

Full stress tensor determination using the CCBO method, sensitive to the variable stiffness matrix of the same rock material

Alice Petrlikova, Lubomir Stas, Petr Konicek

*The Czech Academy of Sciences, Institute of Geonics
alice.petrlikova@ugn.cas.cz*

Abstract

The objective of quantifying the in-situ stress of a rock mass is to achieve the greatest possible precision, which can be accomplished by overcoring methods. However, it should be noted that this process is not without its difficulties, chief among these being the necessity of determining the deformation parameters of anisotropic rock material. In order to determine the full stress tensor, it is necessary to ascertain both the full stiffness matrix of the rock material and the orientation of the principal axes of the anisotropy material with respect to the measuring probe.

The collection of oriented samples from the vicinity of the measurements, in conjunction with their subsequent testing and the research study, was undertaken to ascertain the full stiffness matrix of the rock material. The oriented stiffness matrices of the rock material were subsequently implemented in a numerical model to determine the distribution matrices, which represent the relationships between measured strains and the sought-after stress tensor for each stiffness matrix originally.

The outcomes of the uniaxial compression test and the special dynamic ultrasonic wave measurement technique demonstrated that the stiffness of the same rock material is subject to variation. It is acknowledged that each approach possesses a unique set of strengths and limitations; however, it is important to note that the findings obtained from these methodologies should not be disregarded as erroneous. Rather, they are employed in the sensitivity analysis of the resulting tensor to the input data.

The article presents the variable results of principal axes directions of full stress tensors, which are determined from measured strains using the variable stiffness matrices of the Grimsel granit. The strain measurements were conducted using the Compact Conical Overcoring Method (CCBO) in the vicinity of the underground work at the Grimsel Testing Site in Switzerland.

Keywords

CCBO method, stiffness matrix, anisotropic rock material, full stress determination, Grimsel Testing Site

1 Introduction

1.1 Overview

The CCBO method is utilised to ascertain the stress within the rock mass, employing a special cone probe that is positioned at the end of the borehole. This strain gauge probe is overcored to relieve the stresses that were initially present at the site, and the full stress tensor at the idealised point is ascertained on the basis of the measured strains on strain gauges placed in specific directions on the surface of the probe. An integral part of determining the stress tensor is determining the stiffness of the rock material. This is the input to the numerical modelling, the output of which is the necessary stress-strain relationship, here called the distribution matrix (Petrlikova 2024). The numerical model simulates the overcoring of a probe placed in a borehole under certain stresses.

The rock environment of the probe is assumed to be homogeneous and linearly elastic. Most commonly, deformation characteristics are determined in laboratories assuming that the material is isotropic. However, in most cases, the rock material exhibits signs of anisotropy, and the utilisation of an isotropic material model may introduce errors into the process of determining the resulting stress (Amadei 1983; Nunes 2002). Scientific outputs worldwide are changing the view of what is the fundamental standard, and this has resulted in the use of the full stiffness matrix of a transversely isotropic material in determining the full stress tensor in a rock mass (Hakala and Sjöberg 2006). The most recent trend in this area is the sensitivity analysis of the resulting stress tensor on hypothetical variations of the rock material stiffness matrix (Krietsch et al. 2018).

1.2 Laboratory testing of the anisotropic material

Laboratory tests are utilised to ascertain the deformation and strength characteristics of rock material. The most frequently employed tests are uniaxial compression test (UCT), triaxial compression test, uniaxial tensile test, indirect tensile test or biaxial test directly in situ on drilled cores (Barla 1972; Dambly et al. 2019; Hakala and Heikkilä 1997; Krietsch et al. 2018;). In addition, ultrasonic wave measurements are also widely used to determine the deformation characteristics of rocks (Liao et al. 1997; Aminzadeh et al. 2022). The uniaxial compression test is the simplest and most widely used method for evaluating the strength and deformation characteristics of rock material.

