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On the possible effect of roughness on hydraulic fracturing pressure

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Abstract

Hydraulic fracturing is currently one of the most popular well stimulation techniques. Analytical and numerical models are used to design hydraulic fracturing treatments and to interpret their results. In terms of flow modelling, hydraulic fractures are usually conceptualized as smooth-walled conduits in these models.

Surfaces of real hydraulic fractures are not smooth. When the fracture faces are not perfectly matched, e.g. because of some shear displacement making the fracture deviate from pure mode I, roughness results in a random aperture distribution. This must reduce the accuracy of the cubic law usually employed to describe flow inside hydraulic fractures. The effect of roughness is to increase the flow resistance, thereby increasing the fracturing pressure.

The objective of this study is to provide some engineering estimates of the possible effect of roughness on hydraulic fracturing pressure. Two extreme situations are considered: (i) root-mean square variation of the aperture being independent of the fracture length; (ii) root-mean square variation of the aperture increasing with the fracture length in accordance with the scaling law valid for roughness of natural fractures. In both scenarios, the KGD model is used to estimate the effect of roughness on the net fracturing pressure.

In scenario (i), the effect of roughness on the net fracturing pressure is found to be limited to the first few meters of the fracture propagation, in the example studied. In scenario (ii), the effect is found to be stronger than in scenario (i), and to increase with the fracture length.

Simulations conducted with different values of the scaling exponent in scenario (ii) indicate that the effect of roughness on the fracturing pressure may be difficult to predict since it is influenced by the (generally *a priori* unknown) scaling exponent.

Keywords

Hydraulic fracturing, pressure, model, roughness, fracture

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1 Introduction

Hydraulic fracturing is a widespread well stimulation technique used to reduce the skin by connecting the well to the rock beyond the damaged zone. Numerical models employed for designing hydraulic fracturing treatments and evaluating their results usually make use of various simplifying assumptions. In the past few decades, the degree of realism of these models has been gradually improved by including additional factors such as plasticity (Papanastasiou 1997), stochastic nature of rock properties (Oyarhossein and Dusseault 2024), and the presence of natural fractures (Kresse et al. 2013; Lavrov et al. 2014; Janiszewski et al. 2019; Jia et al. 2022). Yet some factors have remained outside the mainstream modelling efforts. These factors include those related to fracture roughness.

The objective of this work is to provide first-order estimates of the effect that fracture roughness may have on hydraulic fracturing pressure. To this end, a simple one-dimensional model of hydraulic fracturing is used, *viz*. the Khristianovich-Geertsma-de Klerk (KGD) model. The KGD model is reviewed in depth e.g. in (Valkó and Economides 1995) and is briefly summarized towards the end of Section 2.

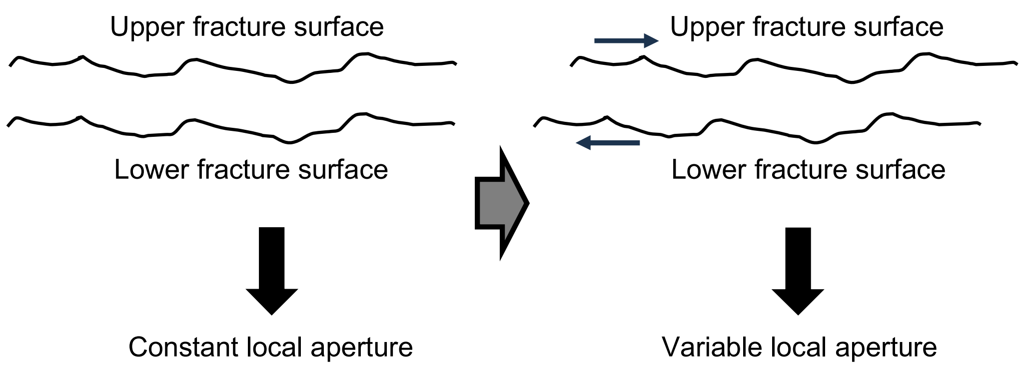


Fig. 1 Rough-walled fracture before (left-hand panel) and after (right-hand panel) shear displacement. Shear displacement (slip) results in a random aperture field (right-hand panel).

