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Evaluation of Analytical Models in Estimation of the Breakdown Pressure in Hydraulic Fracturing

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Abstract
Accurately predicting breakdown pressure in hydraulic fracturing is critical for optimizing operational parameters and ensuring safety across its diverse applications. This paper explores various theoretical models designed to estimate breakdown pressure in the hydraulic fracturing process. By focusing on published experimental data, the study evaluates the predictive capabilities and comparative performance of these models. The efficiency and success of hydraulic fracturing depend on precise modelling to predict fracture initiation and propagation. This paper organises breakdown models based on key theoretical principles that govern fracture mechanics: tensile strength, energy release rate, and stress intensity factor. A comprehensive meta-analysis of experimental data, categorized by rock types such as igneous, sedimentary, and metamorphic, provides a strong basis for comparison. This paper highlights each model's strengths and limitations, examining their suitability across various geological conditions and discussing the ease of their application based on the parameters they use. The findings highlight the robustness of stress intensity factor models, the practical improvements in poroelastic models, and the variability in energy release rate-based models. This work provides a systematic evaluation that offers practical guidance for selecting breakdown pressure models in different geological conditions.

Keywords
Hydraulic fracturing, Breakdown pressure, Analytical models, Stress intensity factor, Poroelatic model

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1 Introduction

Hydraulic fracturing has been a transformative technique since its early development, initially utilised to enhance production from low-permeability reservoirs. Over the decades, its application has expanded significantly across various industries, including oil and gas extraction, geothermal energy production, and mining (Adams & Rowe, 2013). The process involves injecting pressurized fluids into boreholes to induce fractures in rock formations, increasing permeability and facilitating the flow of fluids. A critical aspect of hydraulic fracturing design is accurately predicting the breakdown pressure, the pressure at which fractures initiate and propagate.

Fractures typically develop along planes perpendicular to the direction of minimum in-situ stress, following the path of least resistance (Hubbert & Willis, 1957). In theory, wellbore placement and hydraulic fracturing operations are carefully designed to align with the stress field, ensuring predictable fracture propagation. However, in practice, stress heterogeneity, drilling limitations, and geological variability can significantly influence fracture initiation and growth (Warpinski & Teufel, 1987). These complexities underscore the importance of accurate modelling fracture behaviour under varying conditions, such as different stress anisotropies, fluid properties, and operational parameters.

Breakdown pressure plays a pivotal role in this process, as it governs the initiation of fractures and directly impacts the efficiency and safety of hydraulic fracturing operations. An underestimation of breakdown pressure can prevent the initiation of fractures, undermining the operational goals. Conversely, overestimations can result in excessively large or complex fractures, leading to challenges such as fluid loss, proppant settling, and increased operational costs. The accurate determination of breakdown pressure thus serves as a key risk mitigation strategy, helping to prevent well failures and optimize the overall economic viability of fracturing operations. Numerous theoretical models have been developed to estimate breakdown pressure, each incorporating different aspects of fracture mechanics and rock behaviour. These models often rely on parameters such as tensile strength, energy release rate, stress intensity factors, and fluid-rock interaction. Despite their theoretical advancements, their performance varies widely depending on geological conditions and input parameters, highlighting the need for systematic evaluation.

This paper critically evaluates existing breakdown pressure models, focusing on their theoretical foundations and applicability to different rock types. By integrating experimental data and systematically categorizing the models based on their governing principles, this study provides a detailed analysis of their predictive performance.

2 Theoretical Breakdown Pressure Models

Many theoretical models have been proposed over the years to estimate breakdown pressure in hydraulic fracturing, each developed under different governing criteria for fracture initiation, such as tensile strength, energy release rate or stress intensity factor (Kiss, 2015; Sampath et al., 2018; Wu et al., 2020). Among many well-regarded published datasets in the literature, the structure and description of these models in this paper are inspired by the comprehensive review by Sampath et al. (2018), which provides further insights and additional models beyond those discussed here.

