Impact of Joint Orientation on Rock Mass Erosion: Insights from Model Testing

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Abstract

Rock mass erosion is a critical factor posing threat to the long-term stability of hydraulic structures, especially in unlined spillways. The erosion extent is determined by the hydraulic pressure generated around rock blocks by the flow during flood events. The hydraulic pressure is significantly affected by various on site hydraulic and geomechanical conditions. Among important parameters, joint orientation demonstrates a significant effect on the hydraulic pressure. Determination of hydraulic pressure at a spillway site, especially within the channel and within the joints, is complicated and physical model testing becomes a reliable approach to study the impact of individual parameters on the fluctuating hydraulic pressure. In this study, physical model tests are carried out on nine model blocks arranged in 3x3 configuration. Three different block orientations are tested at a constant joint opening of 10 mm and the hydraulic pressure is analyzed at different locations around the central block. The results are used to analyze the possibilities of block instability under the action of hydraulic pressure, the weight of the block and the joint shear resistance. The resulting non-dimensional coefficient of uplift was observed around 0.2, indicating a potential for block uplift. The results of stability analysis indicate that the blocks oriented against the flow exhibit a tendency to topple and tend to realign with the flow. In contrast the blocks aligned with the flow readily demonstrated a higher potential for block uplift. The blocks oriented perpendicular to the flow without any block protrusion at the surface show more stable behavior. The results provide key insights into the impact of joint orientation on the rock mass erosion.

Keywords

Rock mass erosion, Pilot-plant physical spillway model, Block Instability, Joint orientation, Non-dimensional coefficient of uplift





1 Introduction

Hydraulic rock mass erosion is found to be more prominent in the unlined dam spillways threatening the safety and sustainability of the whole hydraulic structure. The flood water with huge hydraulic energy flowing over unlined spillways, interacts with the discontinuities in the rock mass resulting in noticeable quantities of rock mass erosion. The complex interaction between the hydraulic power transferred to the rock mass and the resistance offered by the rock-mass, which depends on various geomechanical properties, renders the comprehension of the erosion process difficult (Pells et al. 2017; Boumaiza et al. 2019; Kashtiban et al. 2021). Historic examples of spillway erosion cases, as observed from Oroville dam spillway (Zhang et al. 2024), Ricobayo dam spillway (Pells 2016; Liu and Kieffer 2021) and Copeton dam spillway (Pells et al. 2016, 2024), presents the devastating losses experienced. Existing rock mass erosion assessment methods can be broadly classified into two categories: semi-empirical and semi-analytical (Kashtiban et al. 2021).

Literature presents different erosion mechanisms, namely, plucking/uplift (instantaneous and timedependent), abrasion, fracturing (brittle and fatigue) (Kashtiban et al. 2021), out of which block uplift is determined to be the most common mechanism in rock mass erosion, especially with blocks of size less than 1 m (Pan et al. 2014; Wahl et al. 2019). This uplift mechanism is highly dependent on the hydraulic pressure distribution on the top of the block and within the joints. Different flow characteristics such as flow rate, degree of aeration of the water, flow turbulence and formation of hydraulic jump (Wilkinson et al. 2018; Kote and Nangare 2019) and different geomechanical characteristics such as joint opening, joint orientation, protrusion of blocks, joint shear properties, block size and spillway surface characteristics (Bollaert 2002; Pells 2016; Boumaiza et al. 2019) determine the hydraulic pressure distribution around an intact rock block which may result in the possibility of block dislodgement. The hydraulic pressure is often found to be highly fluctuating and the determination of these fluctuating pressures at actual spillway sites would be highly challenging due to its associated cost and complexity. In this scenario, physical reduced-scale models can be useful tools to estimate these pressures. Physical models allow studying the effect of individual hydraulic and geomechanical parameters. Many researchers developed various reduced scale models, however, most models are using a single large block (George 2012, 2015) or a group of very small blocks (Sawadogo 2010). The joint orientation is one of the important parameters influencing the erodibility process as it determines the dislodgement of a block kinematically under the combination of destabilizing and resisting forces. Reinius (1986) studied the effects of block protrusion and joint orientation on the hydraulic pressures and presented non-dimensional uplift coefficient (C_{up} ') values for different conditions. These values are used for the development of the Quasi-Static Impulsion (QSI) method of Comprehensive Scour Model (CSM) by Bollaert (2012). The measurements from Reinius (1986) are based on piezometric measurements and the fluctuations in the hydraulic pressure are taken into account in the development of C_{up} 'values for different arrangements.

