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Estimation of the Failure Probability of Attenuator Rockfall Protection Structures Using a Reliability Analysis Approach (RBD) and Kriging Metamodel

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Abstract

In rockfall protection structures, the dynamic process of stopping or slowing down a falling block is significantly complex and influenced by multiple variables. The application of design approaches based on Reliability-Based Design (RBD) enables the analysis of how the variability of parameters describing a structure affects its performance and provides the structure with a uniform probability of failure. In this article, a specific attenuator system was considered, for which a series of explicit dynamic analyses were conducted using a finite element code. The system's response must be studied considering the variability of the impact conditions and evaluating: the volume, the speed and the rotation of the block, and the inclination and position of impact. To reduce the computational cost of these numerical simulations, a metamodeling procedure has been introduced, using a mathematical operator to describe the response of the attenuator. In this paper, the system behaviour was analysed by varying six parameters that describe the block's kinematics and using a Kriging metamodel to perform reliability analysis. By substituting the metamodel for the original performance function, the probability of failure is calculated, while accounting for the metamodel accuracy.

Keywords

Attenuator, Numerical model, Reliability-Based Design, Kriging metamodel

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1 Introduction

Rockfall mitigation has become a pressing concern to safeguard infrastructure and mountainous regions prone to risk. Among the available solutions, attenuators are gaining prominence due to their ability to intercept and decelerate falling rocks. These systems employ deformable nets to dissipate kinetic energy and guide debris safely to the ground. However, their complex dynamic behavior requires a comprehensive examination of key performance factors, including impact velocity, rock volume, and collision characteristics.

Conventional protective structure design, governed by Limit State Design (LSD) and Eurocode 7, evaluates the kinetic energy of falling rocks against the structure's maximum energy absorption capacity. While effective, this approach is less suited to addressing the variability of conditions typical of rockfall events. In contrast, Reliability-Based Design (RBD) offers a more advanced framework by incorporating uncertainties through a reliability index and a uniform failure probability. This method is particularly advantageous for cutting-edge systems like attenuators.

Numerical modelling has become an essential tool for predicting the behaviour of these systems under dynamic impacts and identifying the factors that most significantly influence their performance. This research analysed a specific attenuator using three-dimensional simulations in Abaqus (Abaqus 2023), taking into account the statistical variability of parameters such as rock size, speed, rotation, impact position, and angle. While the Monte Carlo method provides insights into the probabilistic behaviour of the system, it comes with a high computational cost. To streamline the process, a Kriging-based metamodeling technique was implemented. This approach replaces the detailed performance function with a simplified model, allowing the probability of failure to be calculated efficiently while preserving accuracy (Dubourg et al., 2013; Depina et al., 2016; Lambert et al., 2021).

2 Attenuator numerical model

The attenuation system under study is composed of three modules, each spaced 10 meters apart. The structure is supported by 6-meter-high posts with an H-shaped cross-section (HEA180) and cylindrical hinge constraints at the base. It is reinforced with seven uphill anchoring cables and two lateral cables, all featuring a diameter of 16 mm. The deformable panel is supported by an upper cable, while the lower edge is left free to rest on a slope inclined at 45°.

To optimize the computational effort and element count, the numerical model in Abaqus employed an equivalent membrane to replicate the behaviour of the net panel. This approach relied on experimental data from punching and tensile tests (Mentani et al., 2018; Thoeni et al., 2013). The 3D geometric representation of the model is illustrated in Fig. 1, and Table 1 provides a detailed breakdown of the elements assigned to each part of the system in the Abaqus simulations.