2.1 Tensile strength-based models

Tensile strength-based models are grounded in the principle that fractures initiate when the circumferential stress around the wellbore surpasses the rock's tensile strength. While standard recommendations for measuring rock tensile strength are commonly followed in these models, it is important to note that each testing method has its limitations and should not be considered a one-size-fits-all approach for all rock types with varying brittleness indices (Masoumi et al., 2017; Mutaz et al., 2021; Serati et al., 2014; Serati et al., 2017). The foundational framework for predicting breakdown pressure was introduced by Hubbert and Willis (1957), who assumed linear elastic conditions and impermeable rock behaviour. This framework is widely regarded as the conventional breakdown pressure model (See Eq. 1).

$$P_b = 3\sigma_h - \sigma_H + \sigma_t - P_o \tag{1}$$

where P_b Breakdown pressure

σ_h Minimum effective horizontal principal stress

σ_H Maximum effective horizontal principal stress

σ_t Tensile strength of the rock
P_o Pore fluid pressure

The poroelastic model was introduced by Haimson and Fairhurst (1967) (See Eq. 2) to consider the permeability of rocks in predicting breakdown pressure. This model assumes that breakdown occurs at the wellbore wall and incorporates Biot's coefficient to account for poroelastic effects. However, it does not explicitly consider the fluid pressure distribution within the rock.

$$P_b = \frac{3\sigma_h - \sigma_H + \sigma_t - \alpha\left(\frac{1-2\nu}{1-\nu}\right)P_o}{\left(2 - \alpha\frac{1-2\nu}{1-\nu}\right)} \tag{2}$$

where α Biot's poroelastic coefficient
υ Poisson's ratio

For this study, four variations of the poroelastic model (I to IV) have been specifically defined based on different empirical equations used to estimate Biot's coefficient, as detailed in Table 1.

Table 1 Empirical equations for Biot's coefficient estimation

Reference	**Formula**
Krief et al. (1990)	$\alpha = 1 - (1-\phi)^{\left(\frac{3}{1-\phi}\right)}$
Laurent et al. (1993)	$\alpha = 0.98469 + \frac{-68.7421}{1 + e^{\left(\frac{\phi + 0.40635}{0.09425}\right)}}$
Lee (2002)	$\alpha = 1.75\phi^{0.51}$
Sijing et al. (2001)	$\alpha = 1 - e^{(-3.8\phi - 0.86)}$

where ϕ is the porosity of the material.

The point stress model introduced by Ito and Hayashi (1991) provides insights into the relationship between wellbore diameter, pressurisation rate and breakdown pressure. This model posits that breakdown initiates when the maximum effective stress exceeds the rock's tensile strength at a specific location within the rock, known as the characteristic distance. It also accounts for the influence of pore pressure distribution, which is governed by the constant rate of wellbore pressurization. However, due to the mathematical complexity of the model, two simplified scenarios are considered for practical application. In the first scenario, where the wellbore pressure increases at an extremely high rate, fracture initiation occurs before fluid penetration significantly alters the stress state around the wellbore (Eq. 3). In the second scenario, with a very low rate of wellbore pressurization, pore pressure within the rock rises concurrently with the wellbore pressure (Eq. 4). These scenarios provide a more accessible framework for understanding the model's implications.

$$P_{b,upper\ limit} = \left(1 + \frac{d}{r_w}\right)\left\{\sigma_t - \left[\frac{\sigma_H + \sigma_h}{2}\left(1 + \frac{r_w^2}{(r_w + d)^2}\right) - \frac{\sigma_H - \sigma_h}{2}\left(1 + \frac{3r_w^4}{(r_w + d)^4}\right)\right]\right\} + P_0 \tag{3}$$

$$P_{b,lower\ limit} = \frac{\sigma_t - \left[\frac{\sigma_H + \sigma_h}{2}\left(1 + \frac{r_w^2}{(r_w + d)^2}\right) - \frac{\sigma_H - \sigma_h}{2}\left(1 + \frac{3r_w^4}{(r_w + d)^4}\right)\right]}{1/2\left\{1 + \left(1 + \frac{d}{r_w}\right)^2\right\}\left(2 - \alpha\frac{1-2\nu}{1-\nu}\right)} + P_0 \tag{4}$$

where $P_{b,upper\ limit}$ Maximum breakdown pressure
$P_{b,lower\ limit}$ Minimum breakdown pressure
d Characteristic distance
r_w Wellbore radius

