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Uncertainty and probability analysis for rock slopes empowered by 3D modelling

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Abstract

This paper examines the use of uncertainty and probabilistic analysis in assessing rock slope stability and rockbolting design using 3D modelling techniques. The rationale for conducting probabilistic analysis is to align with the upcoming second generation of Eurocode 7 for rock slope design. This research is applied to two case studies in Norway. The first case involves a wedge failure from a recent road cut in 2019, while the second case study examines a potential wedge failure in a road construction project. Both probabilistic analysis and the partial factor principle are applied to these case studies to evaluate rock slope stability and design rockbolting solutions. Additionally, we compare the results of using simple tetrahedral wedges and realistic wedges in slope stability assessment. Digital mapping of the rock slopes is performed using point clouds from photogrammetry as a supplementary tool to collect sufficient data for uncertainty analysis alongside fieldwork. From the comparison of results obtained from the two stability analysis approaches applied to the case studies, it can be concluded that different model representations have varying impacts depending on the specific case. In addition, the probabilistic method results in 4% to 29% higher Factor of Safety than the deterministic method with partial factors. The findings underscore the importance of integrating probabilistic methods in rock slope stability analysis to better manage uncertainties and enhance safety. This paper aims to highlight the benefits of such integration in anticipation of the new Eurocode 7 standards. The conclusions drawn from these Norwegian case studies provide valuable insights for engineering geologists and practitioners in the field, advocating for the adoption of probabilistic analysis as a standard practice in future rock slope stability assessments.

Keywords

Geotechnical design, rock engineering, digital mapping, partial factor method, reliability analysis

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1 Introduction

Rock slope stability is crucial to engineering geology, especially in regions where infrastructure development requires cutting through rock masses. Slope failures can pose significant safety risks and economic losses, making stability assessment an essential component of the design and construction phases and during the maintenance and operational phases. Traditionally, deterministic method using an approximate wedge model have been used to evaluate rock slope stability; however, this approach often fails to fully account for the actual geometry or the inherent uncertainties in the input parameters. With the rapid advancement of 3D model acquisition methods such as LiDAR scanning and photogrammetry, we can capture high-resolution surface models of rock slopes that provide realistic geometry of the rock slope surfaces and the exposed rock discontinuity surfaces. With sufficient geometric data supplemented from 3D models, probabilistic analysis based on statistical data has become more feasible.

The motivation for this research aligns with the changes in the upcoming second generation of Eurocode 7, which provides more attention and guidance on using probabilistic methods in geotechnical design (Commission et al. 2024). This paper addresses two research questions: (1) how does 3D model representation influence slope stability assessment? (2) how different are the results compared using the deterministic method with partial factors versus the probabilistic method? Via two case studies in Norway, we conduct a comparative study by assessing the stability of unstable wedges using different 3D model representations and calculation methods.

2 Methodology

2.1 Limit equilibrium analysis and input data sources

We calculate the stability of rock wedges at unsupported and supported conditions using limit equilibrium analysis using the software SWedge v7.018 and RocSlope3 v1.005. The design Factor of Safety (*FS*) is assumed 1. We use SWedge to study the simple models, whereas RocSlope3 is for more realistic models. We conduct deterministic analysis with partial factor for both model representations and compare using probabilistic analysis.

For the supported condition, we simulate using pretensioned 33 mm-diameter combined bolts. Assuming a pretensioned load of 60% of the bolt yield load, each bolt is assumed to apply 0.333 MN active force to the wedge. Following Li and Høien, (2023), we calculate the *FS* for shear failure under pretension *T* (active force) using Eq. 1:

$$\text{FS for shear failure} = \frac{Resisting\ force + T_N tan\phi + T_S}{Driving\ force} \tag{1}$$

Where
FS Factor of Safety
T_N Normal component pretension
T_S Shear component of pretension
ϕ Joint friction angle

Input parameters for stability analysis are obtained using various methods. Via fieldwork, we identify the extent of the wedges and study their associated sliding surfaces. We measure the orientations of the sliding surfaces if possible. In addition, we assess the groundwater conditions. We collect rock samples to determine the joint friction angle using tilt tests if possible. Using high-resolution 3D point clouds, we use Maptek PointStudio to extract the waviness and orientation over the exposed sliding surfaces. Some input parameters are assumed due to a lack of data from the field, laboratory or 3D point clouds.

