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Monte Carlo Simulation of the Cohesion Weakening Friction Strengthening Approach for Assessment of Brittle Failure Around Underground Excavations

L.F. Gomez de Alba
BGC Engineering, British Columbia, Canada
fgomezdealba@bgcengineering.ca

A.G. Corkum & N. Bahrani
Dalhousie University, Halifax, Canada

D. Mas-Ivars
SKB (Swedish Nuclear Fuel and Waste Management Co), Stockholm, Sweden KTH (Royal Institute of Technology), Stockholm, Sweden

Abstract

The Cohesion Weakening Friction Strengthening (CWFS) approach is a robust continuum numerical modelling technique used to simulate the depth and shape of brittle failure around underground excavations. The CWFS method is based on the underlying mechanisms of the mobilization of shear strength components of rock, cohesion and friction, which change gradually as a function of plastic strain. In practice, it is often applied using deterministic parameters. However, given the inherent uncertainty associated with the variability and heterogeneity of rock properties, the strength parameters required to implement the CWFS approach for design purposes may be better characterized using a stochastic approach. In this paper, a Monte Carlo Simulation (MCS) was utilized to assess the CWFS parameters for the well-known Mine-by Experiment (MBE) at the Canadian Underground Research Laboratory (URL). The MCS was applied to the CWFS strength parameters using two different sets of statistical distributions. The first set consisted of uniform distributions, and the second included more complex distributions based on observed brittle rock parameters. The models were run for 10,000 iterations, after which the depth, area, and angle of failure were characterized and compiled. The results were then analysed and graphically synthesized to produce a probabilistic-based failure profile. It is shown that applying an MCS for the CWFS approach is not only viable, but also provides insight seldom revealed by deterministic methods. The results were also analysed to better understand the sensitivity of CWFS parameters by plotting correlations between model response (e.g., failure measurements) and input strength parameters. These results enable a more informed risk assessment of potential field conditions, allowing for improved risk management and ground support design.

Keywords

Brittle Rock, Monte Carlo Simulation, CWFS, Tunnelling

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1 Introduction

The Cohesion Weakening Friction Strengthening (CWFS) approach is often used as a deterministic method to simulate brittle failure around underground excavations (Hajiabdolmajid et al. 2002; Gomez de Alba et al, 2024). However, rock strength characterization is more accurately described as a probabilistic endeavour. This paper proposes to use some probabilistic techniques in combination with the CWFS approach to develop a methodology that provides rock mechanics modellers with a better representation of ground response. This methodology can help engineers identify probabilistic scenarios relevant to their specific situations. To achieve this, the Monte Carlo Simulation (MCS) was applied alongside the CWFS method. The Mine-By Experiment (MBE) case scenario was used to test this approach with MCS, resulting in a more informative probabilistic failure profile and a corresponding probability of failure profile. These outcomes can be used for ground support design and risk assessment. The results can be further analysed to improve the understanding of the CWFS method itself. The correlation between the failure profile and input parameters were calculated to identify which parameters have the greatest impact on the results of the CWFS method.

2 Background

Predicting and simulating brittle rock failure around underground excavations is a challenging endeavour. Traditional strength characterization methods used for other types of failure, such as the Hoek-Brown failure criterion, cannot be directly applied to brittle rocks (Kaiser et al, 2000). To address this limitation, different techniques have been developed over the years. One of the most well-established methods is the Cohesion Weakening Friction Strengthening approach. This method was first used by Hajiabdolmajid et al. (2002) and Hajiabdolmajid et al. (2003) to model brittle failure around underground excavations, building on the experimental findings of Martin and Chandler (1994) and the theoretical framework proposed by Martin (1997). The CWFS method is based on the principle that intact rock behaves as a fully cohesive material with negligible frictional resistance. In the field, the intact rock strength is primarily governed by its cohesion, which corresponds to the crack initiation strength (σ_{ci}) of the rock. Once cracks initiate, the cohesion progressively decreases while friction increases. As normal stress continues to develop, friction further mobilizes until a fully frictional state is reached. These behaviours are strain-dependent, which is a critical aspect of the CWFS approach. This method is often used in continuum numerical programs that allow for strain-dependent parameter evolution, such as *FLAC2D* and *FLAC3D*. Several key parameters are required to define the CWFS: cohesion is characterized by its peak (c_p) and residual (c_r) values, while friction is described by the initial (ϕ_i) and mobilized (ϕ_m) friction angles. There are critical plastic strain parameters that control the evolution of strength properties: one governs cohesion (e_c^{ps}) and another governs friction (e_ϕ^{ps}). The dilation behaviour is also described by a dilation angle (ψ). To facilitate the effective selection of these parameters, Walton (2019) developed a set of guidelines for the use of the CWFS method.

