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Evaluating Wedge Stability Using Analytical and Numerical Approaches

Hare Ram Timalsina[1,2], Sanyam Ghimire[1], Krishna Kanta Panthi[1], and Naba Raj Neupane[2]

[1]*Norwegian University of Science and Technology (NTNU), Trondheim, Norway*
[2]*Tribhuvan University, Institute of Engineering (IoE), Pashchimanchal Campus, Pokhara, Nepal*
harert@stud.ntnu.no

Abstract

Wedge failure is a common mechanism of failure in jointed rock slopes, which poses significant challenges for slope analysis due to its inherent three-dimensional nature. The major parameters that affect the stability of rock slopes are the geometry of slopes, rock mass quality, and conditions of discontinuities. Various methods are available for examining the stability of rock slopes, such as stereographic projections, the limit equilibrium method, analytical method, numerical modelling, and the probabilistic approach. In this manuscript, a wedge failure case from Nepal Himalaya is evaluated using both limit equilibrium and numerical modelling approaches, and the results are interpreted and compared.

To do so, data are collected by extensive geological field mapping and measured jointing systems are assessed in the stereographic projection to identify the mode of failure. The factor of safety (FoS) is calculated using the analytical approach and limit equilibrium method (LEM) with 3D rock slope analysis. In addition, the distinct element method (DEM) is used to calculate FoS of the jointed rock mass. The Mohr-Coulomb (M-C)/Barton-Bandis (BB) failure criterion is applied for the critical failure plane.

It is found that the FoS is consistent with each other. However, the DEM approach yielded with lower FoS value providing slightly conservative evaluation. The analytical method is useful for a quick assessment of stability, while numerical methods provide deeper insights into slope analysis. The DEM is suggested for jointed rock masses, as it incorporates characteristics of discontinuities in its calculation. It is also recommended to evaluate rock slope stability under dynamic conditions, which are especially crucial for seismically vulnerable zones like Nepal.

Keywords

Analytical, Distinct element, Jointed rock, Limit equilibrium, Wedge failure

1 Introduction

The stability analysis and design of rock-cut slopes is a key issue and a challenging area of rock mechanics and rock engineering (Goodman, 1976). The stability of rock slopes depends predominantly upon rock mass quality, condition of discontinuities, and geometry of rock slopes. In most of the cases, the failure plane is governed by the orientation of discontinuities. The failure patterns and instability along the discontinuities are influenced by frictional properties, infilling, groundwater conditions, and ground shaking due to earthquakes (Panthi, 2021). Shape and roughness properties of discontinuity directly affect the shear strength of the critical failure plane. Various failure mechanisms have been implemented to investigate rock slopes by different researchers (Abramson et al., 2001; Hoek, 2009; Varnes, 1958; Wyllie & Mah, 2004). There are four major failure mechanisms of rock slopes, namely, planar, toppling, wedge, and circular failures (Hoek & Bray, 1981). Wedge failure is the most common failure mechanism that occurs when two joints intersect, and the line of intersection is sub-parallel and has smaller angle than the slope face (Wyllie & Mah, 2004). The suitable conditions for wedge failure have been highlighted in various research in rock slope engineering (Hoek et al., 1973; John, 1970; Wyllie & Mah, 2004).

This manuscript presents an analysis of a potential wedge failure case where analytical, limit equilibrium, and numerical approaches of stability assessment is comprehensively used and the achieved results are discussed and compared.

2 Analytical formulation

The analytical formulation can be used to determine the safety factor (FoS) of a wedge failure under the conditions of various joints. The wedge is moving along the line of intersection formed by two intersecting joints (Wyllie & Mah, 2004). The *FoS* is a ratio of resisting force to sliding force along the failure plane, as given in Eq. (1).

$$FoS = \frac{F_{resist}}{F_{slide}} \tag{1}$$

Where *FoS* is the safety factor, F_{resist} is a global force that prevents sliding, and F_{slide} is a global force that causes sliding. When two planes intersect, an intersecting line should daylight at the rock slope. The trend of the intersecting line should be approximately similar dip directions (($\pm 20^0$), as illustrated Fig. 1a. The trend α_i and plunge ψ_i of the line of intersection of planes *A* and *B* can be evaluated using Eq. (2) and (3).