We conduct a sensitivity analysis to study the influence of each input variable on the *FS*. Following the recommendation by the Norwegian National Annex of Eurocode 7 which suggests the use of design method 3 for slope stability (Nilsen et al. 2011), in the deterministic analysis, a partial factor of 1.25 has been applied to the joint friction ($\tan\phi$) and cohesion. The random variables in the probabilistic analysis include joint orientation (only available in SWedge), joint waviness, joint friction angle, joint cohesion. Among all the random variables, 10000 values generated via Monte

Carlo sampling are used in the probabilistic analysis. We assume the unstable wedge is homogeneous (i.e. constant unit weight). Seismic load is not included in this study.

2.2 3D modelling to obtain representative SWedge and RocSlope3 models

Based on the high-resolution 3D point cloud of the slope surface, we interpolate the best-fit planar sliding surfaces for each wedge in the case studies via a developed script, using the visual programming plug-in Grasshopper in McNeel's Rhino3D software. In the Grasshopper script, a tetrahedron encompassing the studied wedge will be obtained with user-specified intersection points between both joint planes, the slope face, and the upper ground face. This tetrahedral wedge will be identical to the SWedge model.

In RocSlope3, a simplified mesh (< 1000 mesh faces) triangulated from the slope surface point cloud is used. The slope mesh is extruded to form a closed volume. The orientation (dip and dip direction) and centre coordinates (X, Y, Z) of the joint planes obtained via the abovementioned Grasshopper script are input parameters to the "measured joints" in RocSlope3. In RocSlope3, ensuring large input values of the joint radii, the planar joint discs will intersect each other and across the slope mesh volume, forming a 3D wedge model with a realistic slope face and upper ground face.

3 Case studies results

We have analysed two wedges from two different road cuts in Norway (Fig. 1):