In our previous article (Karnati et al. 2024), we have presented the impact of joint orientation on the hydraulic pressure distribution around the block using physical model tests under different flow rates. The current article focuses on obtaining the C_{up} ' values based on the fluctuating pressures and analysing the stability of the blocks under the action of hydraulic forces in different block orientations. The paper initially presents a description of reduced scale model, then describes the methodology employed to obtain the hydraulic pressures and C_{up} ' values, and finally determines the stability of the blocks inclined with the flow direction.

2 Reduced-scale physical spillway model

The reduced-scale physical model presented in Fig. 1 is constructed at Université du Québec à Chicoutimi, Canada which represents a scaled down version of Romaine IV Hydro-Québec dam spillway at a level of 1:40 following the Froude similarity criterion (Koulibaly et al. 2023). This model allows for studying the individual effect of several important parameters mentioned in the previous section. The functional details of this model are presented in Wisse et al. (2023). The flowrate in the channel is controlled by the operating frequency of the submersible pump and the opening of the sluice gate (Fig. 1). This system allows for the measurement of the flow height and the flow velocity in 3-dimensions of the flow. The downstream end of the channel is provided with an opening to support metal boxes containing the model blocks (Fig. 1(b)). Nine prismoid concrete blocks, each of size 15 x 15 x 30 cm are used to represent a network of rock-mass with intact blocks having connected joints. Lightweight concrete is used for the preparation of concrete blocks to facilitate the placement of

the blocks in the metal box. A constant aperture of 10 mm is provided between the blocks at all levels using Teflon bolts. The dimensions of the metal boxes are such that the top of the blocks always coincides with the spillway surface unless blocks are provided with a protrusion. The central instrumented block consists of 12 internal connected copper rods. The outlets of all the copper rods are provided on the top of the block which are connected to calibrated sensors. The water inlet level is supported with elbows that are placed facing the flow of water acting as a minute pitot tube allowing the accurate measurement of dynamic head (Wisse et al. 2023b). The details of the instrumented block are presented in Karnati et al. (2023, 2024). Each face of the instrumented block is provided with two water inlets. The different faces of the block and corresponding water inlets are termed as mentioned in Table 1.

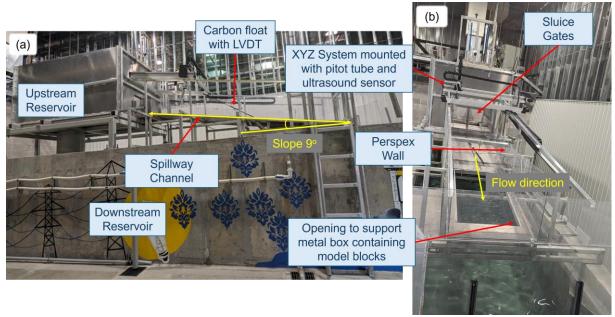


Fig. 1 Reduced scale physical spillway model: (a) Different elements of the physical model; (b) View of the channel surface from model downstream (modified from Karnati et al. (2024)).

Table 1 Different face of the instrumented block and corresponding water inlets

Face	Description	Water inlet	Description
A	Top foce of the block	Ac	Water inlet close to the face C
	Top face of the block	Ad	Water inlet close to the face D
В	Bottom face of the block	Bc	Water inlet close to the face C
	Bottom race of the block	Bd	Water inlet close to the face D
C	II	CI	Water inlet close to the face A
	Upstream face of the block	C2	Water inlet close to the face B
D	Downstream face of the block	D1	Water inlet close to the face A
		D2	Water inlet close to the face B
Е	Face of the block towards the right side of the flow	EI	Water inlet close to the face A
E		E2	Water inlet close to the face B
F	Face of the block towards the left	FI	Water inlet close to the face A
Г	side of the flow	F2	Water inlet close to the face B