$$\alpha_i = tan^{-1}\left(\frac{tan\psi_A\ cos\alpha_A - tan\psi_B\ cos\alpha_B}{tan\psi_B\ sin\alpha_B -\ tan\psi_A\ sin\alpha_A}\right) \tag{2}$$

$$\psi_i = tan\psi_A\ cos(\alpha_A - \alpha_i) = tan\psi_B\ cos(\alpha_B - \alpha_i) \tag{3}$$

Where α_A and α_B are the dip directions of planes *A* and *B* and ψ_A and ψ_B are the dip of these respective planes. The safety factor of the critical wedge can be calculated by using Eq. (4) and normal reaction at planes *A* and *B*, as illustrated in Fig. 1c, evaluated by resolving weight $W\ sin\psi_i$ along two planes using Eq. (5) and (6).

$$FoS = \frac{(N_A + N_B)\ tan\phi}{W\ sin\psi_i} \tag{4}$$

$$N_A\ sin(\beta - 0.5\ \text{ǯ}) = N_B\ sin(\beta + 0.5\ \text{ǯ}) \tag{5}$$

$$N_A\ cos\ (\beta - 0.5\ \text{ǯ}) + N_B\ cos(\beta + 0.5\ \text{ǯ}) =\ W\ sin\psi_i \tag{6}$$

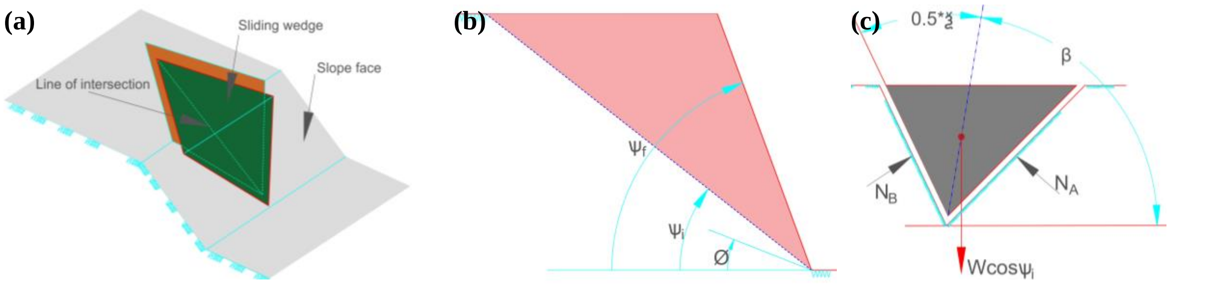

Fig. 1. Geometry of wedge failure: a) pictorial view of wedge failure; b) side view of wedge failure with line of intersection between two planes; and c) resolution of wedge with showing angles β and ǯ and respective normal reactions.

Assuming that the sliding wedge can be prevented by mobilization of the frictional angle at both planes only and combining Eq. (5) and (6), the final relation, as given in Eq. (7) and (8) is obtained by considering same frictional angle for both planes.

$$N_A + N_B = \frac{W\, cos\psi_i\, \sin\beta}{sin\left(\frac{\text{ǯ}}{2}\right)} \tag{7}$$

$$FoS = \frac{\sin\beta}{sin\left(\frac{\text{ǯ}}{2}\right)} * \frac{tan\phi}{tan\psi_i} \tag{8}$$

Therefore, the wedge stability assessment can be carried out using Eq. (8).

3 Case study

The road section selected as a case is located along the Pokhara-Butwal Highway situated at Bhalupahad of Syanja district of Nepal having geographical coordinates of 28° 08' N and 83° 51' E. This area lies in the Lesser Himalayan region and comprises metasandstone rock with predominantly two joint sets, as shown in Fig. 2a.