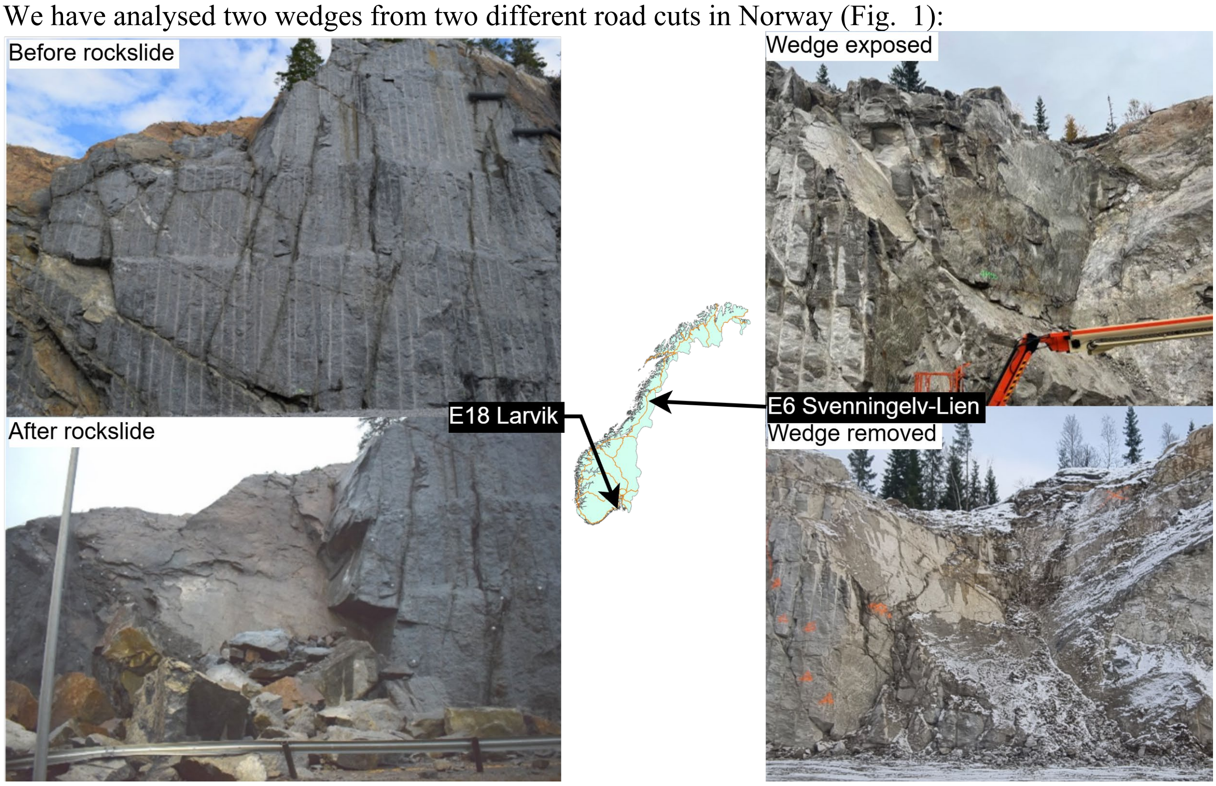

Fig. 1 Location of the two case studies in Norway and overview pictures when the wedges were still in place and slided/removed Image sources: Map – Kartverket; E18 Larvik images: Norwegian Public Road Administration; E6 Svenningelv-Lien image (wedge removed): NGI.

3.1 E18 Larvik

The E18 Larvik case involves analysing a wedge failure along a rock cut on an operating highway that occurred in December 2019, about 180 m from the eastern portal of the Larvik Tunnel. The bedrock consists of larvikite, a coarse-grained monzonitic igneous rock with large-sized feldspar crystals. The wedge was identified as being formed by two intersecting joints (Joint 1 and Joint 2), with a height of approximately 23 meters. The wedge is found to be around 1100 m³ (Nilsen et al. 2020). A 3D point cloud of the rock slope before the slope failure was acquired using photogrammetry based on Google Streetview images, whereas a 3D point cloud with higher resolution and relative accuracy was acquired a month after the failure, using drone images and camera images taken from the lift and the ground (Nilsen et al. 2020). Both point clouds are georeferenced ground control points distributed on the top and bottom of the slope.

The post-failure point cloud revealed the entire surface of Joint 1 and a surface parallel to Joint 2. Joint orientation and waviness data are collected from these surfaces (Mosling 2024). The input parameters and their statistical distributions are listed in Table 1. Joint orientation is assumed to be randomly distributed with Fisher distribution in SWedge, whereas a truncated exponential distribution of joint waviness is more suitable based on the waviness results. Joint friction angle and cohesion are assumed equal for both joints and follow a normal distribution truncated by a larger relative minimum and a smaller relative maximum. The mean friction angle and cohesion values are back-calculated using the mean input values for SWedge probabilistic analysis, assuming the mean wedge *FS* ≈ 1. Standard deviation values of friction angle and cohesion are based on literature values summarised in Mosling (2024). The required pretension force, with the bolting orientation proposed in Mosling (2024), is back-calculated in the same way but aims at mean wedge *FS* ≈ 1.5. The back-calculated total pretension load corresponds to 37 pcs. 33-mm diameter combined bolts. In this study, a mean joint friction angle of 44° is chosen, similar to the back-calculated value of 44.6° in Nilsen et al. (2020). Unlike Nilsen et al. (2020), joint cohesion ~ 0.03 MPa is considered in this study. However, joint water pressure is assumed to be zero. Results from 3D modelling is illustrated in Fig. 2a. The RocSlope3 model's volume is 1079.3 m^3, while its encompassing SWedge model is 1286.1 m^3 at maximum. To compare the stability calculations using the same weight force, the unit weight used in the SWedge analyses is proportionally reduced (see Table 1).