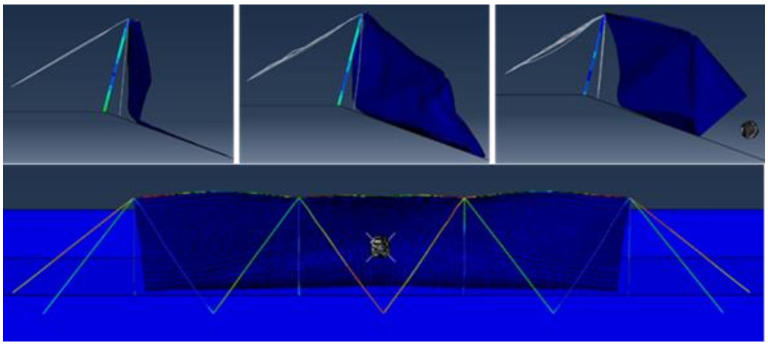

Fig. 1 3D Model of the Attenuator system.

Table 1 Elements Used in the 3D Model in Abaqus

Component	FEM Elements/Beauvoir
Net panel	MEMBRANE - M3D4R: A 4-node quadrilateral membrane with reduced integration and hourglass control. ELASTOPLASTIC
Cables	TRUSS - T3D2: A 2-node linear 3D truss. ELASTIC
Posts	BEAM - B31: A 2-node linear beam in 3D space. ELASTIC
Connection (upper cable/net)	Tie constraints, no relative motion between the surfaces.
Connections (cables/posts)	Join constraints, kinematic constraints u_1=0, u_2=0, u_3=0.
Contact (block/net/slope)	General Contact: Normal Behaviour: Hard Contact; Tangential Behaviour: Penalty – Friction Coefficient 0.4

Block	Rigid Body - C3D8R: An 8-node linear brick with reduced integration and hourglass control.

3 RBD analysis procedure

The Reliability-Based Design (RBD) method provides an advanced framework for the design of passive protective structures against rockfall events. In conventional practice, the design of such structures is based on energy considerations, comparing the kinetic energy of the falling block (action) with the energy absorption capacity of the protective system (resistance).

The application of partial safety factors, as outlined in Eurocode 7 (EC7), is often complex and does not facilitate a direct estimation of the failure probability. In contrast, RBD adds an enhanced layer of precision by introducing a reliability index, which ensures a consistent failure probability across geotechnical systems (Vagnon et al., 2020).

This reliability index, symbolized as β, acts as a measure of structural safety, with the probability of failure (P_f) calculated using the following Eq.1:

$$P_f \approx 1 - \Phi(\beta) = \Phi(-\beta) \tag{1}$$

Where P_f Probability of failure
Φ Cumulative normal distribution function
β Reliability index

This index allows for determining the design point coordinates, x^*, which correspond to the point of tangency between the expanding dispersion ellipsoid and the surface defined by the failure criterion.

3.1 Procedure for Applying RBD to the Considered Attenuator System

To understand and explain the procedure for determining a uniform failure probability based on reliability, a specific attenuator system is considered.

The application of Reliability-Based Design (RBD) in a rockfall protection system follows four main stages, which are outlined in Fig. 2 below. The procedure applied to the chosen attenuator system will be described in detail, leading to the calculation of the probability of failure.

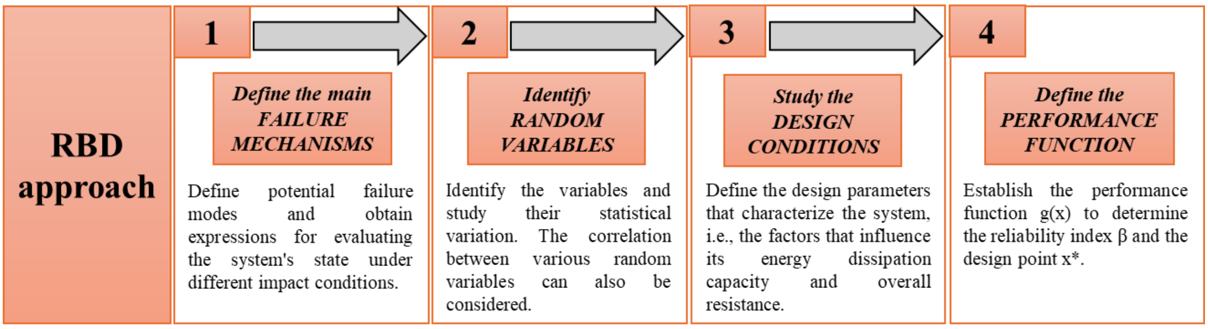

Fig. 2 The main stages for applying the RBD approach.