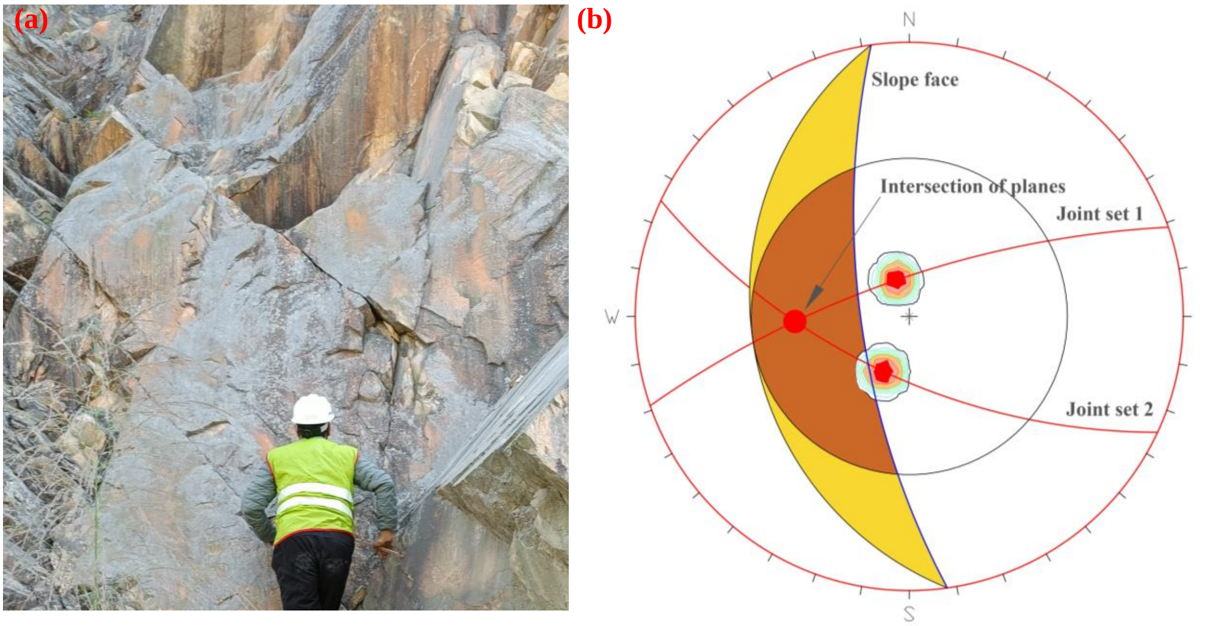

Fig. 2. Case study: a) perspective view of wedge failure case; b) stereography projection of case study site

The failure is structurally controlled and is defined by two discontinuity planes dipping at 55° and 72° with dip directions of 205° and 341° respectively. Kinematic analysis is performed to identify the mode of failure for this case. The observed failure mechanism is a wedge failure, as illustrated in Fig. 2b. The plunge and trend of the intersecting plane are 45° and 264° respectively, given by stereographic projection and analytical equations (Eq. (2) and (3)).

4 Method of wedge failure

4.1 Analytical method

The wedge failure analysis of a rock slope can be analysed using the analytical approach where various input parameters such as the geometry of slope, orientation of joints, and shear strength properties of joints are used (

Table 1). The factor of safety (FoS) is obtained as 1.18 for this rock slope, assuming the same frictional angle for both joints.

$$FoS = \frac{sin81^0 * tan30^0}{sin26.5^0 * tan45^0} = 1.18 \tag{9}$$

Table 1. Information regarding geometry of the rock slope and shear strength properties of joints.

Planes of discontinuity	Dip angle	Dip direction	Joint shear strength parameters	
			Mohr-Coulomb	Barton-Bandis
A	55^0	205^0	$\phi_A = 30^0$, $C_A = 0$	JRC = 8, JCS = 100 (MPa), $\phi_r = 24$
B	72^0	341^0	$\phi_B = 30^0$, $C_B = 0$	JRC = 8, JCS = 100 (MPa), $\phi_r = 24$
Slope face	71^0	263^0		

4.2 Limit equilibrium method

Limit equilibrium is another approach that is used for stability analysis of rock slopes. In this method of rock slope analysis, the factor of safety (FoS) is calculated to assess whether the slope is stable or not. Initially, the deterministic method of limit equilibrium analysis was introduced to analyse the rock slope (Jaeger, 1971; Kutter, 1972). The input parameters such as slope geometry, jointing conditions, rock mechanical properties, and frictional properties of joints are used to calculate the FoS. Since the mapped and assessed parameters have some degree of variations, the factor of safety itself becomes also a random variable. Therefore, a single FoS value does not consider the degree of variation in the input parameters. However, RocSlope3 (Rocscience) has the possibility to include this variation while calculating FoS. The Mohr-Coulomb failure criterion is used as a constitutive model for the wedge analysis where the shear strength properties and orientation of discontinuities as given in Table 1 are used. A critical geometry model of the case slope is developed in Rocslope3 which is shown in Fig. 3.

