

Factor of safety analysis in fractured multi-layered slopes

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Abstract

Several causative factors influence slope stability and the resulting factor of safety, including lithology, discontinuities, variation of water table level, and weak mechanical levels. We have investigated how rock discontinuities, which facilitate the initiation and influence the geometry of landslides, control the slope stability of multi-layered slopes combining a strong cap rock overlying weak and strong layers. The Shear Strength Reduction method was used to estimate the impact of fractures and other causative factors on the stability of multi-layered slopes with different water table levels as potential triggering mechanisms.

The study highlighted that although major vertical discontinuities can localize failures reducing the potential mobilized rock volume, they have little influence on the factor of safety. Weak mechanical levels have shown the most significant importance in localizing the failure surface in depth. In contrast, the water table level strongly influences the factor of safety and the resulting geometry and failure mechanism: high water levels favor toppling, while low water levels favor circular failure surfaces.

Keywords

Slope stability, causative factors, landslide, factor of safety, modeling

1 Introduction

Landslides occur all over the planet in various terrestrial environments (Froude and Petley 2018). Each year, these phenomena cause significant economic losses and material damage and are responsible for the loss of several thousand human lives (Clague and Roberts 2012; Davies and Rosser 2022).

Landslides play a considerable role in the evolution of landscapes by altering landforms, changing the shape of catchments, modifying river profiles, and contributing to the production and export of sediments (Korup et al. 2010). Indeed, landslides are linked to denudation, fluvial, and glacial processes, acting as slope erosion agents (Schmidt and Montgomery 1995). Some landslides are classified as slow-moving, with speeds ranging from a few millimeters to several hundred meters per year (Lacroix et al. 2020). However, they can experience acceleration in their movement rate due to external forces. The acceleration of these slow landslides is often a precursor to a highly probable major failure.

Landslides are triggered when one or more preconditioning factors (internal attributes of the rock mass) combine with one or more external forces. Generally, the internal attributes introduce weaknesses in the rock mass strength, and the external forces alter the distribution and intensity of the stresses applied to it. Rock mass destabilisation leading to landslides can be viewed as an imbalance between resisting and driving forces. The factor of safety often quantifies this balance (Pradhan et al. 2019; McColl 2022) and is used in geotechnics to assess landslide likelihood. Slope failure occurs when plastic deformations exceed the rock's elastic behaviour. Often, when we talk about the factor of safety, it refers to limit equilibrium methods (Duncan 1996). However, other methods allow the determination of the factor of safety without making assumptions about the forces between slices and the position of the failure surface. In a particular case, the expression of the factor of safety represents a reduction factor for the strength associated with a critical surface, as described by Dawson et al. (1999); Diederichs et al. (2007); Krabbenhoft and Lyamin (2015).

Factors like lithology, mechanical discontinuities such as fractures, and variations in pore pressure affect the force balance, with gravity as a driving force countered by cohesion and frictional strength. Different studies emphasize that combining these factors is crucial for understanding landslide initiation and slip surface geometry (Brideau and Stead 2012; Stead and Wolter 2015; Lacroix et al. 2020). Our study examines the relative influence of different causative factors on the initiation of landslides through a case of a very common multilayer environment: a strong layer (cap rock) overlying a weak layer resting on a strong layer. It explores how lithological diversity, fracturing, variations in hydraulic load, and weak layers influence slope stability.

2 Methods

The 3DEC software used to model the influence of causative factors is a three-dimensional program based on the distinct element method to model discontinuous media. The discontinuous medium is represented by an assembly of discrete rigid or deformable blocks cut by discontinuities. The deformable blocks are similar to a continuous medium with a finite difference mesh. As for the discontinuities, they are treated as boundary conditions between the blocks, which can lead to large displacements along these discontinuities, block rotations, and separations between blocks (Lorig and Varona 2004; Gasc-Barbier and Guittard 2009; Itasca Consulting Group, Inc. 2020). The forces and displacements are determined by solving the equations of motion, formulated using an explicit scheme (Gasc-Barbier and Guittard 2009), meaning a procedure that breaks the resolution into a succession of iterations in "time steps" called cycles. Each cycle allows for the calculation of forces and displacements for all elements and contacts within the model.

2.1 Factor of safety calculation

For the 3DEC software, failure analyses are based on the required reduction in the strength of the geological medium (model) needed to induce failure, expressed as a factor of safety (FS) also known as strength reduction analysis. This analysis involves bracketing failure between a lower limit, where force equilibrium is maintained, and an upper limit, where this equilibrium is no longer sustained. The failure criterion considered here for the model layers as well as the joints follows a Mohr-Coulomb type behaviour law.