4 Application of RBD analysis

This section provides a detailed description of the steps involved in applying the RBD approach to a specific attenuator system. The analysis required numerous numerical simulations, optimized by combining Python scripts and Abaqus simulations for model definition, result extraction, and interpretation. Fig. 3 illustrates the flow chart used for these analyses, which can also be applied to subsequent simulations.

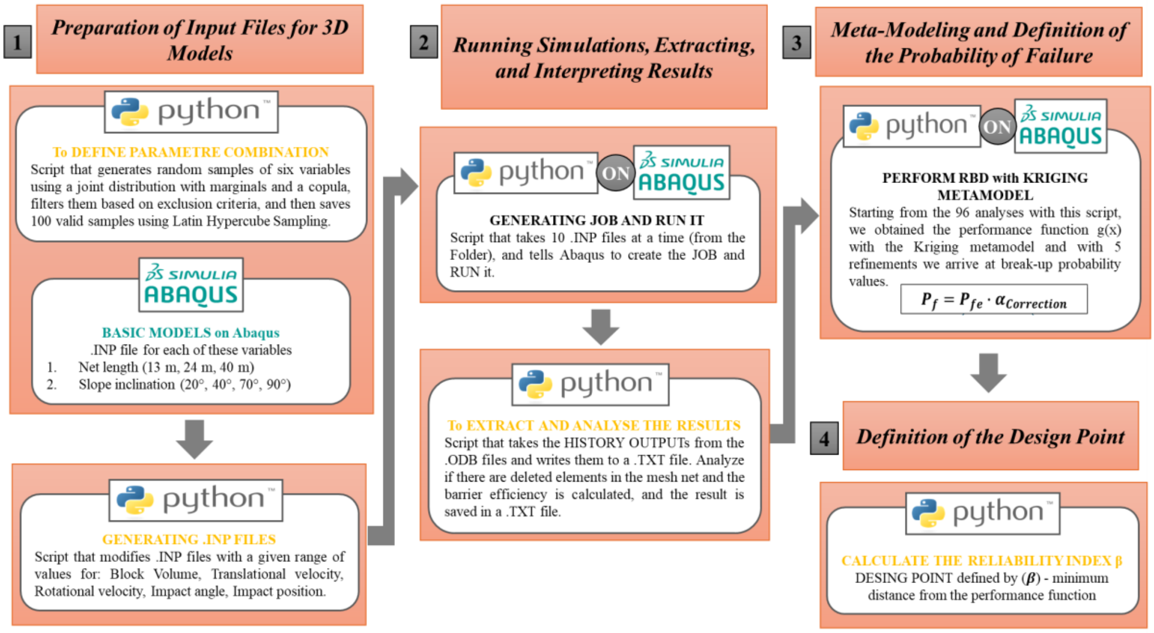

Fig. 3 Flow chart for RBD analysis in Python and Abaqus

4.1 Main Failure Mechanisms

The main failure modes of the system were identified by considering impact scenarios and conditions that could lead to structural collapse. These failure modes, based on the system's response after impact, are:

- The block passing through the net, resulting in the rupture of the panel.
- The attenuator's efficiency falling below 20%.

Attenuator efficiency refers to the percentage of kinetic energy dissipated in the phase occurring approximately 0.5 seconds after the impact, during which the kinetic energy of the block is gradually dissipated by the barrier until it stabilizes. The percentage of energy dissipation was calculated using the following Eq. 2:

$$\Delta E_{\%} = \frac{E_{imp} - E_{t=0.5\,s}}{E_{imp}} \quad [\%] \tag{2}$$

Where $\Delta E_{\%}$ Percentage of kinetic energy dissipated
E_{imp} Kinetic energy of the block at the moment of impact
$E_{t=0.5\,s}$ Kinetic energy of the block at 0.5 seconds after the impact

In the RBD approach, it is important to define the objectives of the numerical simulations before they are conducted. Kinematic histories of the block (position, velocity, acceleration, and kinetic energy) during the impact were extracted from the Abaqus models. The $\Delta E_{\%}$ value was calculated, and panel rupture was analysed using a Python code. For simulations where the panel ruptured, a value of 0 was assigned to $\Delta E_{\%}$ enabling the simultaneous evaluation of both failure modes in the structure.