The resulting mean wedge *FS*, mean *FS*, and the probability of failure (PF) from probabilistic analysis are listed in Table 1. For the Larvik case, probabilistic analyses give 4-6% and 17-29% higher mean *FS* in RocSlope3 and SWedge, respectively, than deterministic analyses with partial factor. The deterministic method yields a higher mean *FS* (5-7%) in RocSlope3 compared to SWedge, but the probabilistic method shows an opposite trend (4-14% difference). The steep curves for Joint 1 dip and dip direction from the sensitivity analysis indicate that Joint 1 orientation is very influential on the *FS* for the Larvik wedge. The shape of the cumulative *FS* curve is similar to the RocSlope3 model with or without rock support. However, the cumulative *FS* curve for the supported SWedge model is significantly flatter than without support and the other scenarios, suggesting a different distribution of the *FS* for the supported SWedge model.

Table 1 Larvik wedge stability analysis using deterministic and probabilistic methods. List of input parameters and results. μ = mean, σ = standard deviation, rel min = relative minimum, rel max = relative maximum.

Input Parameters	**Deterministic, SWedge**	**Deterministic, RocSlope 3**	**Probabilistic, SWedge**	**Probabilistic, RocSlope3**
Slope dip/dip direction (°)	80/134	-	80/134	-
Upper face dip/dip direction (°)	33/180	-	33/180	-
Joint 1 dip/dip direction (°)	56/118	56/118	μ: 56/118 | σ: 7	μ: 56/118
Joint 1 Waviness (°)	3.7	3.7	μ: 3.7 | σ: 3.7 rel min: 3.7 | rel max: 28.0	μ: 3.7 | σ: 3.7 rel min: 3.7 | rel max: 28.0
Joint 2 dip/dip direction (°)	73/173	73/173	μ: 73/173 | σ: 9	μ: 73/173
Joint 2 Waviness (°)	6.7	6.7	μ: 6.7 | σ: 6.7 rel min: 6.7 | rel max: 40.3	μ: 6.7 | σ: 6.7 rel min: 6.7 | rel max: 40.3
Joint 1 & 2 Friction angle (°)	37.7	37.7	μ: 44.0 | σ: 4.5 rel min: 10 | rel max: 2	μ: 44.0 | σ: 4.5 rel min: 10 | rel max: 2
Joint 1 & 2 Cohesion (MPa)	0.024	0.024	μ: 0.03 | σ: 0.012 rel min: 0.03 | rel max: 0	μ: 0.03 | σ: 0.012 rel min: 0.03 | rel max: 0
Wedge height (m)	11.9	-	11.9	-
Unit weight (kg/m^3)	0.0234	0.0279	0.0234	0.0279
Total pretension load (MN), at trend/plunge 325.8°/0.4°	9.24	9.24	9.24	9.24
Results				
Mean Wedge *FS* (Mean *FS*), Unsupported	0.85	0.91	1.05 (0.99)	1.13 (0.95)
PF, Unsupported	-	-	0.5580	0.6369
Mean Wedge *FS* (Mean *FS*), Supported	1.26	1.32	1.51 (1.62)	1.59 (1.40)
PF, Supported	-	-	0.1140	0.0030

