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Application of Digital Image Correlation and strain inversion to determine the five elastic constants of transversely isotropic rock from a single sample by a strip load

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Abstract

This study presents application of digital image correlation (DIC) and strain inversion to determine the five elastic constants of transversely isotropic rock from a single sample by a strip load. Strain values under various stress conditions were measured using DIC during strip loading, and the five elastic constants were determined using strain inversion, which minimizes the difference between the measured strain and the strain obtained from numerical modeling. The proposed method was validated through numerical and laboratory experiments. Numerical validations were conducted under heterogeneous conditions, and laboratory experiments were performed on Onyang gneiss, Paju gneiss, and Boryeong shale using uniaxial compression tests and strip load tests. Through numerical validation, it was confirmed that the proposed method provides more accurate elastic constants at all bedding angles compared to the method using eight strain gauges and additional Brazilian test presented in the previous study. In the laboratory tests, it was validated that a similar distribution of the five elastic constants was obtained as in the uniaxial compression tests. This method provides an efficient and accurate approach for applying DIC to determine the five elastic constants of transversely isotropic rock using a single sample.

Keywords

Digital image correlation, Strip load, Strain inversion, Transversely isotropic rock, Single sample

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1 Introduction

Anisotropy is a fundamental characteristic of rock, influencing its mechanical behavior based on orientation (Amadei 1996). The most pertinent type of anisotropy in rock mechanics is the transversely isotropic model, characterized by five elastic constants demonstrating symmetry around a single axis (Amadei 1996). This model is commonly employed in sedimentary and metamorphic rocks, where minerals, pores, and other features contribute to the formation of structures such as bedding and foliation (Jaeger et al. 2009). Various studies have been conducted to determine these elastic constants using different core types of rocks. First, methods utilizing cylindrical cores drilled in various directions under uniaxial compression (Amadei 1996, Cho et al. 2012) and Brazilian tests (Chen et al. 1998, Chou et al. 2008) were proposed. A second method involving torsional tests on hollow cylindrical samples was also suggested (Talesnick et al. 1999, Nunes 2002). Lastly, a method using a single cylindrical sample has been explored, where Nejati et al. (2019) determined four elastic constants using a single sample and employed Saint-Venant's empirical formula to calculate the second shear modulus (Saint Venant 1863). However, these methods may require coring in multiple directions, specialized experimental setups, or use empirical formulas that could compromise the accuracy of the determined elastic constants.

To address these limitations, Yim et al. (2022) proposed a method using a single-orientation core of transversely isotropic rock under strip loading, which eliminates the need for empirical formulas and allows for the determination of all five elastic constants from single core. However, the accuracy of the determined constants can be sensitive to the rock's heterogeneity and influenced by the bedding angle. To overcome this, the application of Digital Image Correlation (DIC) was considered. DIC involves taking photographs of a loaded target and analyzing the movement in the captured images to measure displacement and strain, providing more extensive strain data than traditional strain measurement methods (Peters et al. 1982).

This study proposes a method to apply DIC and strain inversion to determine the five elastic constants of transversely isotropic rock from a single sample under strip load. By applying a strip load to create various stress conditions and measuring the strain field with DIC, strain inversion is then used to calculate the elastic constants. The application of a strain field reduces deviations caused by heterogeneity and achieves more accurate elastic constants. This research introduces the theoretical background, methodology, validation through numerical experiments, experimental setup, and validations through laboratory experiments.

2 Methodology

2.1 Constitutive model of transversely isotropic rock

Transversely isotropic rocks have five independent elastic constants according to the generalized Hooke's law. E and E' are the elastic moduli in the plane of transversely isotropic rock plane and normal to it, respectively. ν and ν' are the Poisson's ratio that characterize the lateral strain response in the plane of transverse isotropy to a stress acting parallel and norm al to it, respectively. The term G' is the shear modulus in the plane normal to the plane of transverse isotropy (Cho et al. 2012).