4.2 Main Failure Mechanisms

The identification of random variables affecting the energy dissipation of the system is a key aspect of the RBD approach. These variables, including block volume, translational and rotational velocities, impact angle, and location, can vary significantly depending on the site and are not known beforehand. Since the system's overall response cannot be captured by a single load condition, it is necessary to consider the statistical variability and assume probabilistic distributions and ranges of variation for each variable. Defining these distributions accurately is a complex task and must be based on studies regarding block trajectory and volume (Umili et al., 2020).

The variation ranges and statistical distributions for the random variables used in this study are provided in Table 2. Fig. 4 shows the marginal distributions, which were combined using a copula to create a joint distribution representing the entire system. No correlation between the parameters was assumed in this case.

To enhance computational efficiency and reduce costs, a metamodeling approach based on Kriging was applied in this study. The construction of the model relied on simulation data obtained through a Design of Experiments (DoE) methodology using Latin Hypercube Sampling (LHS) (Toufigh et al., 2018). This approach stratifies the input space, ensuring a well-distributed sampling of parameters while minimizing the number of samples compared to traditional Monte Carlo methods. A total of 100 valid samples were generated, and automated numerical analyses were performed using Abaqus (Figure 5).

Table 2 Ranges and statistical distributions of the random variables.

Parameter	**Min; Max Range**	**Statistical Distribution**	**Statistical Parameters (Mean; Std. Dev.)**
Block Volume V [m³]	0.03; 3	Log-normal	-1.20; 0.76
Block Impact Velocity v [m/s]	5; 40	Log-normal	2.65; 0.34
Impact Angle α [°]	-80; 45	Uniform	-17.5; -
Impact Height Z [m]	0.82; 4.5	Normal	2.66; 0.61
Impact Position Y [m]	-14.18; 14.18	Normal	0; 4.73
Block Rotation at Impact ω [rad/sec]	0; 35	Uniform	17.5; -

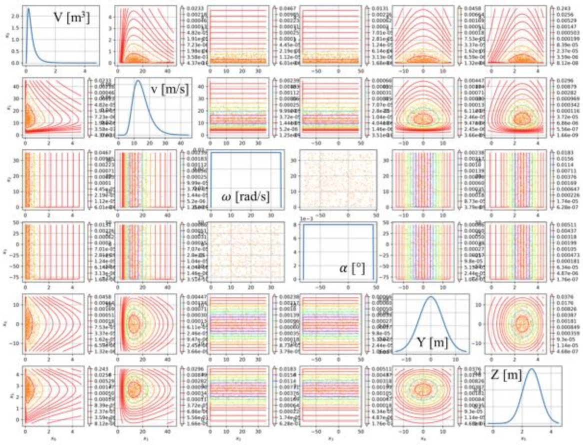

Fig. 4 Copula distribution of the random variables.

4.3 Design Conditions of the Structure

When designing the protection system, structural parameters such as the panel length, the deformability of the mesh, the slope angle, and the height of the posts must be considered, as these factors influence both energy dissipation and structural strength. Unlike the random variables, which exhibit statistical variability, these parameters are site-specific and define the geometry of the model. Therefore, the design configuration must be established upfront, and for each design setup, the probability calculations need to be repeated. The analysis presented focuses on a specific attenuator with a 13-meter-long mesh, medium deformability, and a 45° inclined slope, while maintaining the other characteristics described earlier for the 3D model.