3 Methodology

To obtain a probabilistic representation of the failure profile around an underground excavation, an MCS was applied to a case study. The first step involved running *n* numerical models using the CWFS approach in FLAC2D with different probabilistic input parameters. The strength parameters were selected in two ways: first, by assuming a uniform distribution for all parameters, and second, by employing more complex distributions that correlate with field and laboratory observations of the rock. After running the models the results are stored and analysed. The failure profiles generated were then compared with field observations from the case study. The case study used in this analysis is the Mine-by Experiment (MBE) at the Underground Research Laboratory (URL) in Manitoba, Canada. The laboratory properties of rock are reported by Martin and Kaiser (1996) and Martin et al. (1997).

3.1 Monte Carlo Simulation

The Monte Carlo Simulation followed standard methodologies for geotechnical problems, as outlined by Martin and Christiansson (2009) and Hoek (2023). The MCS was executed in *Python* via *FLAC2D*'s integrated *Python* interface. The process begins with iteration *i*, where all input parameters of interest are randomized, within the specified ranges, using the *NumPy* library (Harris et al., 2020). These randomized parameters are then applied to the material model in the domain of interest in *FLAC2D*. The *FLAC2D* model runs until equilibrium is reached, after which all failure metrics and state of each zone are compiled and stored in *Python* arrays. The *Python* script proceeds to the next iteration (i + 1), repeating the same process. For this research, 10,000 iterations were conducted for each MCS. Once all iterations were completed, the results were analyzed to create failure distributions and probability of failure profiles.

3.2 Numerical model

The numerical models were run in *FLAC2D* (v9.0.164). The model geometry is shown in Fig. 1. To reduce running computational time, the MBE was modelled as a quarter circle, leveraging symmetry to represent the full circle. The effects of this simplification should be negligible, as the behaviour of the boundaries should produce the expected symmetric response. The model dimensions are 20 m × 20 m, with the excavation located at the bottom left corner of the model. The excavation is circular, with a radius of 1.75 m. Near the excavation boundary, there is a refined radial mesh with square zones of 2.5 cm × 2.5 cm adjacent to the boundary extending to 10 cm × 10 cm zones at 5 m from the centre of the excavation. A coarse mesh exists from 5 m away from the centre of the excavation to the model extent at 20 m from the excavation. The left and bottom boundaries are rollers to recreate the symmetry, and the top and right boundaries are fixed.

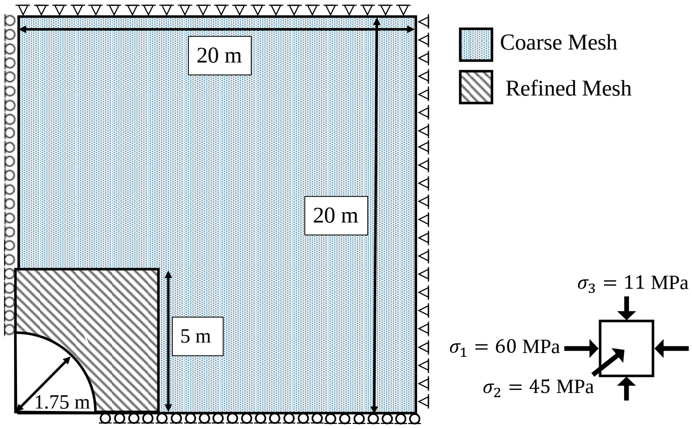

Fig. 1: Numerical model set up for the simulation of the MBE using the CWFS method and MCS

The stresses are introduced to the model by initializing them in all zones and allowing the model to reach equilibrium prior to the removal of the excavation material. The stress regime is based on that described by Martin and Kaiser (1996), where $\sigma_1 = 60$ MPa, $\sigma_2 = 45$ MPa, and $\sigma_3 = 11$ MPa. The stresses were rotated to have σ_1 a plunge of 0° and σ_3 a plunge of 90°, while σ_2 is aligned with the direction of the excavation. The excavation was simulated using *FLAC2D*'s "zone relax" command to avoid dynamic loading from sudden removal of material. The material behavior was modelled using the Strain-Softening model to allow for strain-dependent evolution of the strength parameters. For the analysis of the results, the state of the zones was used as an indicator of instability. Any zone that has reached a failure state is regarded as instability in the area surrounding the excavation.