2 Background and methodology

Fracture roughness in combination with shear displacement (slip) of the fracture faces results in a randomly varying local aperture. Consider two rough surfaces. Even if at the moment of fracturing they were perfectly matched and thus created a constant aperture along the fracture, a shift will result in their mismatch and hence a variation in the local aperture. This is illustrated in Fig. 1. As demonstrated by Brown et al. (1986), a perfect alignment of both rough surfaces is required in order to eliminate longer wavelengths in the aperture profile. Such alignment cannot be always expected in hydraulic fracturing for several reasons, in particular because of the heterogeneity in the stress field (due e.g. to the presence of natural fractures and lithological boundaries) and in the rock properties. Furthermore, breaking off small pieces of rock from hydraulic fracture surfaces is not uncommon (Briggs et al. 2014). Once dislodged, those pieces will affect the aperture locally, even if the fracture surfaces remain perfectly aligned with each other suggesting a constant aperture. It should be noted that the degree of mismatch between the fracture surfaces depends on the amount of shear displacement and/or surface damage. In the case of a small shear displacement / small surface damage, the aperture profile remains strongly correlated. As the amount of displacement or surface damage increases, the degree of correlation decreases. The loss of correlation inhibits the flow, as demonstrated e.g. by Méheust and Schmittbuhl (2003).

As a result of randomly varying local aperture caused by shear displacement and surface damage, the (local) hydraulic aperture of the fracture is smaller than the (local) mean aperture. We use the adjective 'local' here because the aperture of a hydraulic fracture decreases from the fracture mouth to the fracture tip. The 'mean', i.e. average, aperture (also sometimes called 'mechanical aperture') is understood here as the aperture obtained by averaging the aperture over a length scale sufficiently large compared to the wavelengths of asperities but sufficiently small (thus 'local') compared to the fracture length. 'Hydraulic aperture' is defined as the aperture of a smooth-walled fracture that would result in the same flow rate as the real aperture distribution in a rough-walled fracture, given the same applied pressure gradient.

In most cases, roughness works to inhibit the flow, i.e. to reduce the hydraulic aperture below the mean aperture (Méheust and Schmittbuhl 2003). In order to account for the difference between the mean and hydraulic aperture, modifications have been proposed to the cubic law. 'Cubic law' refers to the relation between the flow rate and the pressure gradient in a smooth-walled fracture, whereby the flow rate is proportional to the third degree of the aperture (Raven and Gale 1985). For a comprehensive review of modifications to the cubic law intended to account for fracture roughness see e.g. (He et al. 2021).

Laboratory experiments by van Dam and de Pater (2001) revealed that surfaces of hydraulic fractures are rough, and their roughness, measured as the root mean square variation (RMS) of the surface elevation, becomes independent of the fracture propagation length after the fracture front has left the near-well region. The authors attributed this roughness to the existence of a near-tip plastic zone and a combination of pure mode I fracturing and shear failure preceding it. As a result, roughness increases with the difference between the maximum and minimum in-situ stresses. This interpretation is consistent with the analysis by Papanastasiou and Thiercelin (1993), according to which the size of the plastic zone depends largely on the deviatoric in-situ stress and the rock properties.

Lack of dependence of a hydraulic fracture's roughness on the fracture size was claimed by van Dam and de Pater (2001) based on their laboratory experiments. The dimensions of real hydraulic fractures are much larger than the fractures generated in the lab, typically by 2-3 orders of magnitude larger. It is known that the RMS variation of *natural* fracture surfaces increases with the fracture size L as L^{γ} where $0 < \gamma < 1$ (Brown 1995; Méheust and Schmittbuhl 2003). This power law scaling is in line with scaling laws established for other properties of natural fractures and faults, such as shear displacement and aperture (Bonnet et a. 2001). At the time of writing, it is not clear whether the surface RMS of *field-scale hydraulic* fractures is indeed independent of their length (as suggested by lab experiments) — or rather scales with the length in a similar way as that of natural fractures. For this reason, two scenarios are considered in this study (see below): (i) aperture RMS being constant and (ii) aperture RMS increasing with the fracture length according to a power law. Notice that we distinguish between surface RMS (i.e. RMS variation of the surface elevation) and aperture RMS (i.e. RMS variation of the aperture) in this study, see also below.