Thus, for a Mohr-Coulomb criterion, the safety factor is represented by a strength reduction factor (SRF), which is applied simultaneously to the cohesion and the tangent of the friction angle of the material. The critical reduced parameters thus provide the safety factor:

$$FS = SRF = \frac{C}{C_{red}} = \frac{\tan\varphi}{\tan\varphi_{red}} \quad (1)$$

Where C Mean cohesion
 φ Friction angle
 SRF Strength Reduction Factor
 FS Factor of safety

A FS greater than 1 indicates a stable system, while a factor below 1 indicates system instability.

2.2 Numerical parameters of the model

For 3DEC modelling, the boundary conditions on the outer faces of the model are set so that the basal surface is fixed, movement along the y-axis is restricted on the lateral surfaces, and movement is allowed only along z (vertical) along the upstream and downstream surfaces. The models are pseudo-2D because they are extruded 2D models. A coarse and regular mesh, composed of tetrahedrons with 50 m sides, was used. The parameters and mechanical properties implemented in the 3DEC simulations (Table 1) include Young's modulus, Poisson's ratio, normal and shear stiffness of joints, dry and saturated density, cohesion, and friction angle. Some models include a weak mechanical layer. A range of friction from 10° to 30°, varying in increments of 5°, and a low cohesion set arbitrarily at 1 MPa were used for this weak layer.

Table 1 Parameters and mechanical properties implemented in the simulations using 3DEC. pd: dry density, psat: saturated density, c: cohesion, φ : friction angle, E: Young's modulus, and ν : Poisson's ratio, cb: cohesion of discontinuities, φ_b : friction angle of discontinuities, Ks: shear stiffness, and Kn: normal stiffness.

Units	pd (kg.m ⁻³)	psat (kg.m ⁻³)	c (MPa)	φ (°)	E (GPa)	ν (-)
Strong (cap rock)	2600	2800	3.4	44	26.6	0.21
Weak	2000	2200	2	20	7.5	0.23
Strong	2400	2400	3.8	41	14	0.3
Discontinuity	cb (MPa)	φ_b (°)	Ks (GPa.m ⁻¹)	Kn (GPa.m ⁻¹)		
	0	30	14	39		

2.3 Instability scenarios

Four failure scenarios were modelled (Table 2). Scenario A allows a comparison between a slope composed of a single strong unit (cap rock) (single-layer geotechnical medium) and a slope composed of three units (three-layer media). Scenario B is based on scenario A but consider ten vertical discontinuities spaced 100 m apart within the cap rock. Scenario C incorporates a water table into the three-layer model (scenario A) with arbitrary (piezometric) water table level was simulated. Scenario D incorporates a high-water table level in the three-layer model along with 10 discontinuities in the cap rock unit, as well as a weak mechanical layer with an arbitrary thickness of 20 m. The friction angle of this mechanical layer varies from 10° to 35°, in increments of 5°.

3 Results

Table 2 presents synthetic results giving all calculated factors of safety with their respective scenarios and Figure 1 presents modelling results of the scenarios considered.

Table 2 Summary of all the tested scenarios and their calculated factor of safety values. The errors of the factor or safety factor range from 0.01 to 0.02 for all scenarios and models.

Scenarios	Number of Layers in the Model	Number of Discontinuities	Water table	Weak mechanical level/ prop.	Model name	FoS
A	1	-	-	-	Mo	5.6
A	3	-	-	-	T	3.7
B	3	10	-	-	T_10	3.8
C	3	-	Medium	-	T_M	3.4
C	3	-	High	-	T_H	2.6
D	3	10	High	present/ $\phi=10^\circ$	T_H_10_e_f10	1.6
D	3	10	High	present/ $\phi=15^\circ$	T_H_10_e_f15	1.8
D	3	10	High	present/ $\phi=20^\circ$	T_H_10_e_f20	2
D	3	10	High	present/ $\phi=25^\circ$	T_H_10_e_f25	2
D	3	10	High	present/ $\phi=30^\circ$	T_H_10_e_f30	2.2
D	3	10	High	present/ $\phi=35^\circ$	T_H_10_e_f35	2.3