1.2.1 Uniaxial compression test

The methodology employed for the determination of the deformation and strength characteristics of isotropic materials is that outlined in Bieniawski and Bernede (1979). This methodology involves the execution of uniaxial compression tests, which permit the determination of deformation characteristics through a variety of methods, particularly with regard to the selection of the range of values from which the modulus of elasticity is to be evaluated. The methodology stipulates that the part of the stress-strain curve that appears linear can be used to evaluate the modulus of elasticity. Alternatively, the tangent or secant method can be used. However, if the result is to be taken as the elastic modulus of an anisotropic material, no standard can be referred to. In the majority of cases, the authors rely on the determination of the elastic modulus for an isotropic material model according to Martin and Chandler (1994), which describes the procedure of evaluating uniaxial compression tests for an isotropic material and they provide an upgrade for anisotropic rock material (Dambly et al. 2018; Hakala and Heikkilä 1997; Hakala et al. 2005; Nejati et al. 2019).

1.2.2 Velocity anisotropy measured on a spherical specimen

The measurement of velocity anisotropy under hydrostatic pressure loading on spherical specimens is a specialised procedure that involves the use of a prototype pressure chamber. This apparatus was developed at the Geological Institute of the CAS (Petruzalek et al. 2023). The instrument is capable of measuring both P and S wave velocities due to the special highly viscous layer that surrounds the spherical rock sample. The instrument is equipped with automatic stepper motors, which can be used to rotate the spherical sample and thus detect ultrasonic waves at different positions. This allows the full stiffness matrix of the dynamic constants of the rock material to be determined as the pressure gradually increases. The spherical rock sample is loaded by the hydrostatic pressure of the fluid and thus deforms only due to the inherent deformation characteristic of the rock.

Utilising this method enables the determination of the full stiffness tensor of the rock material under increasing pressure. This approach is distinct from conventional mechanical tests in that it is not constrained by theoretical assumptions or boundary conditions, which often limit the results of basic mechanical tests. The full stiffness tensor provides insights into the anisotropy of the rock material,

categorised as isotropic, transversely isotropic, or orthotropic. The determination of the stiffness matrix of the rock material is then facilitated, along with that of the principal axes directions of the material constants.

1.3 CCBO method

1.3.1 The procedure

The CCBO method is one of the methods for measuring stresses in rock mass (Fig. 1). Its practical installation procedure, evaluation method, stress tensor calculation and associated errors are described in the Sugawara and Obara (1999). The CCBO method allows the determination of the full stress tensor from a single borehole, and the application of the whole method could be divided into three phases. The first phase is the actual production of the probe, the second phase is the installation of the probe into the rock mass. This is followed by the process of overcoring the probe. The third phase is the analysing of the results by a geomechanics who assesses the condition of the borehole by viewing the video inspection, evaluates the measurements and excludes the measurements from the calculation if there are erroneous measurements. Subsequently, a laboratory test is conducted to evaluate the elastic parameters of the rock material, thereby determining its degree of anisotropy and the necessary deformation characteristics. This is followed by the calibration of the raw strain gauge data against temperature. It is only from this sorted and calibrated data, in conjunction with the results of the laboratory test, that the mathematical procedures can be followed to determine the full stress tensor, together with the errors of the method.