Tensile strength-based approaches offer a simplified framework for understanding fracture initiation, relying on assumptions such as isotropic and homogeneous rock conditions, uniform stress distribution, linear material behaviour, and limited coupling with other processes. While these assumptions make models like the conventional, poroelastic, and point stress frameworks computationally efficient and conceptually clear, they fail to capture the gradual, energy-driven progression of fracture propagation.

2.2 Energy released -based models

Griffith (1921) introduced the energy balance criterion, which forms the basis of this approach. According to the theory, the fracture process is primarily controlled by the extension of existing cracks rather than the initiation of new ones. As a crack propagates, the associated energy increases due to the growth of the crack's surface area. At the same time, the formation of the new internal surface enhances the material's capacity for elastic deformation, thereby reducing the free energy under external stresses.

The theory relies on several fundamental assumptions. It assumes that the material is isotropic, homogeneous, and behaves as a linearly elastic solid. Additionally, it presupposes the existence of numerous micro-cracks, either on the material's surface or within its structure, and focuses on the propagation of these pre-existing cracks. Moreover, the theory assumes that the size of the elastic body far exceeds the dimensions of the crack being studied. It also simplifies the analysis by disregarding the influence of other cracks, under the assumption that the stresses and strains they produce diminish rapidly with distance. Finally, it assumes that one of the principal stresses acts as a tensile force perpendicular to the crack plane.

Building on Griffith's explanation of unstable fracture propagation, energy release rate-based models apply this framework to hydraulic fracturing. In this context, the energy driving crack propagation is derived from the fluid pressure within the fracture. These models establish relationships between fluid pressure, material characteristics, and the pressure required to initiate fractures. Orowan (1934) and Sack (1946) developed formulations for two-dimensional and penny-shaped fractures, represented by Eq.5 and Eq.6, respectively. Subsequently, Daneshy (1978) extended the approach to include three-dimensional fractures, resulting in the expression shown in Eq.7.

$$P_b = S_p + \sqrt{\frac{2\gamma E}{\pi c(1-\nu^2)}} \tag{5}$$

$$P_b = S_p + \sqrt{\frac{\pi \gamma E}{2c(1-\nu^2)}} \tag{6}$$

$$P_b = S_p + \sqrt{\frac{3\gamma E}{2(1-\nu^2)} \cdot \frac{(c^2+b^2)[E(k)]^2}{c[2(c^2+b^2)E(k) - c^2 K(k)]}} \tag{7}$$

where
- S_p Total principal stress perpendicular to the fracture plane
- γ Fracture surface energy of the solid
- E Young's modulus of the solid
- c Half width of the crack
- b Fracture height
- $E(k)$ Complete elliptical integral of the second kind
- $K(k)$ Complete elliptical integral of the first kind
- k Parameter for the elliptical integrals

The energy release rate-based approach provides a fundamental assessment of breakdown pressure by defining the energy required for fracture initiation. It is particularly relevant for brittle rock formations, where failure is influenced by energy dissipation. However, its accuracy depends on how well the model assumptions align with actual reservoir conditions. This approach has inherent limitations that can reduce the accuracy of its predictions. For instance, it assumes that the borehole behaves like a circular fracture under internal pressure and experiences uniformly distributed stresses from all directions. In contrast to tensile strength-based models, this approach overlooks variations in the minimum and maximum horizontal principal stresses. Additionally, it does not account for fluid leak-off into the surrounding rock formation, further limiting its applicability in complex scenarios.