4.4 Definition of the Performance Function

The reliability analysis of the considered attenuation system is performed to quantify the effects of variability in the set of random variables $X = [V;\ v; \omega; \alpha; Y; Z]^T$on the ultimate limit state. The performance function represents the relationship between the structural resistance capacity and the impact energy of the block. The function is defined as (Eq. 3):

$$g(X) = R_B - E_i = 0 \qquad (3)$$

Where $g(X)$ The performance function
R_B Structural resistance capacity
E_i The impact energy of the block

When $g(X) = 0$, the system is at the limit of its resistance capacity, while positive or negative values indicate, respectively, a state of safety or failure of the system. Defining this function requires a computationally intensive process and is time-consuming. One approach to reduce computational demands is to approximate $g(X)$ with a computationally less expensive metamodel. The metamodel is typically built by implementing statistical learning methods on the set of observations obtained through Design of Experiments (DoE), mapping the inputs into a standard space to facilitate probabilistic analysis. In this study, a Kriging metamodel, implemented in a Python code, is used to approximate the performance function $g(X)$ that describes the system. By using the performance function through the metamodel, it was possible to evaluate the probability of failure of the attenuation system, considering random variables with different distributions.

4.4.1 Construction of the Kriging Metamodel

The Kriging metamodel is built to approximate the performance function $g(X)$ that describes the system. Kriging is an advanced interpolation technique that not only estimates the value of a function at an un-sampled point but also provides an estimate of the uncertainty associated with this prediction. This methodology is widely used in surrogate modeling problems, where the function to approximate is complex or computationally expensive to evaluate (Depina et al., 2016).

Starting from the results of the 100 numerical analyses performed on the samples obtained from the Design of Experiments (DoE), the Kriging metamodel was constructed using the algorithm defined in the OpenTURNS library, imported into Python (Baudin et al., 2015). The metamodel provides an estimate of $g(X)$ for the generated samples, along with the standard deviation of the estimate at each point.

Kriging regression uses a constant trend and a squared exponential covariance model to establish a relationship between the inputs (standardized samples) and the output (performance function results) derived from the Abaqus numerical analyses. The constant trend assumes that the function to be approximated has a constant mean value across the entire input domain. This is the simplest model and is often chosen when the data do not show an evident trend. The covariance model (or kernel) defines the spatial relationship between points and describes how closely the values of $g(X)$ at two nearby points in the input space are correlated. The squared exponential covariance model assumes that closer points will have more similar values, but the correlation decreases exponentially with distance. Fig. 5 shows, as an example, the surrogate performance curve, $\hat{g}(X)$, obtained from the metamodel in a 2-variable space. The function derived from the simulations must be considered in the space of all 6 variables taken into account.

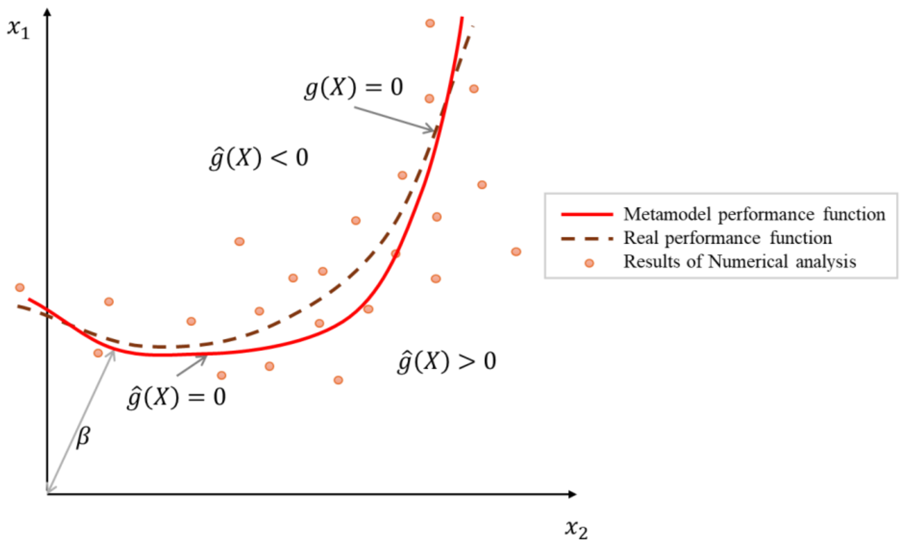

Fig. 5 Scheme of the performance curve obtained with the metamodel, in a 2-variable plane.