3 Methodology

The primary objective of this study is to determine the impact of joint opening on the hydraulic pressure and the corresponding C_{up} values using the physical model tests. Based on the model block aspect ratio (r=1:2), preliminary critical orientations, i.e., -45°, 0° and +45° are chosen for this purpose, where -45° represents the blocks aligned against the flow direction, 0° represents the block with height dimension placed perpendicular to the flow direction and +45° represents the blocks aligned towards the flow direction. The block alignment under different orientations selected is presented in Fig. 2. Triangular wedge blocks are used to support the blocks under inclined conditions. The block set-up representing the fractured rock-mass is subjected to different flowrates ranging from 0.18 to 0.34 m³/s, however, only the results of 0.34 m³/s are presented in this article. A data collection frequency of 100 Hz is used for the collection of pressure data.

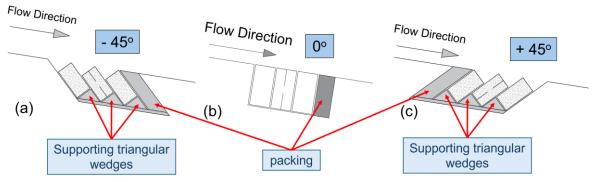


Fig. 2 Schematic of alignment of blocks in different orientations selected: (a) -45°; (b) 0°; (c) +45°.

Determination of C_{up} '

The hydraulic pressure measured as the head of water at different locations of the block, presented in Table 1, is corrected for static and datum pressure heads to obtain the dynamic pressure head. The dynamic head representing mean flow velocity in the channel at selected flowrate (0.34 m³/s) at the instrumented block level is calculated to be 1.25 m. Uplift pressure head acting on the instrumented block, can be calculated as the difference between the dynamic heads at the bottom and the top of the block (Eq. 1). Bollaert (2002) defined the C_{up} as the ratio of the uplift head to the mean velocity head of the flow in the channel at critical conditions (Eq. 2). The hydraulic force acting on each face is calculated as the product of unit weight of water, γ_w , dynamic head and the length along which the pressure is acting. Thus C_{up} 'calculations allow the determination of the uplift force acting on a block.

$$h_{up} = h_B - h_A \tag{1}$$

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$$C'_{up} = \frac{h_{up}}{h_{ch}} \tag{2}$$

Where h_{up} Uplift pressure head acting on the block in m

> h_B Dynamic pressure head at the bottom of the block in m

Dynamic pressure head at the top of the block in m

 C_{up} Non-dimensional coefficient of uplift

Mean flow velocity in the channel at the instrumented block level in m

3.2 Stability analysis of blocks

The pressure results are also used to analyse the stability of the blocks against the dislodgement from its initial position. The blocks presented a tendency to topple and align towards the flow when they are arranged in -45° alignment, whereas they presented a tendency to uplift when arranged in +45° alignment during the trail analysis. To determine the pressures at arrangements designed, the blocks are restrained against any displacements. However, the observed instabilities are analysed under model test conditions using the obtained pressure results. For the blocks in -45° alignment, the toppling and resisting moments are calculated about the downstream bottom corner and the factor of safety against toppling, $F_{s, topple}$ is calculated as the ratio of sum of resisting moments to sum of toppling moments. For the blocks in +45° alignment, the uplift force is determined for critical condition (Eq. 1) and the factor of safety against uplift, $F_{s, up}$ is calculated as the ratio of the uplift force generated to the sum of resisting forces (generated by weight of the block and shear resistance along the sides). The calculations are presented in Table 3 and Table 4 respectively. The blocks aligned in 0° alignment did not show any signs of instability during the preliminary tests.