2.3 Stress intensity-based models

Irwin (1957) proposed the stress intensity approach, which provided a solution to the computational challenges associated with the energy-based method. This approach examines the stresses at the crack tip, defined by the stress intensity factor (K), which depends on the mode of the crack. In the context of hydraulic fracturing, Mode I fracturing is applicable, as tensile fractures are formed. The stress intensity factor for Mode I (K_I) is calculated by combining the stress intensity contributions from various loading sources, including the principal horizontal stresses, wellbore fluid pressure, and the pressure distribution along the fracture. For the fracture to initiate, the estimated mode I stress intensity factor must match the material's fracture toughness. Building on this principle, Rummel (1987) developed a breakdown model (outlined in Eq. 8), which assumes a penny-shaped, symmetrical double crack extending from a circular borehole, oriented perpendicular to the least horizontal stress.

$$P_b = \frac{1}{h_0 + h_a}\left(\frac{K_{1C}}{\sqrt{r_w}} + S_H f + S_h g\right) \tag{8}$$

where K_{IC} Fracture toughness of the rock

S_H & S_h Maximum and minimum horizontal in-situ stresses ($S_H > S_h$)

h_0 Dimensionless stress intensity representing the effect of fluid pressure within the wellbore

h_a Dimensionless stress intensity function accounting for the impact of fluid pressure distributed along the crack

f & g Dimensionless stress intensity factors related to the effects of the maximum and minimum horizontal in-situ stresses, respectively.

The stress intensity factor-based approach appears to provide more reliable predictions, as it incorporates multiple significant parameters when assessing the breakdown pressure (Sampath et al., 2018). Three stress intensity factor models (I to III) were included in this study. The only difference between these models lies in how the fluid pressure distribution is calculated along the crack. Model I assumes a constant pressure distribution along the crack. Model II uses a reduced constant pressure gradient (25% reduction). Model III applies a reciprocal pressure drop. The equations for these pressure scenarios were sourced from Rummel (1987). These variations allow for a comparative evaluation of their influence on breakdown pressure predictions.

It is important to recognise that the models discussed represent foundational frameworks introduced in the field. Over time, numerous modifications and advancements have been proposed in the literature to enhance these models by incorporating additional parameters and factors that significantly influence hydraulic fracturing behaviour.

3 Performance Comparison of Breakdown Models

A comparison of the presented breakdown models was performed based on a collection of hydraulic fracturing experimental results to identify the accuracy and suitability of the models for different rock types. Following a thorough review of numerous studies presenting experimental data from 1977 to 2023, 20 research papers were carefully selected based on the availability of fracture toughness parameters for the considered rock types(Ali & Karakus, 2022; Cheng et al., 2020; Deb et al., 2020; Gao et al., 2020; Hong et al., 2022; Ito & Hayashi, 1991; Li et al., 2020; Long et al., 2023; Muñoz-Ibáñez et al., 2023; Stöckhert, 2015; Wang et al., 2021; Xie et al., 2018; Zhang et al., 2019; Zhou et al., 2021; Zhuang et al., 2023; Zhuang et al., 2020; Zhuang et al., 2019; Zhuang et al., 2018; Zoback et al., 1977).

The dataset, compiled from these studies, consists of 339 data points, each corresponding to a hydraulic fracturing test. To enhance the relevance and clarity of the analysis, the data was categorised based on the type of rock studied. Igneous rocks form the largest category, with 208 data points representing a variety of rock types such as granite, andesite, and rhyolite. Sedimentary rocks account for 114 data points, covering formations like shale, sandstone, and limestone. Additionally, the dataset includes 17 data points related to metamorphic rocks, including examples such as marble and slate.

The evaluation of predictive capabilities was performed on the models presented in this paper, with particular attention to specific aspects of certain models. For instance, the four poroelastic models (I to

IV) were differentiated based on the equations used for estimating Biot's coefficients. Additionally, the three-dimensional fracture model under the energy release rate-based approach was excluded from the comparison due to its computational complexity. In the stress intensity-based approach, three stress intensity factor models (I to III) were considered, focusing on variations in pressure gradients along the crack.