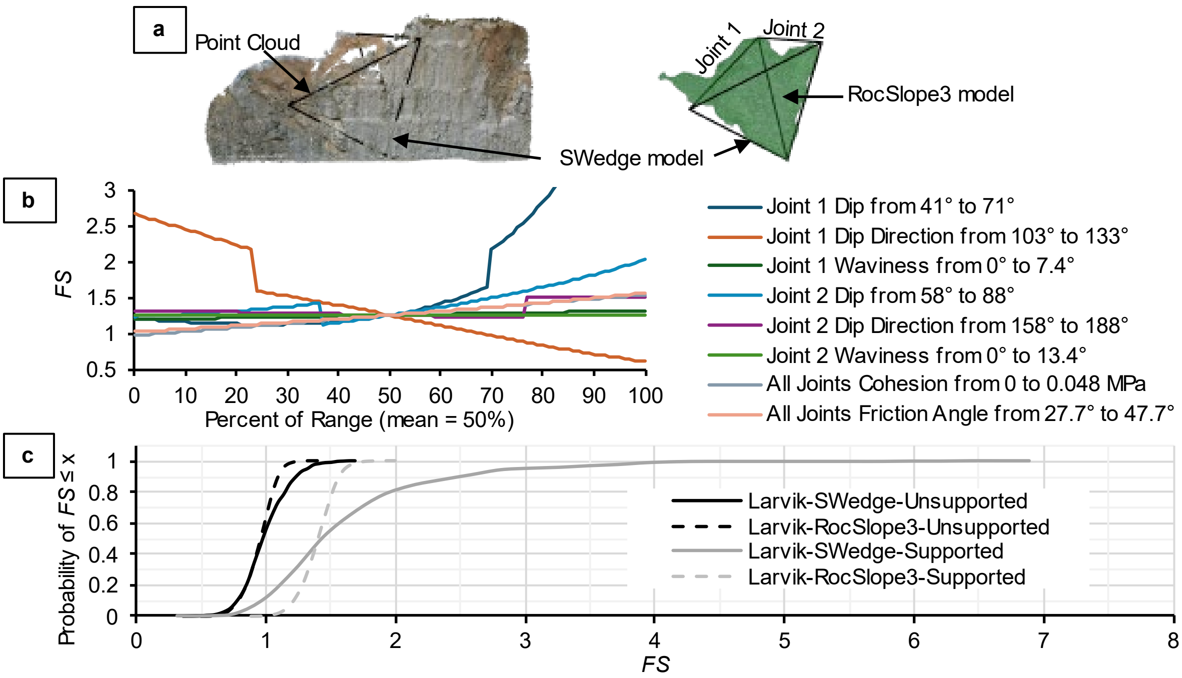

Fig. 2 Results of 3D modelling and limit equilibrium analysis for Larvik case study. (a) The 3D point cloud was used to determine the SWedge and RocSlope3 models. (b) Results of sensitivity analysis of all the input variables. (c) Cumulative plot of *FS* from probabilistic analysis.

3.2 E6 Svenningelv-Lien

E6 Svenningelv-Lien was under construction during this study. The studied wedge is located at around chainage number 9450. The bedrock is marble. A 3D point cloud from drone images acquired about a year before the fieldwork for this study is available. Comparing the wedge exposed in the field image in Fig. 1 and its earlier state in the point cloud in Fig. 3a, the rock cut has been excavated further down, and the wedge is missing its footing. Just a few weeks after the fieldwork, it was decided to remove the wedge due to the potential instability of the wedge.

Two prominent joints forming the studied wedge can be seen in the 3D point cloud and the field. As summarised in Table 2, all the input random variables share the same distribution as those in the Larvik case study. Joint orientation and waviness data are collected from the limited exposure of the surfaces in the 3D point cloud and in the field, while joint friction angles of both joints are determined using tilt tests in the laboratory (Mosling 2024). Cohesion is assumed 0.01 MPa. The required pretension force is back-calculated in the same way as the Larvik case (Section 3.1). Standard deviation values of friction angle and cohesion are based on the same literature values as in the Larvik case study. Joint water is assumed zero. To simulate a wedge daylighting the rock slope, a plane parallel to the slope surface is used to trim the slope model in RocSlope3. The RocSlope3 model has a volume of 400.1 m^3, which is significantly smaller than the SWedge model (maximum 640.5 m^3) (Fig. 3a). As a result, the unit weight used in the SWedge analyses is proportionally reduced for the sake of using the same weight force for all the calculations (Table 2).