3.3 Probability distributions

In the MCS, certain parameters are treated as random variables, that can assume different values in each iteration. For this research, the parameters varied are c_p, c_r, ϕ_i, ϕ_m, and e_c^{ps}. Two MCS approaches were used: one where these parameters followed a uniform distribution (each value having an equal probability of occurrence), and another with more complex distributions based on rock behaviour observations. While some parameters were varied, others were treated as deterministic or dependent on other variables. This simplification reduces computational time and avoids numerical instability. The assumptions are as follows: the tensile strength (σ_t) was fixed at10 MPa for all iterations, the dilation angle (ψ) followed an associated flow rule, meaning that it varies with ϕ, and finally, $e_\phi^{ps} = 2 \times e_c^{ps}$.

3.3.1 Uniform distributions

To first demonstrate the application of the MCS, a simple set of uniform distributions was used. The objective is to assess the capability of the CWFS method to be represented with probabilistic values and to provide a basic representation of the probability of failure. The parameter values were selected according to the guidelines provided by Walton (2019), the model calibration carried out by Hajiabdolmajid et al. (2002) and Hajiabdolmajid et al. (2003), and the sensitivity analyses conducted by Gomez de Alba (2024). The maximum and minimum values selected for the uniform distributions are summarized in Table 3-1.

Table 3-1 Uniform distributions input parameter ranges

Parameter	Minimum	Maximum
c_p (MPa)	39	61
ϕ_i (°)	0	10
e_c^{ps}	0.001	0.003
c_r (MPa)	7.5	30
ϕ_m (°)	47	60

3.3.2 Non-uniform distributions

The strength parameters for the CWFS method can also be represented using more complex, non-uniform distributions, which would allow the input parameters to more closely approximate rock strength observations from laboratory experiments or field studies.

The parameter c_p is correlated with the crack initiation stress of the rock, which is typically assumed to be between 30% and 50% of the rock's UCS (Walton, 2019). Given that UCS from laboratory experiments is often normally distributed, the c_p can also be assumed to follow a normal distribution. For the Lac du Bonnet (LdB) granite, the main rock unit in the MBE, the UCS has a Coefficient of Variation (COV) of 0.11 (Martin, 1993). Previous applications of the CWFS approach to model the MBE have shown that the c_p value of 50 MPa provides the best fit to the failure profile in the field (Hajiabdolmajid et al., 2002, 2003). Based on these observations, c_p is assumed to follow a normal distribution with a mean (μ) of 50 MPa and a standard deviation (σ) of 6 MPa. The parameter c_r is abstract and hard-to-quantify or measure directly (Walton, 2019). Therefore, for this research, a uniform distribution is selected for c_r with a minimum value of 7.5 MPa and a maximum value of 30 MPa. These values are based on a combination of the guidelines provided by Walton (2019) and the calibration and best fit parameters for the MBE case by Hajiabdolmajid et al. (2002, 2003).

The parameter ϕ_i represents the friction angle of the crack initiation strength envelope (Walton, 2019). The laboratory experiments by Martin (1993) suggest that the crack initiation strength envelope for LdB granite is given by:

$$\sigma_{ci}(\text{MPa}) = 49 + 0.39\sigma_3 \tag{1}$$

where σ_{ci} is the crack initiation threshold. This suggests that ϕ_i for this rock is 0°. However, since this is a best-fit line, it can be more adequately represented as a distribution. For this paper, the distribution is assumed to be triangular, with a minimum value of 0°, a mode of 0°, and a maximum value of 10°. This distribution approximately corresponds to a 95% confidence interval for Eq. (1). The parameter ϕ_m is suggested to be a function of the Hoek-Brown value m_i for laboratory specimens (Walton, 2019). Martin (1993) conducted triaxial tests on LdB granite to determine the m_i values, which were then input into RocData by Rocscience to calculate the 95% confidence intervals. Based on these results and the guidelines provided by Walton (2019), the distribution for ϕ_m was determined to be triangular, with a minimum of 47°, a mode of 50°, and a maximum of 60°.