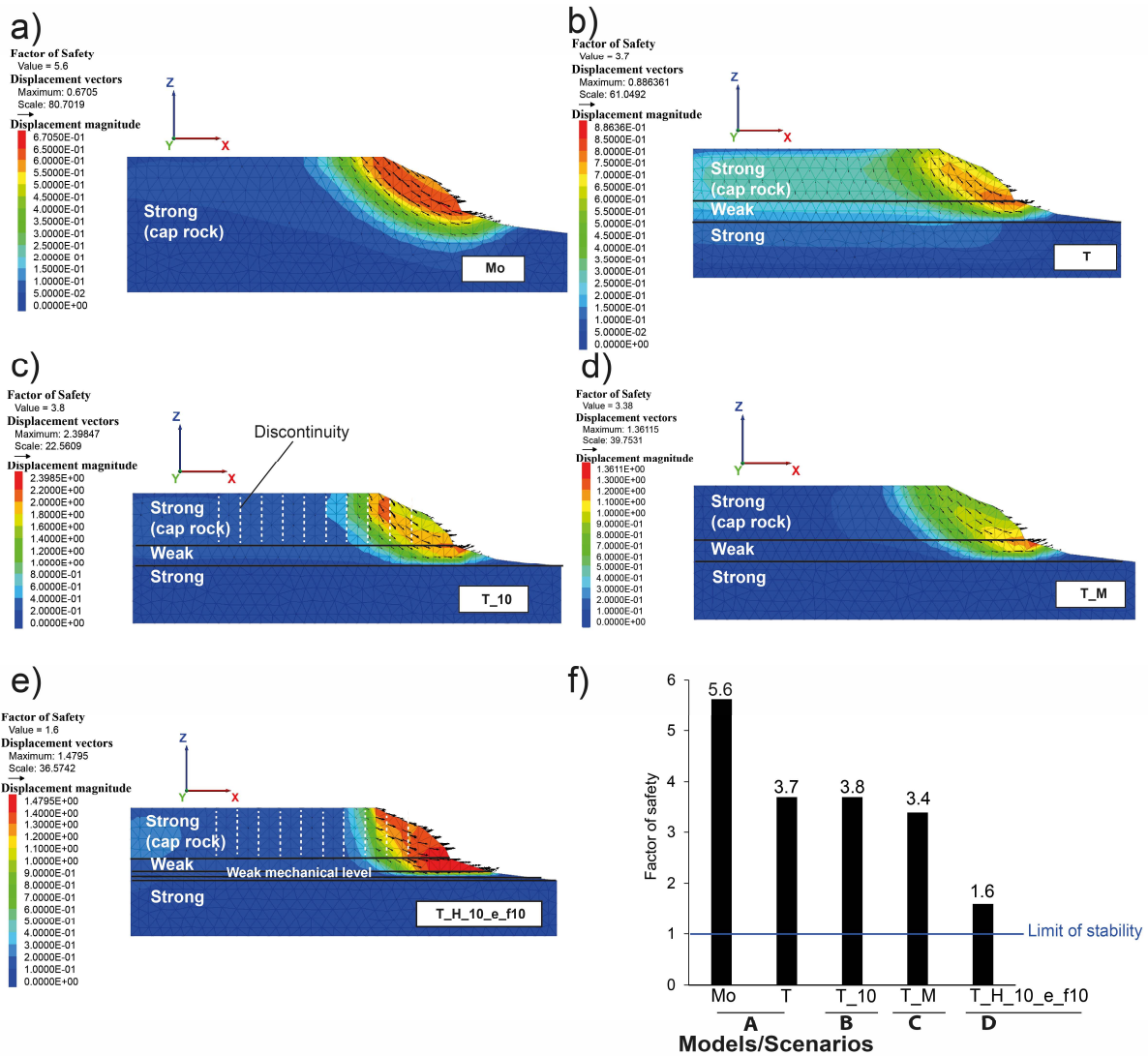


Fig. 1 Models from scenarios A to D with their names, factors of safety, displacement magnitude, and displacement vectors. a) Single-layer model of scenario A. b) Three-layer model of scenario A. c) Three-layer model with 10 discontinuities from scenario B. d) Three-layer model with a medium water table from scenario C. e) Example of a three-layer model with a high water table, 10 discontinuities, and a weak mechanical layer with a friction angle of 10° from scenario D. f) Factors of safety for the models represented.