2.2 Digital Image Correlation (DIC)

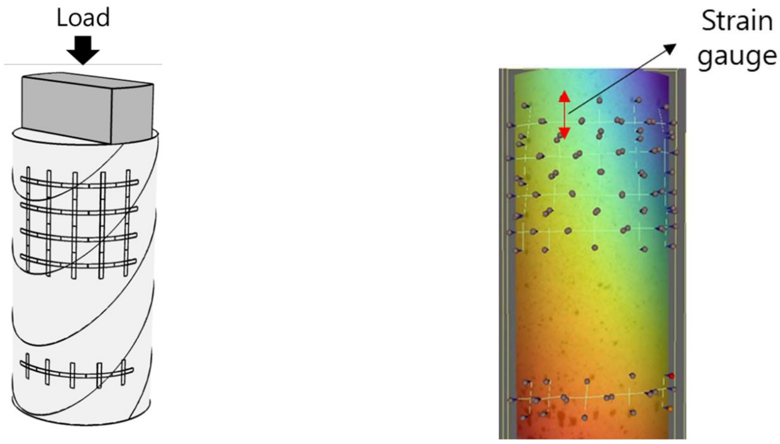
Digital Image Correlation (DIC) is a method that compares a series of digital images of a specimen's surface at various stages of deformation, tracks pixel movements in the region of interest (ROI), and calculates displacement and strain using a correlation algorithm. In DIC, the image taken before loading is called as the reference image, while the image taken after loading and deformation is called the deformed image. Once the ROI is designated, DIC matches the same pixels in the reference image and the deformed image within the ROI and calculates the pixel displacement vectors.

2.3 Strip load test

The strip load test proposed by Yim et al. (2022) was used to determine the elastic constants of transversely isotropic rocks by generating various stress states within a single sample under non-uniform loading. In a strip load test, a non-uniform stress field is formed at the top of the specimen, while at the bottom, a stress field similar to uniaxial compression appears due to the Saint-Venant's principle (Toupin 1965).

DIC can measure the deformation across the specimen surface from top to bottom without any contact, allowing analysis of varying strain field under strip loading. DIC works by defining a region of interest (ROI) within which displacement, strain, and other parameters are analyzed. To determine the elastic constants through strain inversion from the displacement field measured within the ROI, virtual strain gauges must be designated within the ROI, and strain values must be measured at the designated locations.

According to Yim et al. (2022), due to the stress concentration that can occur immediately under the strip load at the top, it is more accurate to measure the strain values at a certain distance below the top. Numerical modeling was used to analyze the strain values according to the strip load height in cylindrical specimens. The results showed that the strain values at all angles were similar to those of uniaxial compression near the bottom of the sample due to Saint Venant's principle. Based on these results, the ROI was set from 1.5 cm to 5.5 cm from the top and 2cm above the bottom. To analyze the strain values measured within the ROI, the ROI was divided into fifty cross strain gauge of 1 cm each, as shown in Fig. 1 (Hong et al. 2024).



(a) (b)

Fig 1. (a) Schematic diagram of strip load test and (b) axial displacement image measured by DIC, in which the region of interest is divided into fifty virtual strain gauges, each with a gauge length of 1 cm

2.4 Strain inversion method

As there is no analytical solution for the stress distribution of transversely isotropic material under strip load, numerical analysis is needed to determine the five elastic constants under strip loading. In this study, finite element method (FEM) was used to calculate the stress and strain of transversely isotropic rock according to the elastic constants. In numerical modeling the positions of fifty strain values within ROI were used to obtain the strain in each square grid. Strain inversion was then carried out to iteratively determine the optimal five elastic constants through the strain field obtained by DIC.

To optimize the elastic constants using the measured strain, the Gauss–Newton method was used to minimize the sum of the squares of the differences between the modeled and measured strains (Lee et al. 2024). The procedure for calculating the elastic constants by strain inversion is given in flow chart in Fig. 4. It starts with measuring the strain field with DIC and gather data for fifty strains in the axial and lateral directions Based on this selected elastic constant, elastic constants are updated, calculating the strain difference and creating a Jacobian matrix. The output condition is to calculate the root mean square error (RMSE) of the strain and ensure it does not exceed the critical value.