4 Results and discussion

Hydraulic pressure distribution

The pressures at inlets Ac and Ad are found to be similar in the case of 0° alignment during the trail tests and hence the dynamic head is measured only at Ac under this alignment and the results obtained at Ac are used for Ad as well. The dynamic pressure head results obtained from the model tests are presented in Fig. 3 for the top of the block for the three block orientations selected. In addition to the pressure fluctuations, 50 period moving average is also presented to identify the trend in the fluctuations. Fig. 3 shows that the pressure on the top of the block is high for 0° alignment, and this

pressure is decreasing with the inclined alignments. However, the fluctuations in dynamic head are increasing under inclined alignments for the inlet close to the spillway surface and are decreasing for the inlet away from the spillway surface. The variation of pressure results for different water inlets is presented in Fig. 4 as variation of mean values with error bars pointing out to the maximum and minimum values of the fluctuating pressures. Fig. 4(a) presents a possibility of block uplift under the critical condition where the dynamic head on the top of the block is minimum and that on the bottom is maximum. Fig. 4(b) shows that the dynamic head within the joints (Faces *B* to *F*; see Table 1) is remaining constant with change in block alignment, however, the fluctuations are increasing under inclined alignments.

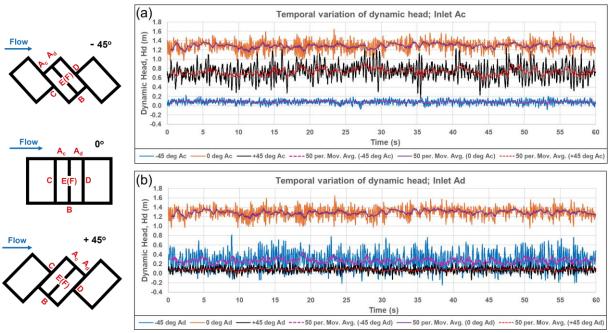


Fig. 3 Temporal variation of dynamic head on the top of the block for different joint orientations: (a) Water inlet Ac; (b) Water inlet Ad.

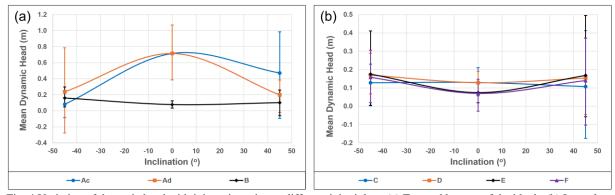


Fig. 4 Variation of dynamic head with joint orientation at different joint inlets: (a) Top and bottom of the block; (b) Lateral sides of the block. (Table 1 explains different faces of the block)

4.2 Non-dimensional coefficient of uplift

A linear variation of the pressure head is considered on the top of the block under inclined alignment conditions. The minimum dynamic pressure on the top of the block is considered zero when it is found to be negative due to the cavitation condition. The value of uplift head and C_{up} are calculated as per Eq. 1 and 2 respectively for all the alignment conditions and are presented in Table 2.

Table 2 Uplift head and non-dimensional coefficient of uplift for different orientations

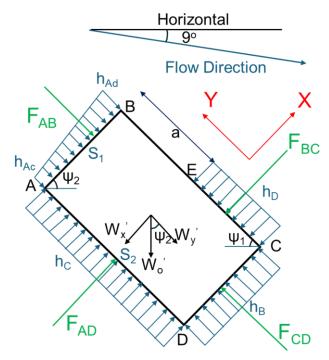
Orientation	-45°	00	+45°
h_A (Minimum value)	0.0000	0.9582	0.0127
h_B (Maximum value)	0.2975	0.1239	0.2585
h_{up}	0.2975	-0.8343	0.2458
C_{up} '	0.24	-0.67	0.20

The negative C_{up} 'value for 0° alignment shows that there is no possibility of block uplift under this condition as observed during the preliminary tests. However, the positive C_{up} 'values under -45° and +45° alignments indicate the possibility of block instability by uplift.

4.3 Stability analysis

4.3.1 Blocks aligned against the flow

The C_{up} ' value shown the possibility of uplift, however, during the trail tests, the blocks tend to topple and align towards the flow direction. Hence the instability arising in this alignment could be a combination of sliding and toppling. The pressures around the instrumented block are considered as presented in Fig. 5. In this alignment, due to the protruded surface developed at the top of the blocks near the triangular edges, a hydraulic jump is being developed creating a negative pressure zone on the leeside of the blocks. This zone is termed as "Exposure zone, a" whose length is taken equal to the length of the block exposed to the flow in the channel, i.e. 16 cm in the model testing scenario. The pressure in this zone is considered zero under critical conditions. The pressures within the joint are found to be similar and hence it is considered uniform within a joint as presented in Fig. 5. The stability against overturning is calculated considering the values presented in Fig. 5, where the head values are obtained from the model testing. The calculation equations and the factor of safety of the block against toppling about point C is calculated as presented in Table 3.