The back-calculated total pretension load corresponds to nine pcs. 33-mm diameter combined bolts (Table 2). Probabilistic analyses in this case study give about 4-6% higher mean *FS* than deterministic analyses with partial factor. Comparing the mean *FS* in Table 2, the RocSlope3 and SWedge models have comparable kinematic stability. Also, the shape of the cumulative *FS* curves is similar for each model with or without rock support (Fig. 3c). However, the supported RocSlope3 model has a lower PF and steeper cumulative *FS* curve than its SWedge counterpart. The gradients of the curves in the sensitivity plot in Fig. 3b are relatively flat, with Joint 1 dip direction and cohesion being the most influential parameters on the *FS*.

Table 2 Svenningelv- Lien wedge stability analysis using deterministic and probabilistic methods. List of input parameters and results. μ = mean, σ = standard deviation, rel min = relative minimum, rel max = relative maximum.

Input Parameters	**Deterministic, SWedge**	**Deterministic, RocSlope 3**	**Probabilistic, SWedge**	**Probabilistic, RocSlope3**
Slope dip/dip direction (°)	84/268	-	84/268	-
Upper face dip/dip direction (°)	06/302	-	06/302	-
Joint 1 dip/dip direction (°)	57/229	57/229	μ: 57/229 | σ: 4	μ: 57/229
Joint 1 Waviness (°)	2.5	2.5	μ: 2.5 | σ: 2.5 rel min: 2.5 | rel max: 1.5	μ: 2.5 | σ: 2.5 rel min: 2.5 | rel max: 1.5
Joint 2 dip/dip direction (°)	59/297	59/297	μ: 59/297 | σ: 6	μ: 59/297
Joint 2 Waviness (°)	7.2	7.2	μ: 7.2 | σ: 7.2 rel min: 7.2 | rel max: 41.3	μ: 7.2 | σ: 7.2 rel min: 7.2 | rel max: 41.3
Joint 1 Friction angle (°)	36.3	36.3	μ: 42.6 | σ: 4.5 rel min: 10 | rel max: 2	μ: 42.6 | σ: 4.5 rel min: 10 | rel max: 2
Joint 2 Friction angle (°)	35.2	35.2	μ: 40.3 | σ: 4.5 rel min: 10 | rel max: 2	μ: 40.3 | σ: 4.5 rel min: 10 | rel max: 2
Joint 1 & 2 Cohesion (MPa)	0.008	0.008	μ: 0.01 | σ: 0.004 rel min: 0.01 | rel max: 0	μ: 0.01 | σ: 0.004 rel min: 0.01 | rel max: 0
Wedge height (m)	13.5	-	13.5	-
Unit weight (kg/m^3)	0.0174	0.0279	0.0174	0.0279
Total pretension load (MN), at trend/plunge 325.8°/0.4°	2.25	2.25	2.25	2.25
Results				
Mean Wedge *FS* (Mean *FS*), Unsupported	0.96	0.95	1.17 (1.01)	1.15 (0.99)
PF, Unsupported	-	-	0.4620	0.5535
Mean Wedge *FS* (Mean *FS*), Supported	1.29	1.28	1.53 (1.37)	1.52 (1.34)
PF, Supported	-	-	0.0020	0.0001