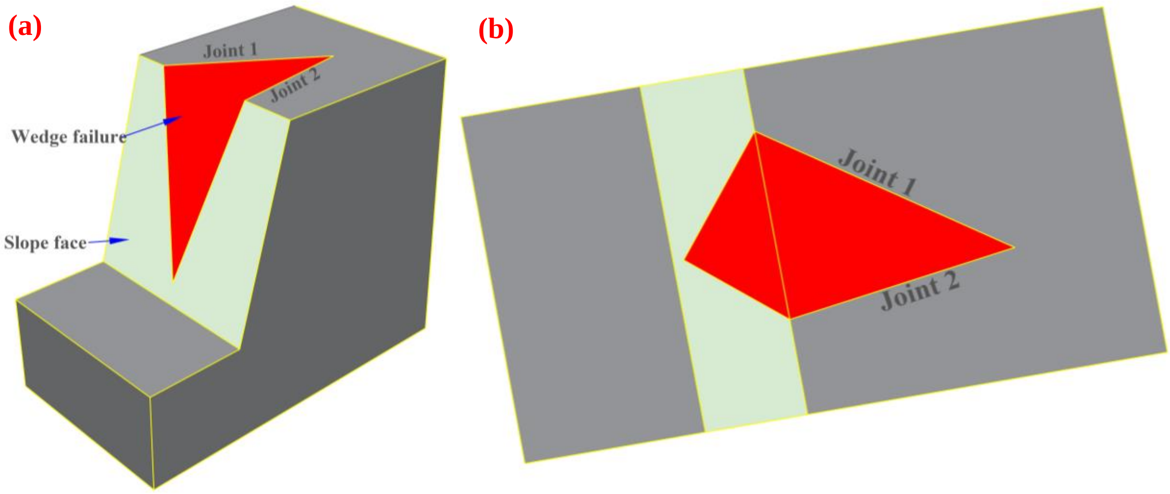

Fig. 3. The model output of Rocslope3 (Rocscience): a) pictorial view of wedge sliding, b) plan of wedge sliding

The factor of safety for the critical wedge calculated using the limit equilibrium method (LEM) in Rocslope3, is found to be 1.17 (Table 2). The sliding wedge intends to move along the trend of 267.5° and plunge of 44.7°.

Table 2. Wedge failure block information given by Rocslope3 (Rocscience).

Block weight (MN)	Resisting force (MN)	Driving force (MN)	Factor of safety	Failure mode	Sliding trend	Sliding plunge	No. of joint sets
1.43	1.17	1	1.17	Sliding	267.5	44.7	2

4.3 Distinct element method

The distinct element method (DEM) is employed to simulate the dis-continuum rock mass because it can incorporate the effect of joints present in the rock mass (ROEST et al., 1990). The Universal Distinct Element Code (UDEC) is widely used in rock slope analysis as it can simulate large deformation conditions of falling blocks. It is more applicable for dynamic analysis of slope, which also allows the effect of seismicity (Board, 1989; Eberhardt & Stead, 1998). Board (1989) has suggested Eq. (10) and (11) for estimating normal and shear stiffness of joints in the rock mass.

$$K_n = \frac{E_m\, E_r}{S(E_r - E_m)} \tag{10}$$

$$K_s = \frac{G_m\, G_r}{S(G_r - G_m)} \tag{11}$$

Where E_m is the deformation modulus; E_r is Young's modulus of intact rock; K_n is the joint normal stiffness; S is the spacing between joints; G_m is the shear modulus of rock mass; G_r is the shear modulus of intact rock; K_s is the joint shear stiffness.

The rule of thumb is that, the joint stiffness in the UDEC model, K_n and K_s should be set to a factor times the equivalent stiffness neighboring zone. Eq. (12) and (13) are used to estimate the normal and shear stiffness (Board, 1989).