The last parameter with a probabilistic distribution is e_c^{ps}, which has been identified as an intrinsic property of the rock and can be measured through cyclic testing (Martin and Chandler, 1994). However, due to high cost and extensive nature of these tests, a simplified uniform distribution was assumed for this study. According to Walton (2019), the range of critical strain for crystalline rock typically falls between 0.001 and 0.003. Therefore, these values were used as the minimum and maximum bounds for the uniform distribution in this study.

3.4 Failure characterization

Three failure metrics were used to characterize the predicted failure profile. The first metric is the depth of failure, d_f, as defined by Martin (1997). The parameter d_f is measured from the crown of the tunnel to the tip of the furthest extent of failure (i.e., notch tip). In the numerical models, this is measured from the excavation boundary to the outermost failed zone. The second metric is the angle of failure, measured from the first failed zone on the right along the boundary of the excavation to the last failed zone. The final metric is the area of failure, calculated by summing the areas of all failed zones. Fig. 2 shows these measures. Each metric is recorded and stored for every iteration.

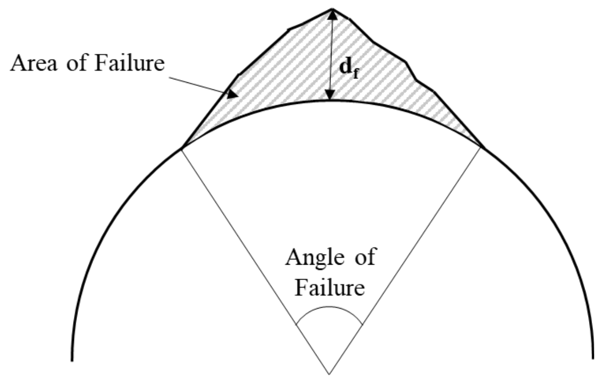

Fig. 2: Representation of the measures of failure d_f, area of failure, and angle of failure

Further analysis can be conducted by examining the failure characterization in relation to the random parameters. By comparing the random parameters of each iteration with d_f, a correlation can be established. For example, the c_p value can be plotted against the d_f, and the resulting plot can be analysed to identify the correlation between failure and its strength. These correlations are assumed to be linear, and their significance are assessed based on the R value and R^2 value. This approach helps identify which parameters have the largest effect on the results. Through these correlations, further insight into the CWFS approach can be gained, providing future users with a clearer understanding of how each parameter affects the model outcomes.

3.5 Probability of failure

As a means of providing guidance for support design and risk assessment, the results of the MCS can be visualized by showing the probability of failure for each zone. The methodology used for this measurement involved recording the state of each zone, failed or not failed, and storing it in an array. This process was repeated for all iterations and stored in *Python*. Once the 10,000 iterations were executed, the average probability of failure of each zone was calculated. This probability of failure was then overlaid onto the numerical model for graphical visualization

4 Results

4.1 Uniform distributions

The statistical failure characterization is shown in Fig. 3. The predicted d_f has μ of 0.50 m and a σ of 0.19 m. This represents a slight underestimation compared to the actual d_f of 0.52 m. As shown in Fig. 3a, the distribution of d_f is right-skewed and approximately normal. The area of failure distribution shown in Fig. 3b is also right-skewed and approximately normal with a μ of 0.44 m^2 and σ of 0.22 m^2. Lasty, the angle of failure, illustrated in Fig. 3c, follows a normal distributed with an μ of 71° and a σ of 9.8°. These results demonstrate that, when using uniform distributions, the mean predicted failure profiles tend to be slightly smaller than expected.

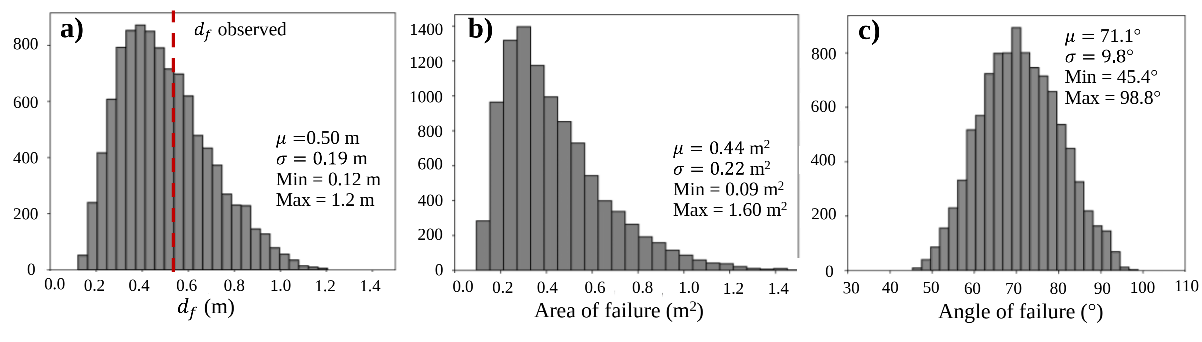

Fig. 3: Distributions of the measures of failure resulting from the MCS with uniform distributions for the input parameters a) d_f, b) area of failure, and c) angle of failure