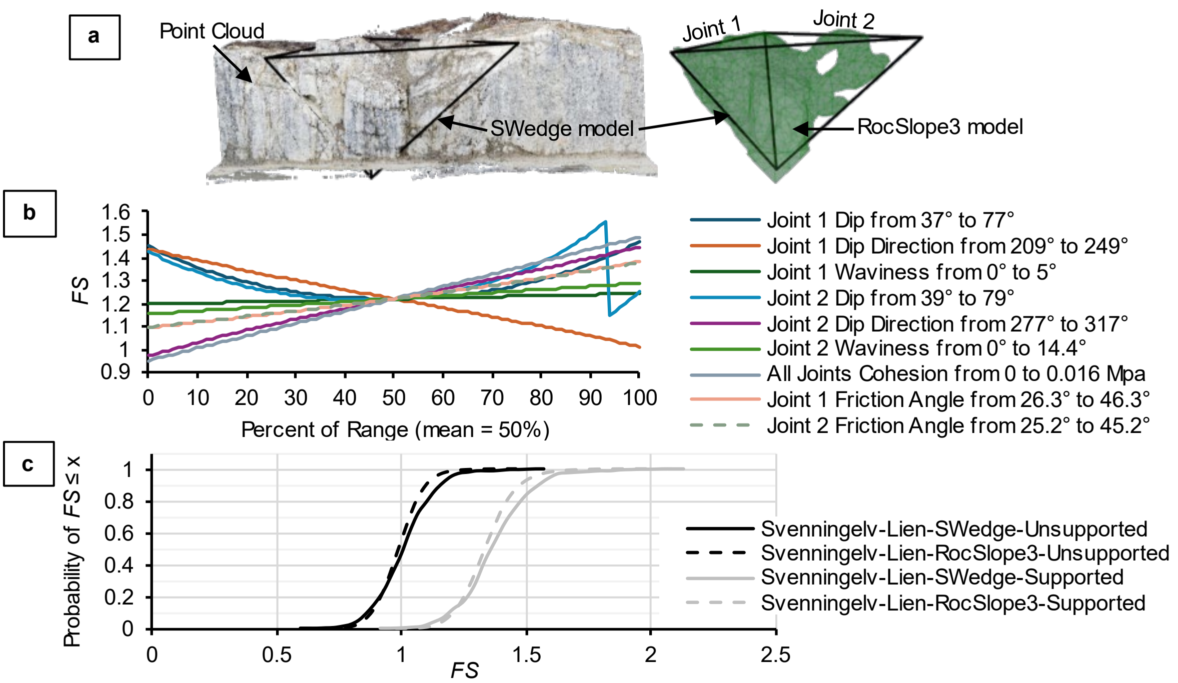

Fig. 3 Results of 3D modelling and limit equilibrium analysis for Svenningelv-Lien case study. (a) The 3D point cloud used to determine the SWedge and RocSlope3 models. (b) Results of sensitivity analysis of all the input variables. (c) Cumulative plot of *FS* from probabilistic analysis.

4 Discussions

4.1 Comparing simple and realistic models

The differences between using simple models versus more realistic representation for stability analysis are polarising for the Larvik case but are less pronounced for the Svenningelv-Lien case. As illustrated in Fig. 2a and Fig. 3a, with the exception of Joint 1 in the Svenningelv-Lien wedge, the joint areas in the SWedge models are larger than those in the RocSlope3 models. These results are unexpected, as the cohesive resisting force is proportional to the joint area. Therefore, it would be anticipated that the models with larger sliding surface areas (primarily the SWedge models) would yield greater *FS*.

The significantly lower mean FS (17-29% difference) for the Larvik SWedge probabilistic analysis compared to its RocSlope3 counterpart, is likely due to the inclusion of joint orientation as a random variable in the SWedge analysis. Variations in joint orientation result in changes to the wedge volume and, consequently the *FS*. In contrast, RocSlope3 does not permit variability in joint orientation input. Accounting for uncertainty in joint orientation is critical for rock slope stability analysis and rockbolting design, particularly when joint surfaces are not directly exposed and can only be inferred from trace observations. Future research should focus on developing efficient computational methods to generate realistic block volumes with random joint orientations.

Considering the rockbolting design, it is more appropriate to use the realistic model to design the placement of systematic bolting, so that the rock support locations can match the real situations better.

4.1 Comparison of Deterministic and Probabilistic Approaches

Collecting statistical values for the input variables for probabilistic analysis demands more time for data acquisition and processing than the deterministic method. However, the resulting input data values are more representative and case-specific, facilitating a more informed decision-making process for assessing slope stability and rock support design.

