen 6 coefficients for each angle history.

Note from Table 1, for the row corresponding to this example, that the PSO is able to satisfy the interception (collision) constraint to O(10^{-11}) AU. Since 1 AU = 1.49 x 10^{11} m
That means the interception "error" is on the order of 1 meter.

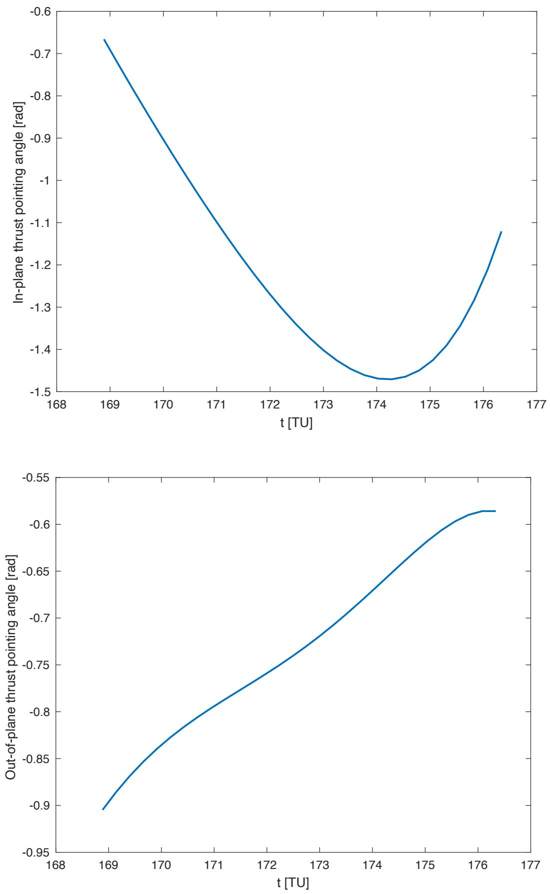

Figure 3. In-plane (left) and out-of-plane thrust pointing angle histories (right) during spacecraft interception trajectory

Additional Results

Table 1 shows the results of other simulations with different S/C thrust magnitude and departure . With one exception, the deflection obtained decreases as the spacecraft becomes less capable, i.e. has lower thrust or lower departure hyperbolic excess velocity. The optimal impact date does not change, which is somewhat surprising, rather the departure date is moved forward to accommodate a vehicle with less capability. The result was tested by changing the bounds of the PSO flight time parameter so as to exclude 1/19/2028 and a successful solution was obtained, but with a smaller deflection.

Confirmation of PSO result with transcription into NLP problem

To confirm the PSO result a small number of cases were also optimized via the qualitatively different numerical optimization method of R-K parallel shooting previously described. The example below, also for deflection of Apophis in 2029, is a direct comparison of the results from the two different numerical optimizers.

S/C Initial thrust accel. = 30 x 10^{-6} g
Exhaust velocity = 29.78 km/sec (I_{sp} = 3035 sec)
$V_{\infty/Earth}$ = 1.8 km/sec

Initial S/C mass = 10000 kg

Epoch date is 1/1/2026

R-K result

Departure date of 12/30/2026
Impact date of 1/19/2028
S/C mass remaining at impact = 6674 kg
Interception (collision) error = 5.3E-8 AU
Impact results in deflection of 1376 km

PSO Result
Departure date of 12/30/2026
Impact date of 1/19/2028
S/C mass remaining at impact = 6674kg
Interception (collision) error = 7.5E-7 AU
Impact results in deflection of 1371 km

The results from the two numerical optimizers are virtually the same. Figures 4 & 5 show the thrust pointing angle histories for the two optimizers. Note again that the PSO result, because PSO can only optimize a modest number of parameters, requires that the histories be described by a small number of parameters, in this case 6 coefficients (each) of a 5th degree polynomial in TOF. On the contrary, the R-K parallel shooting solution requires no *a priori* specification of the form of the solution. The fact that the solutions are yet so similar indicates that the 5th degree polynomial was a good choice for the parameterization of the thrust program.