Where the *factor* is a multiplication factor (generally set as 10), K and G are bulk and shear moduli respectively, and $\Delta_{z\ min}$ is the smallest width of an adjoining zone in the normal directions, as illustrated in Fig. 4.

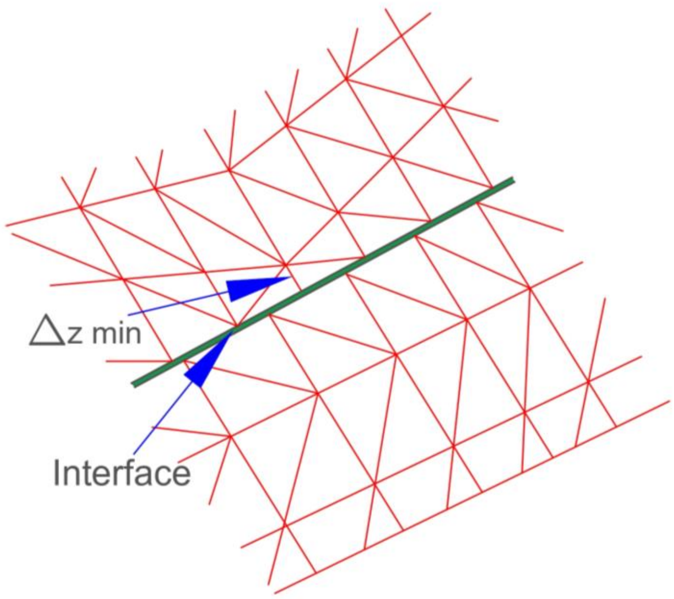

Fig. 4. Zone dimension used in stiffness calculations, modified (Board, 1989).

$$K_n = factor\ x\ max\left[\frac{\left(K + \frac{4}{3}G\right)}{\Delta_{z\,min}}\right] \tag{12}$$

$$K_s = \frac{E}{2(1+\mu)} \tag{13}$$

The physical and mechanical properties of intact rock and Joints given in Table 3 are used for UDEC model for the rock slope.

Table 3. Input parameters for the UDEC model.

Intact rock				Joint properties			
γ (MN)	E (MPa)	ν	ϕ (^{0})	K_n (GPa/m)	K_s (GPa/m)	Φ_j (^{0})	C_j (MPa)
0.026	35000	0.3	40	20	20	30	0

In Table 3, γ is the density of intact rock, E is Young's modulus, ν is the Poisson ratio, ϕ is the friction angle of intact rock, K_n is the normal stiffness of the joint, K_s is the shear stiffness of the joint, Φ_j is the frictional angle of the joint, and C_j is the cohesion in the joint.

The factor of safety of this rock slope is evaluated as 1.14, and the failure wedges vector is shown in Fig. 5. The history of unbalanced force is obtained from the UDEC model, as illustrated in Fig. 6a that ensures the computational time of the rock slope. On the other hand, Fig. 6b highlights the displacement history of critical failure wedge over the computational time.