The results of the correlations between d_f and the probabilistic parameters are summarized in Table 4-1. The two parameters with the highest influence are c_p and ϕ_i with R values of -0.81 and -0.48, respectively. These correlations are shown in Fig. 4. All other parameters exhibit negligible correlation levels. The correlation between c_p and d_f reveals that as cohesion increases, there is a significant decrease in the depth of failure, with a high level of correlation. Therefore, c_p appears to be a critical parameter for calibration when applying the CWFS approach.

Table 4-1 Empirical correlations, R, and R^2 of the strength parameters and the d_f for the uniform distributions

Parameter	Empirical Correlation	R	R^2
c_p (MPa)	$d_f = -0.024c_p + 1.69$	-0.81	0.66
ϕ_i (°)	$d_f = -0.032\phi_i + 0.66$	-0.48	0.23
e_c^{ps}	$d_f = 63.0e_c^{ps} + 0.38$	0.19	0.04
c_r (MPa)	$d_f = -0.004c_r + 0.58$	-0.15	0.02
ϕ_m (°)	$d_f = -0.005\phi_m + 0.78$	-0.10	0.01

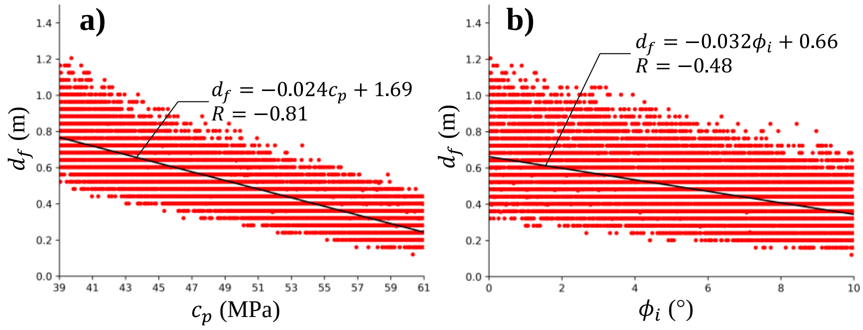

Fig. 4: Correlation for the results of the MCS with uniform distributions between: a) d_f and c_p, and b) d_f and ϕ_i

The last result to observe is the probability of failure for each zone. These results, shown in Fig. 5a, are overlaid onto the observed failure profile from the MBE, as reported by Martin et al. (1997). The probability of failure presents a more rounded profile than the "v-notch" observed in the field. This highlights the variability of potential failure scenarios and demonstrates that, under the given probabilities, the depth of failure is at least 25 cm.

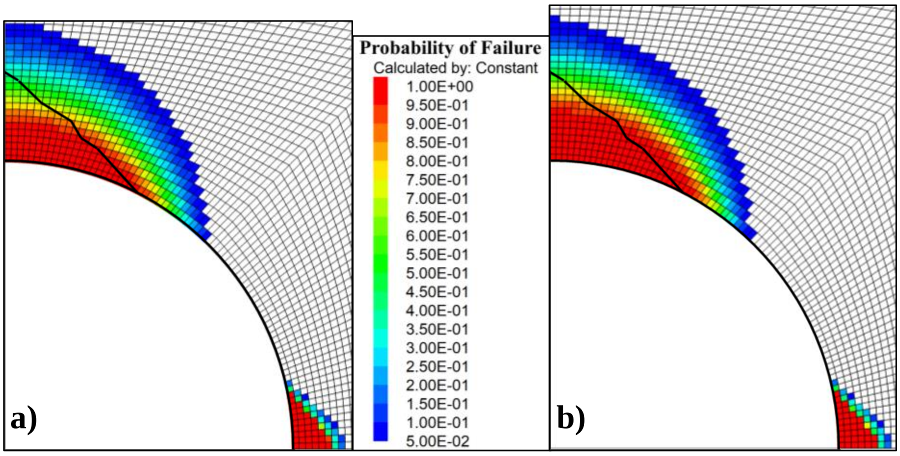

Fig. 5: Probability of failure for each zone from the MCS using: a) uniform distribution for the input parameters compared to the observed failure profile (in black), b) complex distribution for the input parameters compared to the observed failure profile (in black)