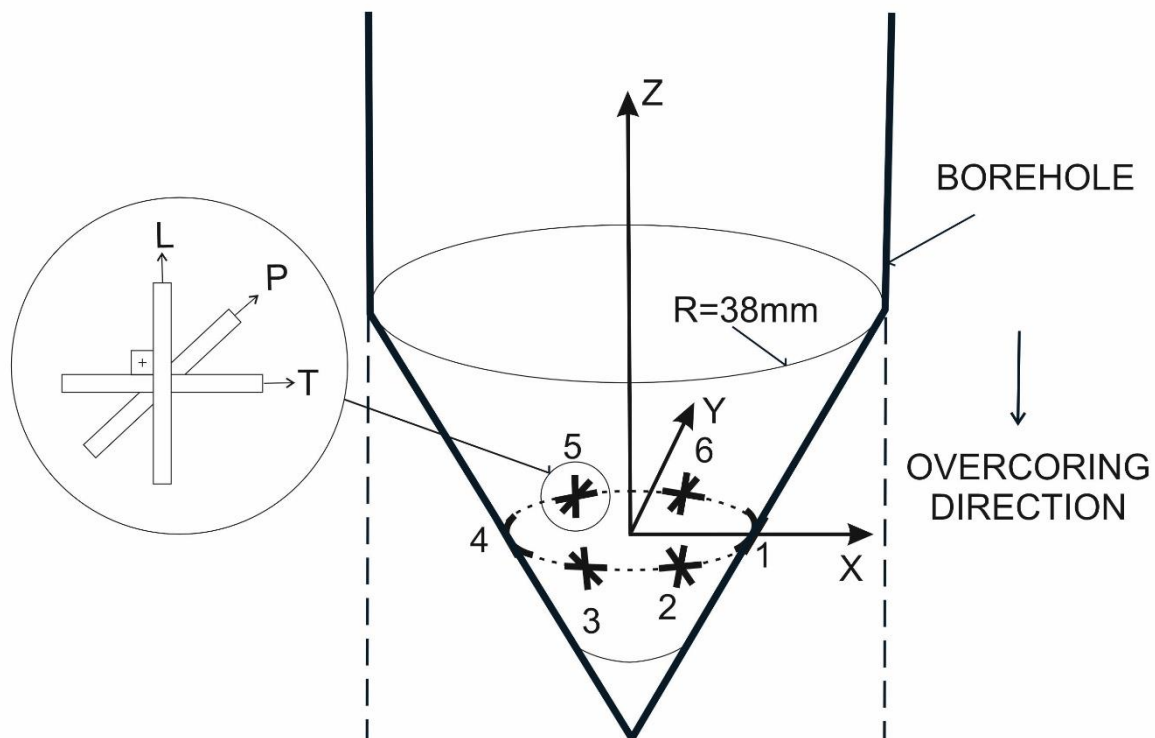


Fig. 1 CCBO probe situated at the end of a borehole and then overcored (ϵ_{ij} – measured strains in directions belonging to individual strain gauges; i – position around the circumference of the probe ($i = 1-6$); j – direction of individual strain at the position (direction T, L or P)).

1.3.2 Theoretical background

The underlying physical principle is the relaxation of the immediate surroundings of the probe, which have been stressed by the original primary stress, through the deformation of these surroundings caused by the overcoring. The method requires a numerical modelling (using Finite Element Method). This is necessary in order to establish the searched for relationship between the strain and the stress and to determine the error in the resulting stress tensor.

The numerical model utilises the superposition principle, whereby unit superposition stress states are applied to the rock model of the probe surroundings. The model simulates the actual overcoring

process, leading to deformation responses of the rock surroundings of the probe through strain gauges. The strain responses are monitored under the action of each superposition state, and by aggregating all the strain responses, a matrix is constructed, which is designated as the distribution matrix.

Eq. 1 introduces constitutive law between measured strains and searched for stress tensor:

$$\varepsilon_{ij} = (D) \cdot \sigma \quad (1)$$

Where ε_{ij} Vector of measured strains [unitless], i – position of the strain gauge around the circumference of the probe, $i = (1-6)$, j – direction of the strain gauge at the i -th position (direction T, L or P; see Fig. 1)
 σ Vector of unknown parameters of the stress tensor, $\sigma = (\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{xz})$ [MPa]
 (D) Distribution matrix, the number of rows is equal to the number of components of the vector ε_{ij} [GPa⁻¹]

The distribution matrix captures information about the stiffness of the rock material, the transformations between the rectangular coordinate system and the system of directions of measured strains through strain gauges on the conical probe, and the redistribution of stresses resulting from the overcoring process in the immediate vicinity of the conical probe.