The comparison between deterministic and probabilistic approaches revealed significant differences in rock slope stability assessment. In all the cases, the probabilistic method gives a higher mean *FS* than the deterministic method with partial factors, ranging from 4% to 29%. This may indicate that the deterministic method, which relies on single-value inputs, often provides a more conservative estimate, thus resulting in a higher need for rock support. On the contrary, probabilistic analysis provides a more informed estimate, incorporating uncertainties and offering a better understanding of the potential failure scenarios. This approach is particularly important in complex geological settings where the inherent variability in rock mass properties can significantly impact stability, such as the Joint 1 orientation in the Larvik case, which is highly influential than any other input parameters.

Considering the target value of the reliability index, β, which equals $-\Phi^{-1}$(PF), suggested in Eurocode (EN1990-1 Annex C), a Consequence Class 2 ("normal consequence") given a 50-year reference period is 3.8. The corresponding PF shall not exceed 0.0001. Only the supported RocSlope3 in the Svenningelv-Lien case fulfils the target β. This suggests that the proposed rock reinforcement using pretensioned bolting is not sufficient.

4.1 Limitations and recommendations for future practice

Many input parameters are inferred from previous literature due to a lack of field, laboratory, or 3D model data. This reliance on assumptions impacts the accuracy of probabilistic analysis. To minimise uncertainties in the results, it is recommended to prioritise the collection of additional data and to thoroughly evaluate the quality of the most influential input parameters. In this study, joint waviness is calculated along the joint dip direction; however, it would be more realistic to evaluate joint waviness along the sliding direction. A simplified mesh is used in RocSlope3, which may amplify the influence of wedge volume on stability, particularly if the wedge being considered is relatively small (e.g., less than 100 m³). A sensitivity analysis to examine the effect of mesh resolution on volume calculations is therefore recommended. Additionally, errors stemming from Monte Carlo sampling due to limited sample size should be accounted for. Increasing the sample size in the probabilistic analysis is recommended to minimise such errors.

This study demonstrates the key differences in input data acquisition and processing steps for conducting limit equilibrium analysis using deterministic and probabilistic methods. The probabilistic method facilitates objective risk quantification, whereas the deterministic method avoids the need for complex statistical analysis. To combine the benefits of both approaches, the conversion procedure proposed by Tsegaye and Gylland (2019) can be used to determine the site-specific partial factor values that considers the target β, the influence of random variables, and their statistical distributions.

Digital tools should be integrated into the workflow for geometric analysis and rock support design. These tools offer significant advantages in terms of efficiency and visualisation, and should be further incorporated with the probabilistic approach for rock support optimisation using realistic geometry.

5 Conclusions

- This study aims to address two main research questions: how 3D model representation influences rock slope stability assessment, and how the results differ when comparing deterministic methods with partial factors versus probabilistic methods.
- The influence of 3D model representation is assessed by comparing simplified tetrahedral wedge models with more geometrically realistic ones. The results vary between the case studies; it shows no significant differences in the stability results for the Svenningelv-Lien case but polarising results for the Larvik case. However, smaller blocks (e.g. < 100 m^3) may be highly affected by the resolution of the realistic model and should be further investigated.
- Probabilistic analysis provided a more representative estimate of rock slope stability than deterministic approaches, resulting in 4% to 29% higher Factors of Safety in the studied cases. This highlights the benefits of probabilistic approaches in capturing variability in geotechnical parameters compared to the deterministic approach and should be further implemented for rock support design optimisation.
- Digital mapping using high-resolution point clouds from photogrammetry effectively supplements traditional field mapping, with detailed geometrical input for reliability analysis and reducing uncertainty in stability assessments. Enhanced data collection efforts and quality evaluation of influential parameters are recommended to improve the reliability the results
- The findings from the E18 Larvik and E6 Svenningelv-Lien case studies underscore the importance of integrating probabilistic methods with digital modelling techniques to better characterise geotechnical variability and enhance the safety of engineered rock slopes.
- With the upcoming second generation of Eurocode 7, probabilistic analysis is expected to become standard practice in geotechnical design. The conclusions drawn here advocate for early adoption of these reliability-based methods by engineering geologists and rock engineers to improve design resilience and reduce uncertainty in future rock slope stability assessments.

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