To facilitate comparison among the analytical breakdown models, error percentages were calculated for each data point, and the results are presented in Fig. 1 as a box plot. The error percentage was calculated using Eq. 9. Rather than relying on absolute error percentages, this study retains the sign of the deviation to distinguish whether a model overestimates or underestimates the experimental results. The models displayed in the plot are foundational frameworks under each approach. Consequently, they exhibit a broader range of error percentages due to simplified assumptions, model-specific limitations, and sensitivity to parameters. Despite these limitations, the plot provides insights into the comparative performance of the models.

$$Error\ Percentage\ (\%) = \frac{P_{b(experiment)} - P_{b(prediction)}}{P_{b(experiment)}} \tag{9}$$

Tensile strength-based models demonstrate a broader range of errors, particularly for the conventional model and the upper limit of the point stress model, reflecting their limited accuracy. In contrast, the poroelastic models display significant improvements, making them more reliable and practical for certain applications. Energy release rate-based models exhibit considerable variability, with higher error ranges, likely due to their complexity and reliance on restrictive assumptions. On the other hand, the stress intensity factor models show strong agreement with experimental data, characterised by consistently low error percentages and minimal variation, highlighting their robustness and reliability for breakdown pressure predictions.

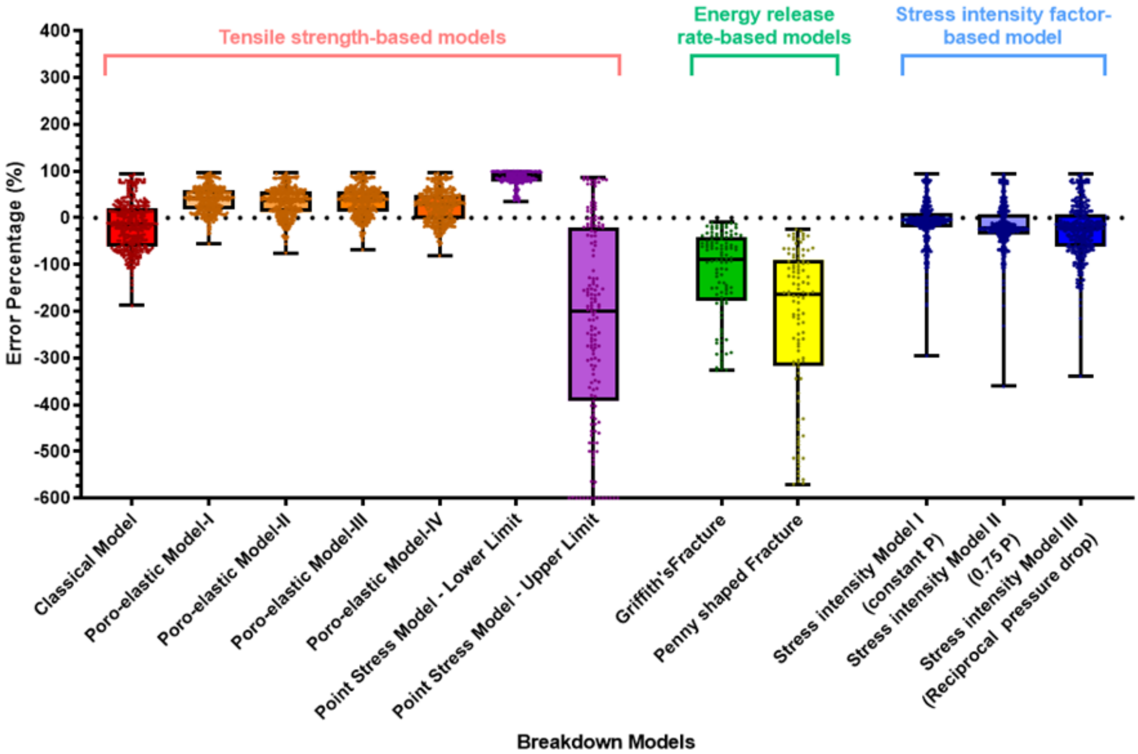

Fig. 1 Performance variability across breakdown pressure prediction models: box plot of error percentage (%) for predicted breakdown pressure

Figure 2 compares the breakdown pressure prediction models across different rock types. While certain models, such as the stress intensity factor models, show consistent performance across different rock types, other models, such as the classical model and energy release rate-based models, exhibit variations in error percentages, highlighting their sensitivity to rock type.