Values used in the stability calculations:

- Unit weight of water, γ_w = 9.81 kN/m³
- 2. Unit weight of the blocks used in model tests, $\gamma_r = 13.5 \text{ kN/m}^3$
- 3. Dip of the top surface of the block, $\psi_2 = 45^{\circ} 9^{\circ} = 36^{\circ}$
- 4. Width of the block, $S_1 = 15$ cm
- 5. Height of the block, $S_2 = 30$ cm
- 6. Exposure length, a = 16 cm
- 7. $h_{Ac} = 0$ (min value is found to be negative and hence considered 0)
- h_{Ad} = 0 (min value is found to be negative and hence considered 0)
- 9. $h_B = 0.298 \text{ m}$
- 10. $h_C = 0.230 \text{ m}$
- 11. $h_D = 0.290 \text{ m}$
- 12. Submerged unit weight of the block per unit length, $W_o' = (\gamma_r \gamma_w) * S_1 * S_2$

Fig. 5 Pressure distribution around the blocks aligned against the direction of flow (Considering unit length along the lateral direction).

The calculations are made for the case of laboratory scenario, where light weight concrete blocks are used to represent intact blocks. Under this scenario, the blocks are unstable and tend to topple as observed from the trail tests. However, in reality the unit weight of rock blocks is about 27 kN/m³. The blocks are found to be stable against toppling under this scenario as observed from the calculations in second row of Table 3 made with realistic unit weight of rock blocks (27 kN/m³).

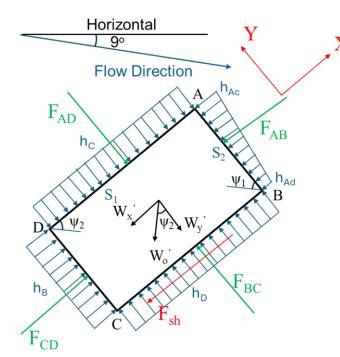
 $Table \ 3. \ Calculations \ involved \ in \ the \ stability \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ involved \ in \ the \ stability \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ involved \ in \ the \ stability \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ involved \ in \ the \ stability \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ involved \ in \ the \ stability \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ analysis \ for \ the \ -45^o \ blocks - Toppling \ about \ downstream \ corner \ point \ Calculations \ analysis \ an$

Toppling moments, M_o (kN-m/m)		Resisting moments, M_R (kN-m/m)				ΣM_o	ΣM_R	$F_{s,topple}$
$F_{CD} * \frac{S_1}{2}$	$F_{AD} * \frac{S_2}{2}$	F _{AB} * eccentricity	$F_{BC} \times \frac{S_2 - a}{2}$	$W_y' * \frac{S_1}{2}$	$W_x' * \frac{S_2}{2}$	(kN- m/m)	(kN- m/m)	$=\frac{\Sigma M_o}{\Sigma M_R}$
0.033	0.102	0	0.028	0.010	0.015	0.134	0.053	0.39
0.033	0.102	0	0.028	0.047	0.068	0.134	0.143	1.06*

^{*} This factor of safety is related to the calculations made using realistic unit weight of rock blocks.

4.3.2 Blocks aligned towards the flow

As observed in the trail tests, the blocks arranged in this alignment are uplifting. Thus, the instability calculations are made for sliding along the surface BC. The pressures are assumed constant along a surface under critical condition, i.e., minimum pressure on the top of the block and maximum pressure within the joints. The pressures within the joint are found to be similar and hence it is considered uniform within a joint as presented in Fig. 6. The stability against sliding is calculated considering the values presented in Fig. 6, where the head values are obtained from the model testing. The factor of safety of the block against sliding along surface BC is calculated as presented in Table 4.