Next, a sampling and refinement process is carried out to improve the estimates of the probability of failure. Before performing the sampling, it is necessary to select an initial set of points that effectively represent the input space. The k-means clustering algorithm is used to divide the initial samples into clusters within the standardized space. The purpose of clustering is to identify "representative" points of the input domain, avoiding redundancy or insufficient coverage. The centers of the clusters (or representative points) are selected as initial seeds for the subsequent sampling process.

Once the initial points are chosen, the Metropolis-Hastings method is applied to generate additional samples in the standardized space. This method belongs to the family of Markov Chain Monte Carlo (MCMC) algorithms and is designed to sample from complex distributions (Au and Beck, 2001). For each selected point, the acceptance probability is evaluated based on the performance function, its standard deviation, and the probability density at the proposed points. In particular, samples near the threshold $g(X) = 0$ are clustered together to identify the representatives that will be used for further simulations in Abaqus.

Five refinement processes were considered, and for each process, 30 analyses were carried out in Abaqus. The results of each process were subsequently imported into the Python code to improve the accuracy of the probability of failure estimate.

4.4.2 Construction of the Kriging Metamodel

Using the results from the metamodel, an initial estimate of the failure probability, p_{fe}, is calculated. In the standardized space, each sample contributes to the failure probability based on its proximity to the safety threshold $g(X) = 0$. If a sample x_i is close to or within the failure region ($g(X) < 0$), its contribution becomes more significant. The failure probability is computed as the sum of the probabilistic contributions of all samples in the standardized space (Eq. 4):

$$p_{fe} = \frac{1}{N}\sum_{i=1}^{N} \Phi\left(-\frac{\hat{g}(x_i)}{\sigma_g(x_i)}\right) \tag{4}$$

Where p_{fe} Failure probability
N Number of sample
Φ Cumulative normal distribution function of the standard normal distribution
$\hat{g}(x_i)$ the metamodel estimate for the sample x_i
$\sigma_g(x_i)$ the standard deviation of the estimate provided by the metamodel

Since the calculation of p_{fe} is entirely based on metamodel estimates, it may contain errors due to inaccuracies in approximating $g(X)$, particularly in critical regions (Dubourg et al., 2013). To address this, a correction factor α is calculated through the five refinement processes. This factor is used to improve the accuracy of the initial failure probability estimate.

The correction factor is the ratio between the accurate probability, calculated through numerical simulations, and the estimate obtained from the metamodel (Eq. 5):

$$\alpha = \frac{p_{acc}}{p_{meta}} \tag{5}$$

Where α Correction factor
p_{acc} Accurate probability
p_{meta} Metamodel probability

The final failure probability is then calculated as (Eq. 6):

$$p_f = p_{fe} \cdot \alpha \tag{6}$$

Where p_f Final failure probability
p_{fe} Uncorrected probability computed with the metamodel
α Correction factor

For the attenuator system considered, after the third refinement process, the corrected failure probability p_f stabilizes at 0.14, indicating an extremely high failure probability for geotechnical structures of this type. To achieve acceptable safety conditions, the analysis will need to be repeated with adjustments to the structural configuration.

5 Conclusions

This paper presents a comprehensive analysis of the application of a reliability-based design (RBD) approach, utilizing advanced probabilistic techniques. The main objective was to overcome the limitations of traditional deterministic models by implementing a methodology based on failure probability, which accounts for uncertainties and variability in design parameters. The Kriging metamodeling method was applied to optimize the failure probability calculation and enhance computational efficiency. The integration of Kriging provided reliable failure probability estimates, optimizing the design process and reducing computational times. This approach represents a significant advancement in the design of protection systems, ensuring greater efficiency in the overall design process.

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