The number of measured strain gauges exceeds the number of unknown components of the stress tensor; this problem is therefore overdetermined, and is typically solved using the Least Squares Method, which yields the following solution:

$$\tilde{\sigma} = (D^T \cdot D)^{-1} \cdot D^T \cdot \varepsilon_{ij} \quad (2)$$

Where $\tilde{\sigma}$ Vector of unknown parameters of the stress tensor,
 $\tilde{\sigma} = (\tilde{\sigma}_x, \tilde{\sigma}_y, \tilde{\sigma}_z, \tilde{\sigma}_{xy}, \tilde{\sigma}_{yz}, \tilde{\sigma}_{xz})^T$ [MPa]
 D^T Transposed distribution matrix [GPa⁻¹]

2 In situ rock stress determination

2.1 Deformation parameters of Grimsel granit

The stiffness matrices of the rock material were initially derived from the laboratory results of oriented samples collected in the immediate vicinity of the CCBO measurement site. The rock spherical specimens were then subjected to detailed measurement of seismic velocity anisotropy using unique apparatus (Aminzadeh et al. 2022). The experiment yielded the full dynamic stiffness matrix, as well as the directions of principle axes of the anisotropic rock material. Aminzadeh et al. (2022) declare that the Grimsel granite is transversely isotropic under atmospheric pressure. The degree of anisotropy of the rock decreases with applied confining pressure due to closing of preferentially oriented cracks. The Grimsel granite is very sensitive to pressure and becomes almost isotropic at high chamber pressures (100 MPa).

A research study was conducted to identify additional laboratory-determined stiffness matrices of anisotropic materials from the same rock mass. These stiffness matrices were determined using uniaxial compression test measurements. It is imperative to acknowledge that these stiffness matrices represent static material constants, in contrast to the dynamic material constants that are derived from wave velocity measurements. The principal axes of the dynamic material constants were assigned to their static counterparts. This was due to the requirement for directional information of these principal axes in order to facilitate stress determination. The underlying rationale for this decision stemmed from the fact that the static material constants had been obtained through research analysis, and thus their orientation to the measuring probe remained unknown.

Nejati et al. (2019) present results of tangent and secant method to evaluate stiffness of the transversal isotropy rock material from the Grimsel Testing Site (GTS) indicating that the rock material is

transversely isotropic. The tangent values of the elastic constants were determined for the purpose of analysing the load-dependency of the elastic constants. The secant values of the elastic constants were obtained to analyse the average deformational behaviour of the rock.

The Table 1 introduces the selected deformation parameters that were used for stress determination. In the ensuing analysis of the resultant stresses, the deformation parameters selected for investigation were (as named in Table 1): 1 – *isotropic*, where the aforementioned deformation parameters were selected on the basis of research analyses (Krietsch et al. 2018). The static deformation characteristics 2 – *UCT tangent* and 3 – *UCT secant*, which are the deformation parameters determined for tangent and secant method respectively, traced from a research analyses based on the uniaxial compression test results by Nejati et al. (2019). The dynamic deformation characteristics, 4 – *dynamic 0.1 MPa*, 5 – *dynamic 12 MPa*, and 6 – *dynamic 50 MPa*, utilise the measurement of ultrasonic waves on spherical specimens, which have been considered for chamber pressures of 0.1, 12, and 50 MPa. The 0.1 MPa pressure is equivalent to atmospheric pressure; the 12 MPa pressure is the assumed in situ stress at the measurement location; and the 50 MPa pressure is the crack-closing pressure for oriented cracks, as reported by the authors Aminzadeh et al. (2022).

In the course of the testing of spherical samples, it became evident that closure of oriented pore space resulted in alterations to the direction of the plane in which the rock material would be considered isotropic. Consequently, three distinct orientations of the principal axes of the material constants were taken into account during the stress tensor evaluation. Consequently, three distinct stiffness matrices consisting of dynamic material constants with three different orientations corresponding to pressures chamber 0.1MPa, 12MPa and 50MPa were selected.