3.1 Scenario A: Lithological influence

The models in Scenario A show that the failure surface is circular for both the single-layer and three-layer configurations, with a smaller surface for the single-layer model (Fig. 1a and b). In the three-layer model (T), the surface is anchored in the weak layer and is limited by the second strong unit. This scenario yields a factor of safety (FS) of 5.6 for the single-layer model and 3.7 for the simple three-layer model (Fig. 1g and Table 2). Thus, considering a three-layer configuration reduces the factor of safety by a factor of 1.9.

3.2 Scenario B: Discontinuity influence

Scenario B considers a three-layer geological medium affected by multiple discontinuities (Fig. 1c). As in Scenario A with the three-layer model, the failure surface is circular, with the failure surface located within the weak layer. However, this failure surface is positioned along a discontinuity (the third one from right to left), reducing the width of the failure surface (and thus the horizontal extent of the slide) compared to the simple three-layer models. The factor of safety is 3.8, showing no difference compared to the model without discontinuities (Scenario A).

3.3 Scenario C: Water Table influence

The result of Scenario C considers an unfractured three-layer model with different water tables such as a medium water table (Fig. 1d and Table 2). The failure surface obtained is identical to that in Scenario A with a three-layer model (Fig. 1d), indicating that hydraulic loading does not change the geometry and failure mode of an unfractured medium. However, the hydraulic load lowers the factor of safety, which is equal to 3.4 (Table 2). Comparing the simple three-layer model, a reduction in the safety factor of 0.3 is observed.

3.4 Scenario D: Influence of Weak Mechanical Level

Scenario D explores a three-layer model with a high water table level and a weak mechanical layer within the weak unit. The weak mechanical layer is characterized by a variable friction angle. The failure surface for these simulations is defined in depth by this weak mechanical layer, which leads to block translation and toppling (Fig. 1f). As with a high water table, the displacement vectors on the discontinuities indicate forward rotation of the blocks and horizontal movements close to the weak mechanical level. The factor of safety ranges from 1.6 to 2.3 for friction angles between 10° and 35° , respectively, with a maximum FS reduction of 2.1 (Table 2). Notably, the model with a 10° friction angle has the lowest factor of safety (1.6) of all the simulations, approaching 1. This is the scenario closest to failure.

4 Discussion

The Fig. 1g shows the FS for each scenario. The modelling of a cumulative effect of multiple causative factors shows that the FS decreases until it reaches a value of 1.6. This decrease in the FS is related to the consideration of a three-layer model, 10 discontinuities, water table, and the existence of a weak mechanical layer) implemented in this study. Multiple lithologies showed a decrease in the FS by 1.9 between a single-layer composed of the cap rock unit and a three-layer model. This FS difference is because the weak unit has much lower strength (more than 50% percent) than the cap rock unit.

Weak units as clays have long been recognized as lithologies with low strength, predisposing them to failure (Skempton 1985; Stead 2016). The implementation of large vertical discontinuities in the three-layer model showed no difference in terms of FS compared to the simple three-layer model perhaps due to the coarse mesh. Considering a water table suggests a significant decrease in the FS. This decrease can be explained by the fact that variations in hydraulic load lead to a reduction in effective stress due to the increase in pore pressure (Keefer et al. 1987; Crozier 2017; McColl 2022). When considering large discontinuities and water table, the FS decreases further. By considering a weak mechanical layer, in addition to large discontinuities and a high water table, the FS is further reduced, dropping from 3.7 for the three-layer model to 1.6 for the model with the weak mechanical layer (friction angle of 10°). These models present the lowest FS. This reduction in the FS is due to the weaker strengths of the weak mechanical layer, suggesting that stability depends on it mostly as suggested by several authors in the literature (e.g. Li et al. 2021).

5 Conclusion

The slope stability simulations revealed significant differences in the factor of safety across the scenarios. The cumulative effect of causative factors reduces the factor of safety, suggesting that failures are better explained when considering the cumulative effects of causative factors. The role of lithology (mechanical layering) appears crucial, with three-layer models showing more extensive failure surfaces anchored in weak layers and lower factors of safety due to the low strength of the clay unit. Vertical discontinuities influence the location of failures without significantly impacting the factor of safety. The water table strongly reduces stability by increasing pore pressure. A weak mechanical layer, combined with discontinuities and a high water table, further intensifies instability, resulting in a failure pattern between translational sliding and forward rotation. Despite the fact the obtained FS are >1 , all the considered factors (mechanical layering, discontinuities, water tables and weak mechanical level) reduce the stability. Their combination needs to be investigated more deeply.

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7 References

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