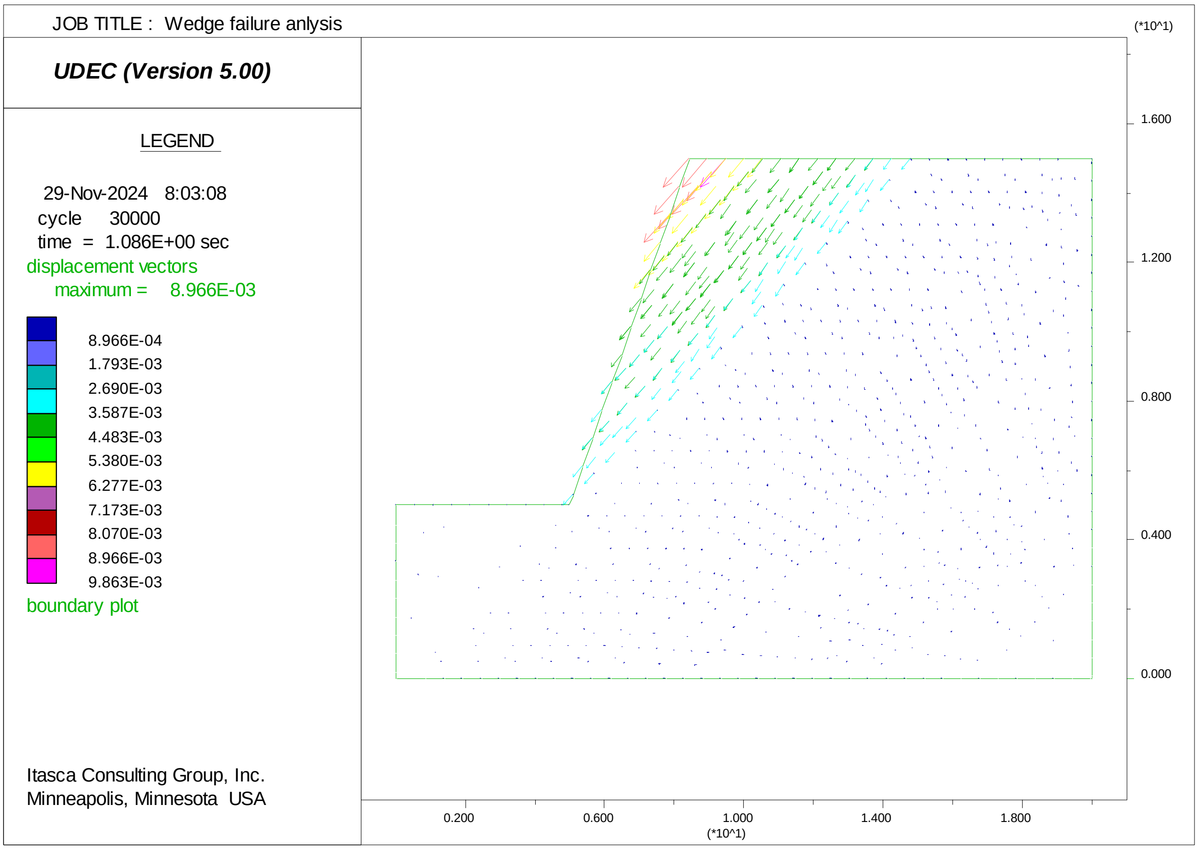

Fig. 5. Pictorial view of UDEC output for displacement vector of failure mass.

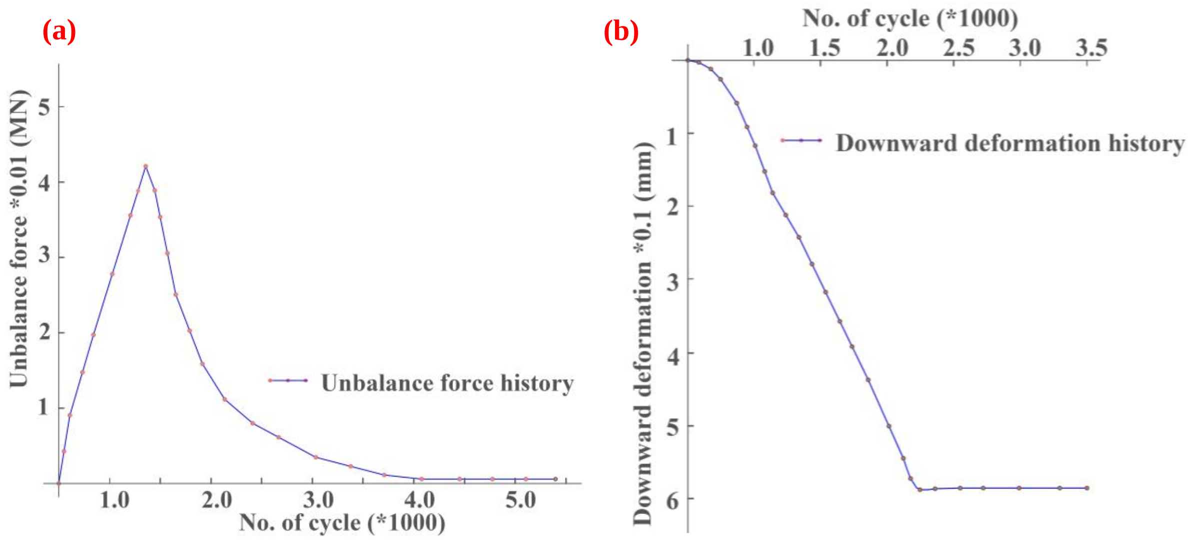

Fig. 6. Graphical representation of UDEC history output: a) trend of unbalanced force history; b) trend of deformation history of critical wedge mass.