Values used in the stability calculations:

- 1. Unit weight of water, $\gamma_w = 9.81 \text{ kN/m}^3$
- 2. Unit weight of the blocks used in model tests, $\gamma_r = 13.5 \text{ kN/m}^3$
- 3. Dip of the top surface of the block, $\psi_1 = 45^{\circ} + 9^{\circ} = 54^{\circ}$
- 4. Width of the block, $S_2 = 15$ cm
- 5. Height of the block, $S_1 = 30$ cm
- 6. C_{up} ' = 0.20
- 7. Mean flow velocity (head) in the channel, $h_{ch} = 1.25 \text{ m}$
- 8. $h_{Ac} = 0$ (min value is found to be negative and hence considered 0)
- h_{Ad} = 0 (min value is found to be negative and hence considered 0)
- 10. $h_C = 0.412 \text{ m}$
- 11. $h_D = 0.370 \text{ m}$
- 12. Angle of shearing resistance, $\phi = 30^{\circ}$
- 13. Uplift force, $F_u = C_{up}' * h_{ch} * \gamma_w * S_2$

Fig. 6 Pressure distribution around the blocks aligned towards the direction of flow (Considering unit length along the lateral direction).

Table 4. Calculations involved in the stability analysis for the +45° blocks – Sliding along surface BC

Uplift force	Forces on lateral sides		Resisting	Resisting forces, F_R		ΣF_u	
ΣF_u	F_{AD}	W_y '	F_{BC}	W_{x}'	F_{sh}	$-\Sigma F_R$	$F_{s,up} = \frac{\Sigma F_u}{\Sigma F_R}$
0.368	1.213	0.134	1.090	0.098	0.149	0.246	0.67
0.368	1.213	0.626	1.090	0.455	0.433	0.887	2.41*

^{*} This factor of safety is related to the calculations made using realistic unit weight of rock blocks.

The calculations are made for the case of laboratory scenario, where light weight concrete blocks are used to represent intact blocks. Under this scenario, the blocks are unstable and tend to slide as observed from the trail tests. However, in reality the unit weight of rock blocks is about 27 kN/m³. The blocks are found to be stable against sliding under this scenario as observed from the calculations in second row of Table 4 made with realistic unit weight of rock blocks (27 kN/m³).

5 Conclusion

In this study, the hydraulic pressure distribution around an intact model block is determined using physical model tests with different block orientation arrangements. The variation of dynamic head indicated that the pressure fluctuations are increasing under the inclined block arrangements which are necessarily creating unstable situations. The non-dimensional coefficient of uplift values show that the blocks arranged in 0° alignment are stable against uplift, and blocks arranged in -45° and +45° alignments are under critical conditions. The primary instabilities observed in -45° and +45° alignments are determined to be toppling and sliding respectively. The stability analysis carried out using the laboratory scenario confirms the instability as observed from the factor of safety less than 1. However, the blocks are found to be stable when the calculations are made under realistic rock block unit weight. Further tests with different inclined conditions, varied block sizes and numerical modelling will help us develop a relationship between the critical pressure heads with the joint orientation which further improves the understanding of the rock mass erosion mechanism.