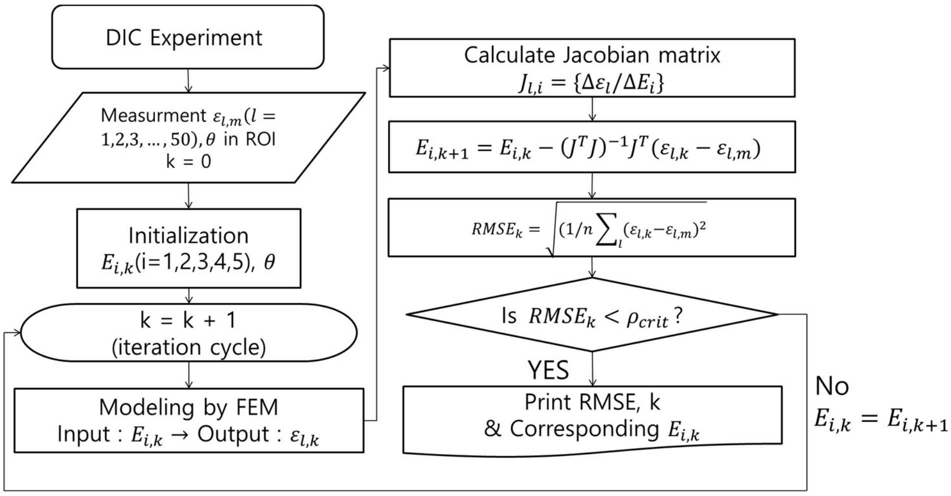


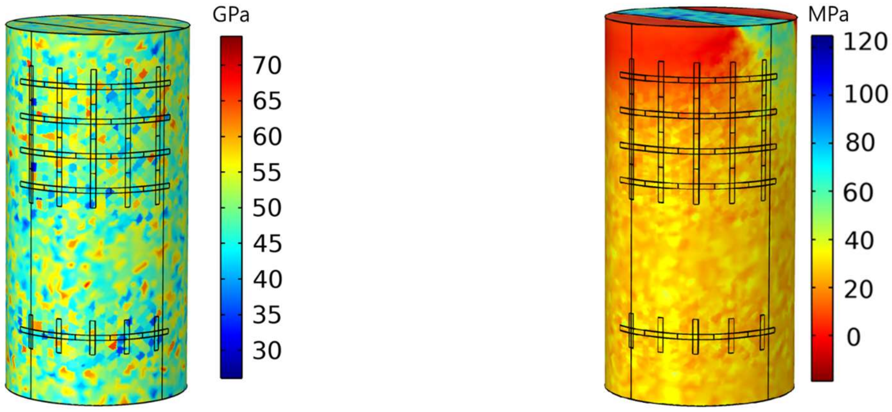
Fig. 2. Flow chart of Gauss–Newton method (m : measured, k : iteration number, $\boldsymbol{E_i}$: i-th elastic constant. $\boldsymbol{\varepsilon_l}$: l-th strain value)

3 Validation by numerical experiment

In order to consider more realistic rock, heterogeneous transversely isotropic rocks were numerically created for validation. The heterogeneous elastic constants followed a normal distribution with the mean and standard deviation as shown in Table 1, and different elastic constants from this distribution were assigned to each 2mm size element to reproduce the heterogeneity of the elastic constants by using Monte-Carlo simulation. Fig. 3 shows examples of samples which have heterogeneous elastic constants and corresponding stress distribution due to strip load (Hong et al. 2024). In the numerical experiments, uncertainties in measuring parameters such as the bedding angle (θ) and dip direction (φ) were also considered. Through these numerical experiments, hundred heterogeneous models were generated, and fifty strain values to be measured in each model were calculated. Using this data, five elastic constants were determined using the Gauss-Newton method as shown in Fig 2. For comparison, the uniaxial compression test method and the strip load method, which were measured by strain gauges, were used (Cho et al. 2012, Yim et al. 2022).

Table 1 Average and standard deviation value for making heterogeneity model

Parameter	E(GPa)	E'(GPa)	ν	ν	G'(GPa)	θ(˚)	φ(˚)
average	45.0	30.0	0.15	0.11	15.0	0-90	0
Standard deviation	4.8	3.5	0.039	0.039	1.7	2	3



(a) (b)

Fig. 3. Distribution of elastic constant E in cylindrical heterogeneous transversely isotropic specimen (b) and stress distribution when the bedding angle is 45˚

Table 2. Averages and standard deviations of five elastic constants for heterogeneous transversely isotropic model determined by suggested method. (θ= 45°)

Core	Measured	E (GPa)	E' (Gpa)	G' (Gpa)	ν (-)	V' (-)
Input		45 ± 4.8	30 ± 3.5	15 ± 1.7	0.15 ± 0.039	0.11 ± 0.039
Two core	gauge	44.74 ± 1.83	29.71 ± 0.52	14.98 ± 0.11	0.145 ± 0.012	0.114 ± 0.0046
Single	gauge	44.51 ± 3.03	30.42 ± 1.25	15.03 ± 0.18	0.169 ± 0.049	0.106 ± 0.0068
	DIC	44.61 ± 2.23	30.11 ± 1.01	15.02 ± 0.15	0.159 ± 0.038	0.106 ± 0.0051

Table 2 shows the average and standard deviations of five elastic constants obtained by each method. The method of measuring strain using DIC under strip load has been verified to determine all elastic constants more accurately compared to the traditional method employing strain gauges. Additionally, when compared to the conventional two-core method, no significant difference was observed. The traditional gauge method, being highly sensitive to heterogeneity, exhibited considerable deviations in E and ν at low angles. To assess whether DIC could reduce the impact of heterogeneity, the coefficients of variation for E and ν were calculated and displayed in Fig 4. This analysis not only significantly reduced the errors associated with E and ν but also proved to be more accurate in determining elastic constants than the results obtained from supplementary Brazilian tests aimed at reducing these errors.