5 Result and discussions

The wedge failure analysis of the rock slope at Bhalupahad shows stability for each case assessment method used under static conditions. The safety factor obtained through analytical, limit equilibrium, and distinct element methods is 1.18, 1.17, and 1.14, respectively. It is noted here that the analytical approach provides a simplified assessment of slopes, while the LEM using Rocslope3 offers a more comprehensive estimate of the FoS where an uncertainty assessment of the varying inputs can be used. The DEM highlights the discontinuous nature of the rock mass and influence of joint behaviour on rock slopes. The study has shown that FoS achieved through this approach is slightly less than other two approaches. The assessment has shown that the rock slope is stable under static and dry conditions and is within the border of failure. It is interpreted that given dynamic and groundwater loading, it is likely that the wedge will occur, which can be seen through the failure that have been occurred in the past as indicated in Fig. 2a.

6 Conclusions

The analytical approach can be applied to assess the initial stability of the rock slope in consideration and the numerical methods provide further insights into the stability condition. The DEM is relatively more suitable for capturing the effect of discontinuities in the rock mass and dynamic loading. Hence, this method should be given priority in stability assessment for discontinuous rock mass. The study also shows that the combined approaches can be used for a comprehensive understanding of wedge failure mechanism. The FoS obtained from different methods are relatively consistent, the DEM yielding the lowest value followed by LEM and analytical approaches, respectively. This study emphasizes the importance of integrating available approaches for reliable slope stability assessment. Further dynamic analysis should be performed to examine the stability of slopes during seismic events and groundwater conditions and predict the needed support system.

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References

Abramson, L. W., Lee, T. S., Sharma, S., & Boyce, G. M. (2001). Slope stability and stabilization methods. John Wiley & Sons.

Board, M. (1989). UDEC (Universal Distinct Element Code) Version ICG1. 5.

Eberhardt, E., & Stead, D. (1998). Mechanisms of slope instability in thinly bedded surface mine slopes. Engineering Geology: A global view from the Pacific Rim,

Goodman, R. E. (1976). Toppling of rock slopes. Proc. Specialty Conf. on Rock Engineering for Foundations and Slopes,

Hoek, E. (2009). Fundamentals of slope design. Keynote address at slope stability, 9-11.

Hoek, E., Bray, J., & Boyd, J. (1973). The stability of a rock slope containing a wedge resting on two intersecting discontinuities. Quarterly Journal of Engineering Geology and Hydrogeology, 6(1), 1-55. doi.org/10.1144/GSL.QJEG.1973.006.01.01

Hoek, E., & Bray, J. D. (1981). Rock slope engineering. CRC press. doi.org/10.1201/9781482267099

Jaeger, J. C. (1971). Friction of Rocks and Stability of Rock Slopes. Geotechnique, 21(2), 97-134. doi.org/10.1680/geot.1971.21.2.97

John, K. W. (1970). Engineering Analyses of Three-Dimensional Stability Problems Utilizing the Reference Hemisphere. ISRM Congress,

Kutter, H. (1972). Analytical methods for rock slope analysis. Rock Mechanics, 197-211.

Panthi, K. K. (2021). Assessment on the 2014 Jure Landslide in Nepal–a disaster of extreme tragedy. ISRM EUROCK, doi.org/10.1088/1755-1315/833/1/012179

ROEST, J., HART, R., & Lorig, L. (1990). Modelling fault-slip in underground mining with the distinct element method. [International congress international association of engineering geology. 6. Symposia. 1990, pp 105-110, 6 p ; Illustration ; ref : 10 ref]. International congress international association of engineering geology. 6. Symposia,

Varnes, D. J. (1958). Landslide types and processes. Landslides and engineering practice, 24, 20-47.

Wyllie, D. C., & Mah, C. (2004). Rock Slope Engineering. CRC Press. doi.org/10.1201/978131 5274980