With regard to the uniaxial compression test results, it was not possible to determine the orientations of the stiffness matrices with respect to the measuring probe; therefore, these static material constants were assigned the orientations specified for the dynamic material constants determined by wave velocity measurement. The tangent deformation parameters were set up to an orientation belonging to atmospheric pressure of 0.1 MPa, named as *tangent (0.1 MPa)* (see Fig. 3), whereas the secant deformation parameters were set up for two different orientations assigned to the orientation of material characteristics belonging to pressure chamber in wave velocity measurement 12 MPa and 50 MPa in, so named in Fig. 3 as *UCT secant (12 MPa)* and *UCT secant (50 MPa)*. This indicates that a total of seven distinct stiffness matrices were utilized for stress determination, with the secant parameters applied in a duplicated manner, each for a distinct orientation of the principal axes of the material constants.

Table 1 Selected deformation parameters of the Grimsel granite laboratory results. Where E, G, ν and E', G', ν' are deformation parameters of the rock material in the plane of isotropy and perpendicular to it respectively. This data was taken and adapted from Aminzadeh et al. (2022) and Nejati et al. (2019).

	E [GPa]	E' [GPa]	G [GPa]	G' [GPa]	ν [-]	ν' [-]	K _E [-]
1 – isotropic	22	22	14.7	14.7	0.25	0.25	1
2 – UCT tangent	27.2	6.9	7.9	11.7	0.17	-0.04	3.9
3 – UCT secant (used twice with different directions)	43	22.5	13.6	17.3	0.24	0.07	1.9
4 – dynamic 0.1 MPa	35.5	18	9.5	14	0.33	0.18	1.9
5 – dynamic 12 MPa	53	41	17	20	0.27	0.24	1.3
6 – dynamic 50 MPa	86	77.5	27	32	0.23	0.32	1.1

As demonstrated in Table 1, the dynamic Young's and shear moduli exhibit higher amplitudes compared to the static deformation characteristics. This phenomenon can be attributed to the boundary conditions imposed during the dynamic test on spherical specimens, wherein the rock specimen is loaded, which restrict its deformation capacity relative to that observed in the UCT. Consequently, the Poisson number ranges are smaller than those obtained in the UCT results. In principle, the rock sample loaded by the dynamic spherical test becomes much stiffer. The last column of Table 1 introduces the degree of anisotropy K_E, which represents the ratio between E and E'.

2.2 Numerical modelling and calculation

Based on the six variations of the anisotropic material model's stiffness matrix from Table 1, seven variations of the distribution matrices were created by analyse of subtracting the strain gauge results

from the numerical model (Fig. 2), when one of them, namely 3 – *UCT secant* (as named in Table 1) was set up to two different orientation relatively to the measuring probe. By fitting the distribution matrixes to Eq. 2, the equilibrium components of the stress tensor $\tilde{\sigma}$ were determined using the Least Square Method.

In the case of the anisotropy theoretical model, the same geometry as that employed for the isotropic theoretical model was utilised. However, it was necessary to create three different versions of this numerical model, taking into account three different positions of the anisotropy with respect to the conical probe. Each of these versions was specific to a different position of the conical probe with respect to the global coordinate system in which the material compliance matrix was defined. In principle, the entire geometry of the model corresponded to the global coordinate system in which the principal axes of the material constants were defined.

It is worth noting that the secant deformation characteristics based on the results of the uniaxial compression test were used for two different orientations of the principal axes of the anisotropic rock material, i.e. the values of the deformation characteristics are the same but defined at different positions relative to the CCBO probe. This means that not only the dependence on different stiffness matrices is observed, but also the influence of the same stiffness matrix applied in two different directions with respect to the probe.