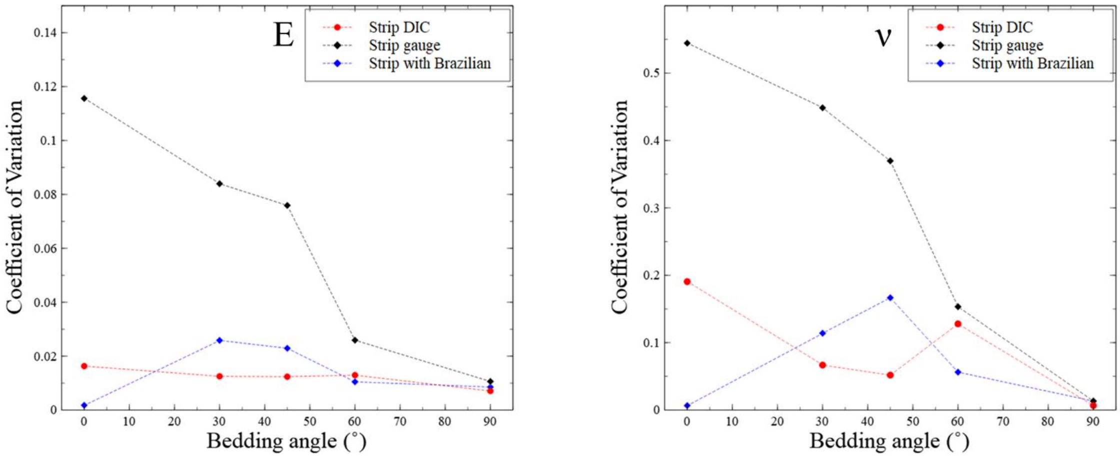


Fig. 4. Variation of the coefficients of variation for E and ν at different bedding angles under strip load, measured by Digital Image Correlation (DIC), strain gauges, and with the addition of Brazilian tests.

Strain is most significantly influenced by the elastic constants defined at the maximum principal stress direction, particularly at low angles where the strain is primarily affected by the constants E' and ν ' defined at 0 degrees. When using strain gauges, only localized deformations are measured, which minimizes the apparent effects of E and ν, and heterogeneity more substantially influences the E' and ν '. This often leads to disturbances in the estimation process of the elastic constants, resulting in considerable errors. However, by using DIC, the measurement of the strain field mitigates the disturbances caused by heterogeneity, enabling a more accurate determination of E and ν.

4 Validation by laboratory experiment

The rock samples used for this study are Paju gneiss, Onyang gneiss, and Boryeong shale from Korea which exhibit clear transversely isotropic planes. A directional coring machine was used to extract rock samples with bedding angles of 0, 30, 45, 60, and 90 degrees with six samples taken for each angle. For each angle, three samples were used for uniaxial load tests and the other three for strip load tests.

Fig. 6 illustrates the displacement field for bedding angles ranging from 0° to 90°, and resulting from these stress distributions and shows box plots for the five elastic constants determined under strip load using DIC and the conventional uniaxial compression test (Cho et al. 2012; Hong et al. 2024). For specimens with bedding angles of 0° and 90°, the displacement field exhibited a symmetric distribution similar to that observed in isotropic rocks. In contrast, for specimens with bedding angles of 30°, 45°, and 60°, the stress distribution was altered along the bedding plane, leading to a curved displacement field that followed the bedding plane. These results confirm that DIC effectively measures the deformation induced by strip loading.

Using the proposed method, the five elastic constants were determined from three sets of rocks for each bedding angle. The ranges of the determined elastic constants were largely comparable for all five constants and three rock types. The strip load test exhibited slightly larger deviations, as it uses a single rock core, making it more susceptible to heterogeneity and experimental errors than the uniaxial compression test, which uses two rock cores. Importantly, the range of elastic constants obtained from DIC, which incorporates 50 strain values, is significantly narrower than that reported in the previous study by Yim et al. (2022), which used only eight strain values.