4.2 Complex distributions

The statistical characterization of the failure profile is shown in Fig. 6. The d_f follows a normal distribution with a μ of 0.56 m and a σ of 0.18 m. This mean d_f is close to the observed d_f at the MBE, giving confidence to the random parameter selection. The distribution is less skewed than that observed for the uniform distributions with a slight overestimation compared to field observations. The area of failure exhibits a right-skewed normal distribution, with a μ of 0.48 m^2 and a σ of 0.25 m^2. The angle of failure is also normally distributed, with a μ of 72.2° and a σ of 8.5°. When comparing these non-linear distribution results to those from the uniform distribution, there is no significant contrast to suggest that one approach offers a substantially better representation of the statistical distribution of failure metrics.

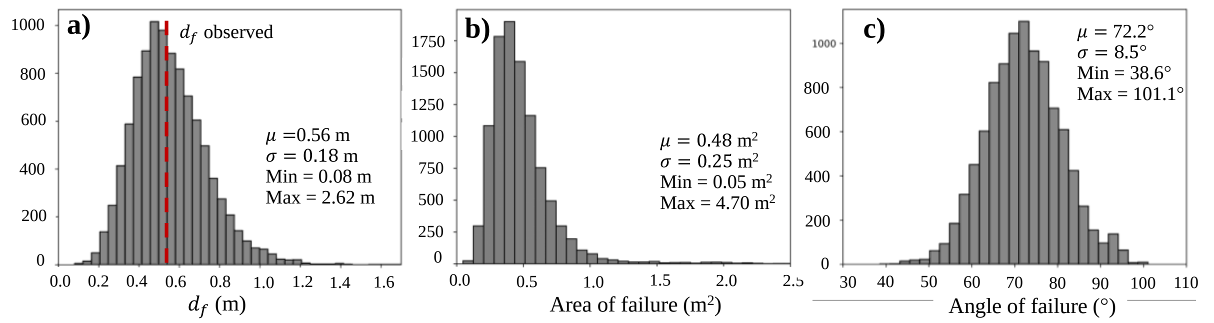

Fig. 6: Distributions of the failure metrics resulting from the MCS with complex distributions for the input parameters: a) d_f, b) area of failure, and c) angle of failure

The correlations between the strength parameters and the failure characteristics are summarized in Table 4-2. Similar to the uniform distribution case, the most strongly correlated parameters are c_p and ϕ_i, with R values of -0.81 and -0.43, respectively, as shown in Fig. 7. These results provide greater confidence that these parameters are critical for calibration for the effective application of the CWFS method.

Table 4-2 Empirical correlations, R, and R^2 of the strength parameters and the d_f for the complex distributions

Parameter	Empirical Correlation	R	R^2
c_p (MPa)	$d_f = -0.026c_p + 1.87$	-0.81	0.66
ϕ_i (°)	$d_f = -0.033\phi_i + 0.67$	-0.43	0.18
e_c^{ps}	$d_f = 76.2e_c^{ps} + 0.40$	0.24	0.06
c_r (MPa)	$d_f = -00.4c_r + 0.63$	-0.16	0.03
ϕ_m (°)	$d_f = -0.005\phi_m + 0.82$	-0.08	0.01

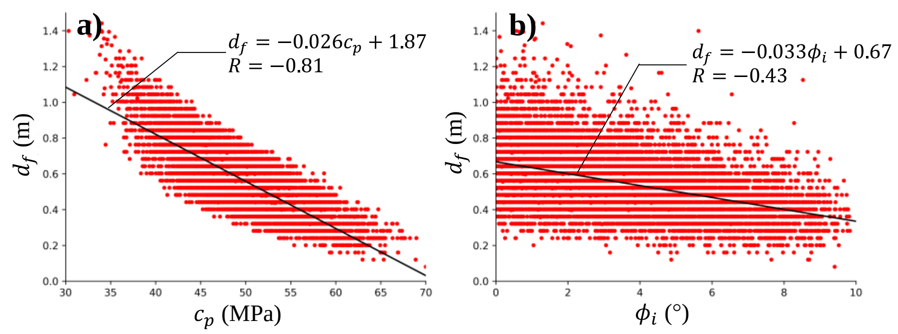

Fig. 7: Correlation for the results of the MCS with complex distributions between: a) d_f and c_p, and b) d_f and ϕ_i