Table 1. Optimal deflections obtained with various S/C thrust magnitude and departure V∞ /Earth

T_{max}/m_0 (10^{-6} g)	V ∞/Earth (km/s)	Deflection (km)	Interception (AU)	Departure	Impact
30	1.80	-1371	7.50E-07	12/30/2026	1/19/2028
24	1.80	-1361	7.40E-12	12/11/2026	1/19/2028
18	1.80	-1267	3.30E-11	11/13/2026	1/19/2028
18	1.65	-1217	2.90E-11	11/10/2026	1/19/2028
18	1.50	-1147	7.80E-12	11/7/2026	1/19/2028
12	1.50	-846	2.80E-10	10/11/2026	1/19/2028
12	1.35	-828	4.00E-11	10/14/2026	1/19/2028
12	1.20	-851	9.00E-12	10/22/2026	1/19/2028

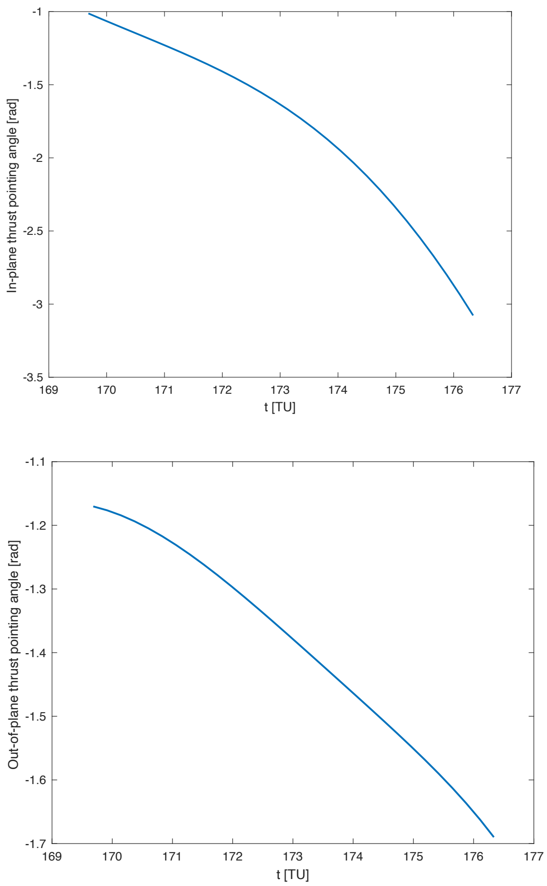

Figure 4. PSO result for in-plane (left) and out-of-plane thrust pointing angle histories (right) during spacecraft interception trajectory

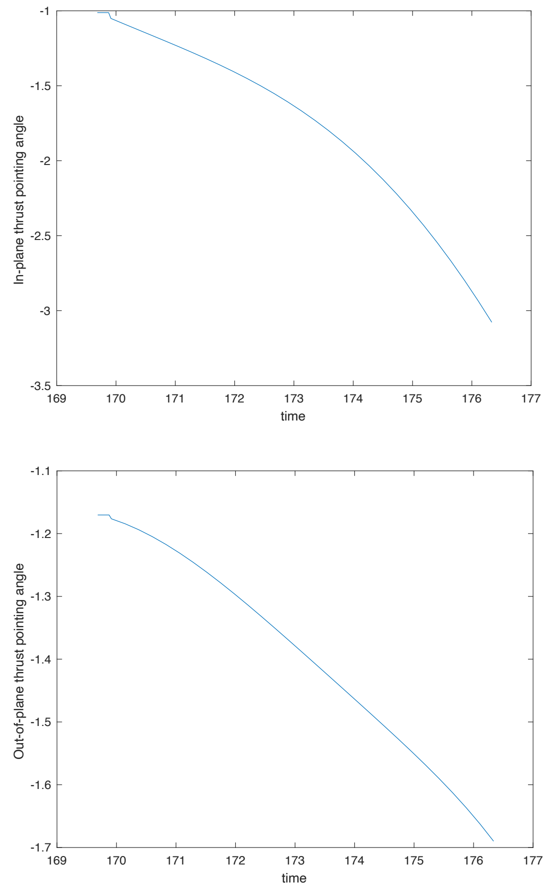

Figure 5. R-K parallel shooting (NLP transcription) results for In-plane (left) and out-of-plane thrust pointing angle histories (right) during spacecraft interception trajectory

Conclusions

A heuristic (PSO) optimizer has successfully found optimal strategies for asteroid deflection missions.

This solution method is straightforward and benefits from not needing to require an initial guess, which can prejudice convergence to a local minimum.

A qualitatively different optimization method, similar to collocation, in which the problem is converted to a (large) NLP problem, has confirmed the solution obtained by PSO.

The use of the system STM is simplifying and also adds to accuracy, since forward integration of the EOM post-collision is numerically difficult because the delta-V caused by the impact is only a fraction of 1 m/sec.

Interestingly, for the case of Apophis, the optimizer chooses a lengthy wait time before departure, in order to improve the relative geometry of Earth and Apophis.

With present technology a 10,000 kg spacecraft, given a lead time of about 2 years, impacting Apophis, can cause a deflection on the order of 1300 km. This is likely a lower bound as it does not assume any benefit from momentum transfer to ejecta.

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