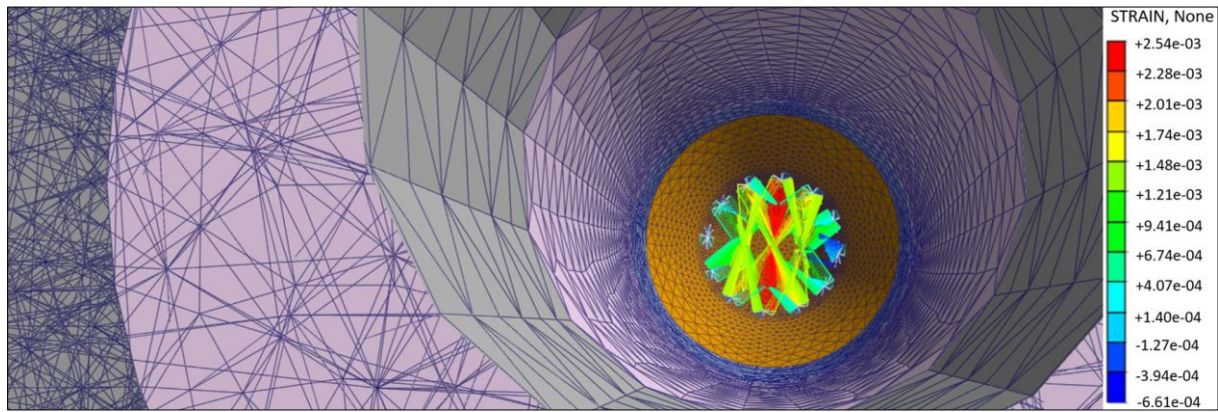


Fig. 2 The numerical model of the borehole, with a particular focus on the resulting strains on the strain gauges at the surface of the CCBO cell.

3 Results

As previously mentioned, the measured stiffnesses of the Grimsel granite are highly variable. The resulting stress tensors are found to be linearly dependent on the material stiffnesses. Consequently, the present study does not address the amplitudes of the resulting principal stresses, but rather focuses on the resulting orientations of the principal stresses. It should be noted that these orientations are independent of the absolute values of the deformation characteristics E , G , ν and E' , G' . However, they are dependent only on the ratios between these eponymous deformation characteristics and Poisson's ratios.

It is noteworthy that all principal stress directions resulting from the analysis are based on the same input data, which are the measured strains and the applied stress tensor to the numerical model, where the components of the stress tensor are all defined as unity. The resulting principal stress directions are presented in Fig. 3 so that the different applied material stiffnesses to the determination process can be compared with the isotropic solution.

The sensitivity analysis demonstrated that variations in the stiffness matrix have a significant effect on the orientation of the axial cross of the principal axes of the material constants relative to the measuring probe. A higher degree of anisotropy leads to a substantial deviation of the axial cross of the principal stresses from the isotropic solution. It is noteworthy that in all instances of this sensitivity analysis, the same rock material is tested by two different laboratory tests, with different pressures applied to the rock samples during testing. It can be concluded that none of the results from case 1 to case 6 (from Table 1) can be considered erroneous; on the contrary, all these results should be given due consideration, and the resulting stresses should be regarded as a quantity that depends on these

variable stiffness matrices of the same rock material. This analysis has facilitated the determination of a dispersion of directions of the principal stresses, which should be considered when designing underground constructions or underground waste disposals.

From Fig. 3 can be stated that low value of degree of anisotropy for the case 1 – *dynamic 0.1 MPa* (see Table 1), the change of the direction from the isotropic solution is neglectable. The more the degree of anisotropy is creasing the more deflection of axial cross of principal axes of material constant is evident. The maximum degree of anisotropy is 3.9 for the case 2 – *UCT tangent* (see Table 1), where the principal axes had deviated by 60 degrees from the isotropic variant. The identical stiffness matrix values, yet a divergent orientation, were utilised for the case 3 – *UCT secant* (see Table 1) used for two orientations, named in Fig. 3 as *UTC secant (12 MPa)* and *UTC secant (50 MPa)*, respectively. It was also observed that a significant deflection of the axial cross of the main stresses was occurring in this instance, reaching up to 40 degrees.