References

- Bollaert E (2012) Wall jet rock scour in plunge pools: A quasi-3D prediction model. Hydropower and Dams 20:
- Bollaert E (2002) Transient water pressures in joints and formation of rock scour due to high-velocity jet impact. Ph.D. Thesis, École Polytechnique Fédérale de Lausanne Laboratory of Hydraulic Constructions (EPFL-LCH)
- Boumaiza L, Saeidi A, Quirion M (2019) A method to determine relevant geomechanical parameters for evaluating the hydraulic erodibility of rock. Journal of Rock Mechanics and Geotechnical Engineering 11:1004–1018. https://doi.org/10.1016/j.jrmge.2019.04.002
- George MF (2012) Block theory application to scour assessment of unlined rock spillways. Masters Thesis, University of California, Berkeley
- George MF (2015) 3D Block Erodibility: Dynamics of Rock-Water Interaction in Rock Scour. Ph.D. Thesis, University of California, Berkeley
- Karnati VR, Saeidi A, Rouleau A, Quirion M (2024) Effect of Joint Orientation on Rock Mass Erosion Based on Experimental Results Using a Pilot Plant Spillway Model. In: 58th U.S. Rock Mechanics/Geomechanics Symposium. ARMA, Golden, Colorado, USA, p ARMA-2024-0753
- Karnati VR, Saeidi A, Rouleau A, Quirion M (2023) Effect of Joint Opening and Block Protrusion on the Hydraulic Pressure Around Rock Blocks in Unlined Dam Spillways
- Kashtiban YJ, Saeidi A, Farinas M-I, Quirion M (2021) A Review on Existing Methods to Assess Hydraulic Erodibility Downstream of Dam Spillways. Water 13:3205. https://doi.org/10.3390/w13223205
- Kote AS, Nangare PB (2019) Hydraulic Model Investigation on Stepped Spillway's Plain and Slotted Roller Bucket. Eng Technol Appl Sci Res 9:4419–4422. https://doi.org/10.48084/etasr.2837
- Koulibaly AS, Saeidi A, Rouleau A, Quirion M (2023) A Reduced-Scale Physical Model of a Spillway to Evaluate the Hydraulic Erodibility of a Fractured Rock Mass. Rock Mech Rock Eng 56:933–951. https://doi.org/10.1007/s00603-022-03101-5
- Liu Q, Kieffer DS (2021) LiDAR Based Three-Dimensional Rock Structure Model for Evaluating Spillway Scour at Ricobayo Dam, Zamora, Spain
- Pan Y-W, Li K-W, Liao J-J (2014) Mechanics and response of a surface rock block subjected to pressure fluctuations: A plucking model and its application. Engineering Geology 171:1–10. https://doi.org/10.1016/j.enggeo.2013.12.008
- Pells P, Pells S, van Schalkwyk M (2016) A TALE OF TWO SPILLWAYS. In: International Symposium on "Appropriate technology to ensure proper Development, Operation and Maintenance of Dams in Developing Countries." Johannesburg, South Africa, pp 279–288
- Pells PJ, Bieniawski ZT, Hencher SR, Pells SE (2017) Rock quality designation (RQD): time to rest in peace. Can Geotech J 54:825–834. https://doi.org/10.1139/cgj-2016-0012
- Pells S, Duwell A, Parmar H (2024) Copeton Dam: Performance of the Great Scour Case Study in the floods of 2022. In: ANCOLD 2023. Cairns, Australia
- Pells SE (2016) Erosion of Rock in Spillways. Ph.D. Thesis, University of South Wales
- Reinius E (1986) Rock erosion. International Water Power and Dam Construction 38:43-48
- Sawadogo O (2010) Scour of unlined dam spillways. Masters Thesis, Stellenbosch: University of Stellenbosch
- Wahl TL, Frizell KW, Falvey HT (2019) Uplift Pressures below Spillway Chute Slabs at Unvented Open Offset Joints. J Hydraul Eng 145:04019039. https://doi.org/10.1061/(ASCE)HY.1943-7900.0001637
- Wilkinson C, Harbor DJ, Helgans E, Kuehner JP (2018) Plucking phenomena in nonuniform flow. Geosphere 14:2157–2170. https://doi.org/10.1130/GES01623.1
- Wisse M-H, Koulibaly AS, Saeidi A, Quirion M (2023a) Small-Scale Physical Model for Studying the Effect of Rock Mass Parameters in the Hydraulic Erosion Process of Unlined Spillways. In: World Environmental and Water Resources Congress 2023. American Society of Civil Engineers, Henderson, Nevada, pp 89–98
- Wisse M-H, Saeidi A, Quirion M, Nilsson C-O (2023b) Effects of joint opening and block protrusion on the hydraulic parameters affecting rock block erosion in unlined spillways using a reduced-scale model. Acta Geotech. https://doi.org/10.1007/s11440-023-02085-y
- Zhang X, Wang C, Chen X, et al (2024) Insights into the cause of the Oroville dam spillway failure, 2017, California. Environ Sci Pollut Res. https://doi.org/10.1007/s11356-024-32462-3