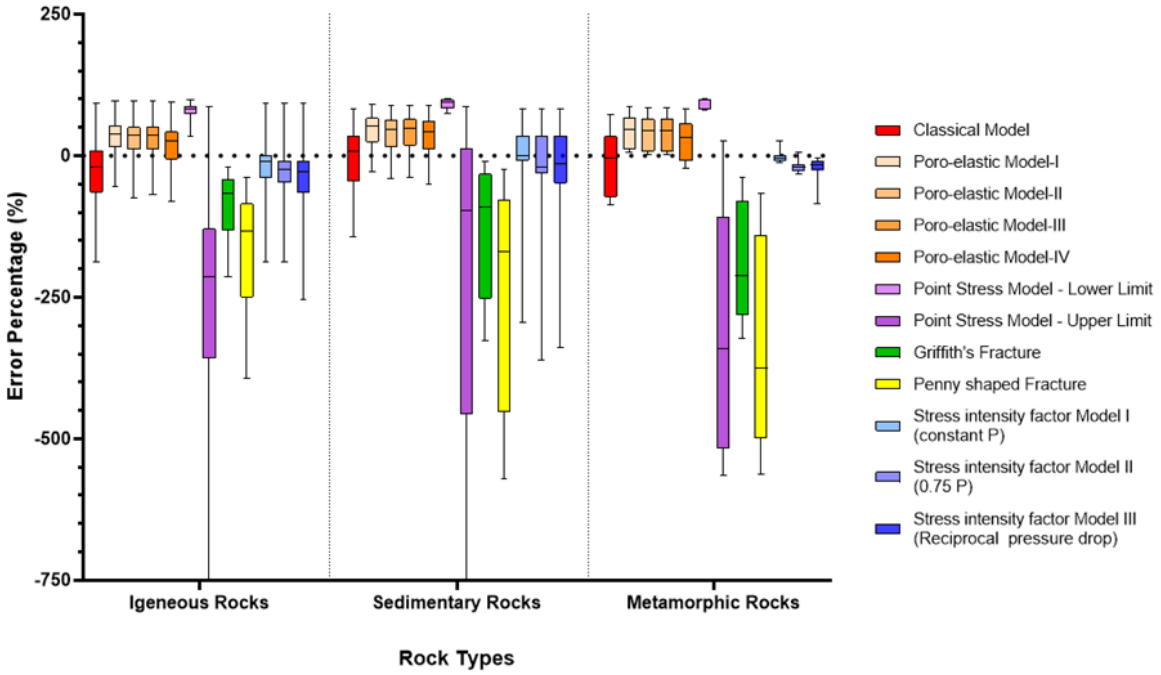

Fig. 2 Comparison of breakdown pressure prediction models across rock types: grouped box plot of error percentage (%) for predicted breakdown pressure by model and rock type

4 Summary

Hydraulic fracturing is a dynamic technique that continues to evolve, with new analytical models being introduced regularly. However, these models have not yet been widely tested or thoroughly investigated. This study focuses on established analytical frameworks that have been widely adopted and validated in the literature. Among them, stress intensity-based models demonstrate the highest reliability, exhibiting consistently low error percentages and minimal variability across different rock types due to their incorporation of fracture toughness and stress concentration effects. Poroelastic models within the tensile strength-based approach also perform well, though they show slightly higher variability. In contrast, energy release rate-based models, while insightful, show broader error distributions and greater computational complexity, limiting their accuracy. These differences arise from how each model incorporates rock heterogeneity, fluid penetration, and stress redistribution. These findings emphasize the importance of selecting appropriate models based on specific geological and operational conditions to ensure reliable breakdown pressure predictions.

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