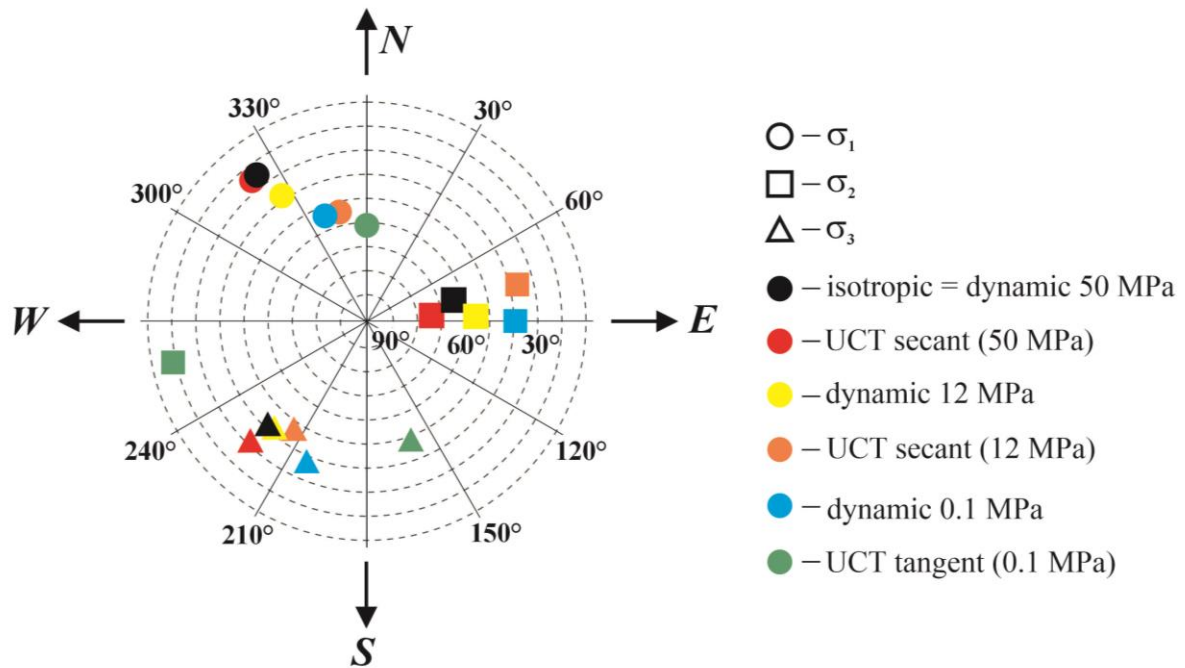


Fig. 3 Resulting 7 variants of principal stress directions. Stereographic projection into the upper hemisphere. Input stiffness matrices are presented in Table 1. (Note that isotropic = dynamic 50 MPa).

4 Discussion

In consideration of the principal stresses at the Grimsel Testing Site, the resulting directions are found to be almost equivalent to those measured by alternative methods for detecting stresses in rock mass. These methods include the CSIRO HI and USM methods, as presented in the paper (Krietsch et al. 2018).

In the event of the CCBO probe being overcored, the immediate probe surroundings are deformed in an attempt to relieve the initial stress to zero. It is for this reason that the resulting stress tensors used in this analysis are based on the material stiffness for different loading conditions. In the next phase of the research, it would be appropriate to analyse a stress tensor that would be based on partial stress values that would be determined based on material stiffnesses that are determined for certain stress intervals.