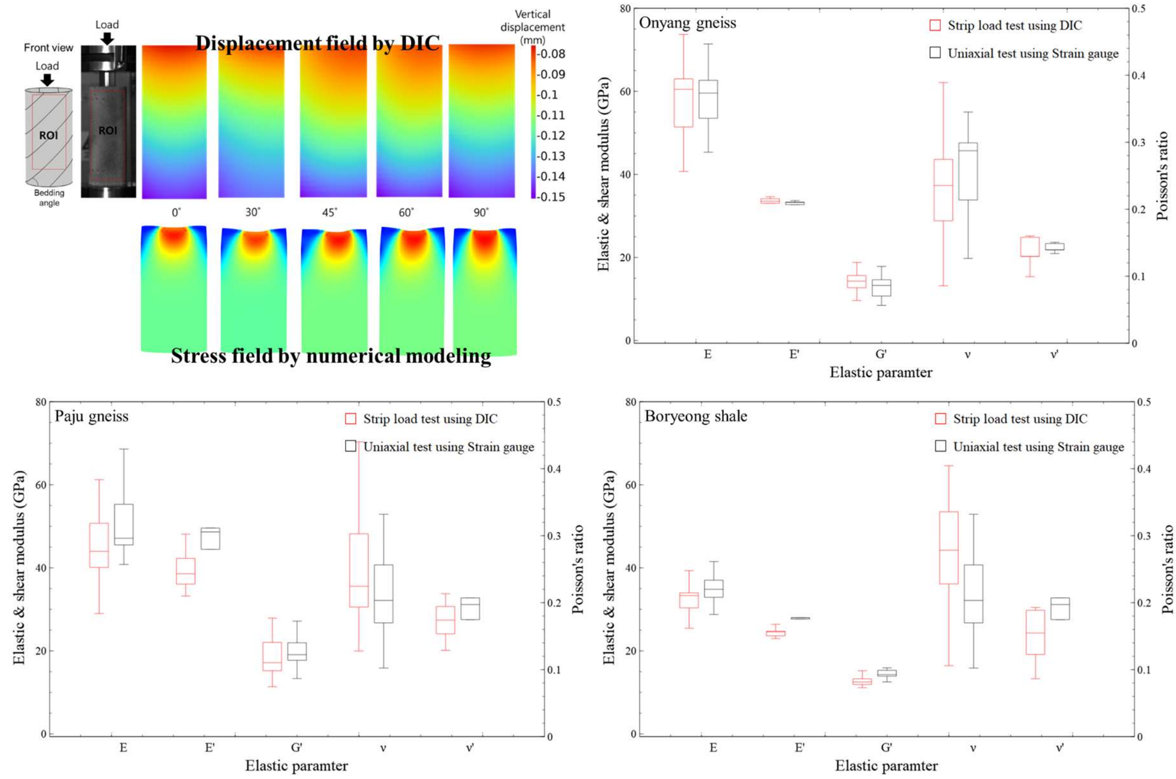


Fig. 5. Displacement field measured by DIC in transversely isotropic rock and the corresponding axial stress distribution at the same angles obtained through numerical analysis and boxplot of the five elastic constants determined from stress-strain values measured by DIC and strain gauges under uniaxial compression for Onyang gneiss, Paju gneiss, and Boryeong shale

5 Conclusion

This study proposed a method to apply DIC to determine the five elastic constants of a single transversely isotropic rock sample under strip loading with the strain inversion method. Strain values under various stress conditions were measured using DIC during strip loading, and the five elastic constants were determined by applying strain inversion, which minimizes the difference between the measured strain and the strain obtained from finite element modeling. The proposed method was validated through numerical and laboratory experiments. In the numerical experiments, the results obtained using eight strain and additional Brazilian test values from Yim et al. (2022), were compared with those obtained using 50 strain values to determine the five elastic constants through strain inversion. It was validated that using more strain values reduces the error in the determined elastic constants.

Laboratory experiments were conducted on Onyang gneiss, Paju gneiss, and Boryeong shale with bedding angles of 0, 30, 45, 60, and 90 degrees, involving both uniaxial compression tests and strip load tests. The distribution of the five elastic constants obtained from the strip load test was compared with that of the five elastic constants obtained from the uniaxial compression test, and similar distributions were observed. This validated that the proposed method can accurately determine the five elastic constants using DIC with a single sample.

The proposed method not only allows DIC to replace strain gauges but also enables the determination of the elastic constants of transversely isotropic rocks using cores drilled in a single direction from the field, making it applicable to various engineering applications. Further research is needed to determine if placing a mirror behind the sample and simultaneously measuring the front and back surfaces using DIC can enhance accuracy, or if conducting additional Brazilian tests and measuring with DIC can more accurately determine elastic constants compared to the gauge methods.

6 AKNOWLEDGEMENT

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