Fig. 5b shows the failure probability for each zone in the MCS using the complex distributions. The results demonstrate a better correspondence with the observed d_f, with a minimum d_f of 0.35 m. It predicts a greater d_f at the lowest probability (5%) compared to the uniform distribution scenario. Similar to the previous case, the failure profile appears more rounded compared to the “v-notch” observed in the field. The roundness in both scenarios is an outcome of the numerical modelling set-up. In a physical tunnel the failed material is removed from the excavation boundary allowing for further redistribution of stresses, while in the numerical models the material, while failed, remains in the model, creating artificial confinement. A finer mesh could also allow for a less round failure profile.

5 Conclusion

The findings of this study demonstrate that using the CWFS approach combined with MCS is a viable technique for simulating underground excavations in brittle rock while accounting for uncertainty and variability. The methodology outlined in this paper can be used by rock engineering practitioners to obtain a quantifiable understanding of risk assessment and support design. A probabilistic-based failure profile offers more valuable design insights than a single deterministic failure profile. The MCS also enhances our understanding of how input parameters affect the CWFS method’s outcomes. It was demonstrated that the most influential parameters are c_p and ϕ_i, while other parameters have a more limited impact on the simulation results. Further investigation on the mesh dependency of the results is required, particularly to investigate the influence of e^{ps} on the results. This work can be further expanded by incorporating probabilistic stress fields and further analyzing the correlations between input parameters and failure characteristics. The use of the methodology outline here can be applied to other case scenarios, improving its validity and its capabilities.

References

Gomez de Alba LF, (2024). A Review and Detailed Investigation of Continuum-Based Numerical Modelling Methods for the Simulation of Brittle Rock Failure Around Underground Excavations. Dissertation at Dalhousie University, Halifax, Canada

Gomez de Alba LF, Corkum A, Bahrani N, (2024) Application of the CWFS method in FLAC2D to model brittle failure around the Qirehataer Diversion Tunnel, in: Hazzard, J., Nelson, M., Katsaga, T., Saftenberg, J., (Eds.) 6th International ITASCA Symposium on Applied Numerical Modeling in Geomechanics: 17-01

Hajiabdolmajid V, Kaiser PK, Martin CD, (2002) Modelling brittle failure of rock. International Journal of Rock Mechanics and Mining Science. 39(6): 731-741.

Hajiabdolmajid V, Kaiser PK, and Martin CD (2003) Mobilized strength components in brittle failure of rock. Geotechnique 53(3): 327-336.

Harris, CR, Millman, KJ, van der Walt, SJ, Gommers, R, Virtanen, P, Cournapeau, D, … Oliphant, TE (2020) Array programming with NumPy. Nature, 585, 357–362.

Hoek, E (2023) Practical Rock Engineering. Rocscience

Martin, CD (1993) The strength of massive Lac du Bonnet granite around underground opening. Ph.D. thesis, Department of Civil Engineering, University of Manitoba, Winnipeg, Man.

Martin, CD, (1997) The effect of cohesion loss and stress path on brittle rock strength. Canadian Geotechnical Journal 34(5): 698 - 725.

Martin, CD, Chandler, NA, (1994) The progressive Fracture of Lac du Bonnet Granite. International Journal of Rock Mechanics and Mining Science. 31(6): 643-659

Martin, CD, Christianson, R (2009) Estimating the potential for spalling around a deep nuclear waste repository in crystalline rock. *International Journal of Rock Mechanics & Mining Sciences* 46: 219-228

Martin, CD, Kaiser PK, (1996) Mine-by experiment committee report Phase 1: excavation response summary and implications. AECL-11382, COG-95-347

Martin CD, Read, RS, Martino, JB (1997) Observation of brittle failure around a circular test tunnel. International Journal of Rock Mechanics and Mining Sciences 34(7) 1065-1073

Walton, G, (2019) Initial guidelines for the selection of input parameters for cohesion-weakening-friction-strengthening (CWFS) analysis of excavations in brittle rock. Tunnelling and Underground Space Technology .84: 189-200.