References

- Amadei B (1983) Rock anisotropy and the Theory of stress measurements. (1983). Springer, <https://doi.org/10.1007/978-3-642-82040-3>.
- Aminzadeh A, Petruzalek M (2022) Identification of higher symmetry in triclinic stiffness tensor: Application to high pressure dependence of elastic anisotropy in deep underground structures. International Journal of

- Rock Mechanics and Mining Sciences (158): article 105168.
<https://doi.org/10.1016/j.ijrmms.2022.105168>.
- Barla G (1972) Rock Anisotropy. in: Rock Mechanics. International Centre for Mechanical Sciences 165.
Springer, Vienna. https://doi.org/10.1007/978-3-7091-4109-0_8
- Bieniawski ZT, Bernede MJ (1979) Suggested methods for determining the uniaxial compressive strength and deformability of rock materials. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 16 (2): 138–140. [https://doi.org/10.1016/0148-9062\(79\)91451-7](https://doi.org/10.1016/0148-9062(79)91451-7).
- Dambly MLT, Nejati M (2018) On the direct measurement of shear moduli in transversely isotropic rocks using the uniaxial compression test. International Journal of Rock Mechanics and Mining Sciences. 113, 220–240 (2018). <https://doi.org/10.1016/j.ijrmms.2018.10.025>.
- Hakala M, Heikkilä E (1997) Development of laboratory tests and the stress-strain behaviour of Olkiluoto Mica Gneiss, Posiva 97–04, Helsinki
- Hakala M, Kuula H (2005) Strength and Strain Anisotropy of Olkiluoto Mica Gneiss, Posiva 2005–61, Helsinki
- Hakala M, Sjöberg J (2006) A Methodology for Interpretation of Overcoring Stress Measurements in Anisotropic Rock. Posiva 2006–99, Olkiluoto
- Krietsch H, Gischig V (2018) Stress Measurements for an in Situ Stimulation Experiment in Crystalline Rock: Integration of Induced Seismicity, Stress Relief and Hydraulic Methods. Rock Mechanics and Rock Engineering, 52 (2): 517–542. <https://doi.org/10.1007/s00603-018-1597-8>.
- Liao JJ, Hu T-B, Chang C-W (1997) Determination of dynamic elastic constants of transversely isotropic rocks using a single cylindrical specimen. International Journal of Rock Mechanics and Mining Sciences 34 (7): 1045–1054. [https://doi.org/10.1016/s1365-1609\(97\)90198-2](https://doi.org/10.1016/s1365-1609(97)90198-2).
- Martin CD, Chandler NA (1994) The progressive fracture of Lac du Bonnet granite. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 31 (6): 643–659.
[https://doi.org/10.1016/0148-9062\(94\)90005-1](https://doi.org/10.1016/0148-9062(94)90005-1).
- Nejati M, Dambly MLT (2019) A methodology to determine the elastic properties of anisotropic rocks from a single uniaxial compression test. Journal of Rock Mechanics and Geotechnical Engineering 11 (6): 1166–1183. <https://doi.org/10.1016/j.jrmge.2019.04.004>.
- Nunes ALLS (2002) A new method for determination of transverse isotropic orientation and the associated elastic parameters for intact rock. International Journal of Rock Mechanics and Mining Sciences 39 (2): 257–273. [https://doi.org/10.1016/s1365-1609\(02\)00025-4](https://doi.org/10.1016/s1365-1609(02)00025-4).
- Petrlikova A (2024) Anisotropy respected in determination of the stress state in a rock mass using the CCBO relief method. Dissertation at Ostrava, VSB- Technical University of Ostrava
- Petrušalek M, Lokajicek T (2023) Velocity anisotropy measured on the spherical specimens: History and applications. Journal of Geodynamics 158: article 102002. <https://doi.org/10.1016/j.jog.2023.102002>.
- Sugawara K, Obara Y (1999) Draft ISRM Suggested Method for in Situ Stress Measurement Using the Compact Conical-Ended Borehole Overcoring (CCBO) Technique. International Journal of Rock Mechanics and Mining Sciences 36 (3) 307–322.