In order to estimate the effect of roughness-induced aperture variation on the pressure gradient, we need estimates of some roughness-related parameters of hydraulic fractures. In particular, we need the RMS variation of the fracture *aperture* that enters several popular versions of modified cubic law. There have been few systematic studies of roughness parameters of hydraulic fractures. In laboratory experiments conducted by van Dam and de Pater (2001), the RMS variation of fracture *surface landscape* (*not* the aperture) was measured to be on the order of 0.1 mm, independent of the fracture size. Let us estimate the range of the *aperture* RMS corresponding to this data. The lower limit of the aperture RMS is obtained when both surfaces of a hydraulic fracture are identical rough surfaces, and there is no shear displacement. In this case, the local aperture is everywhere the same, and the aperture RMS is 0 (Fig. 1, left-hand panel). This is the greatest lower bound for the aperture RMS. To estimate the least upper bound of the aperture RMS, consider two random uncorrelated rough surfaces having the same surface RMS of 0.1 mm, or two identical rough surfaces but displaced a long distance relatively to each other. In this case, the maximum aperture RMS is obtained which, from the theory of probability, is given by $0.1\sqrt{2}$ mm = 0.14 mm. Thus, given the surface RMS of 0.1 mm, the aperture RMS is bound between 0 and 0.14 mm under the assumption that it is independent of the fracture size. This is corroborated by the results from profilometry of natural fractures in several rock types carried out by Brown (1995). These indicate that the aperture RMS varies from 0.25 to 1.0 times the surface RMS. In scenario (i), we assume that the aperture of a hydraulic fracture has a constant RMS variation of 0.1 mm, independent of the fracture propagation distance. This figure, 0.1 mm, has the same order of magnitude as the surface RMS measured by van Dam and de Pater (2001) in their lab experiments.

Naturally, the aperture RMS of 0.1 mm used in scenario (i) is just an example, suggested by laboratory experiments on a limited number of rock types. In other rock types and under in-situ conditions, the fracture surface variation might be larger. For instance, the variation in the surface landscape of individual fracture faces in a shale was found to be on the order of 1 mm or 1 cm, depending on the sample location (Briggs et al. 2014).

In order to capture the size effect on roughness, a second scenario is additionally considered in this study. In the second scenario, we assume that the aperture RMS scales with the fracture size as $\sigma = \sigma_0(L/L_0)^\gamma$ where γ is a positive constant and σ_0, L_0 are fitting parameters. This scaling law is similar to the one known for the surface RMS of natural fractures. We choose L_0 to be the order of the laboratory length scale (L_0 = 0.1 m), σ_0 = 0.1 mm to be the order of the surface RMS reported in van Dam and de Pater's (2001) laboratory experiments.

The main difference between the two scenarios of modelling the aperture RMS described above is that the aperture RMS remains constant and equal to 0.1 mm during the fracture growth in scenario (i), while it increases monotonically with the fracture length in scenario (ii).

In order to make estimates of the effect the aperture RMS has on the hydraulic fracturing pressure, we use the KGD model without leak-off. In this model, a vertical fracture is considered, and plane-strain conditions in the horizontal plane are assumed. Consequently, the model is usually viewed as an approximation of a relatively short but high vertical fracture, with $2L < H$ where L is the fracture length; H is the fracture height (Valkó and Economides 1995). According to this model, the pressure drop along the fracture is

$$\Delta p = p_{n,w} - p_{n,tip} = \frac{12\mu q L}{H}\left(\frac{1}{L}\int_0^L \frac{dx}{w^3}\right) \tag{1}$$

where $p_{n,w}$ is the net pressure at the wellbore; $p_{n,tip}$ is the net pressure at the fracture tip; q is the volumetric flow rate; μ is the dynamic viscosity of the fracturing fluid (Newtonian rheology is assumed for the latter); w is the local fracture aperture; x is the coordinate along the propagation direction. The parenthesized expression on the right-hand side of Eq. (1) is the value of w^3 averaged along the fracture. 'Net pressure' refers to the difference between the fluid pressure inside the fracture and the in-situ stress normal to the fracture plane.

Assuming that $w = 0$ at $x = L$ and that the fracture has an elliptic shape results in an improper integral in Eq. (1). Valkó and Economides (1995) pointed out that one way of regularizing it is to assume the existence of a fluid lag, i.e. an unwetted zone at $0.9123L < x < L$. The existence of a fluid lag is well-known in hydraulic fracturing mechanics, see e.g. (Desroches et al. 1994). Using an elliptic fracture, i.e. $w = \sqrt{1 - x^2/L^2}$, results then in

$$\frac{1}{L}\int_0^{0.9123L} \frac{dx}{w_w^3\left(1 - x^2/L^2\right)^{3/2}} \approx \frac{7}{\pi w_w^3} \tag{2}$$

a frequently quoted result obtained for the KGD fracture using different methods.

In order to make a first-order estimation of the effect of roughness on the pressure drop, we use Eq. (1) with the upper integration limit set to $0.9123L$ instead of L, and with the hydraulic aperture w_h instead of the actual aperture w. Due to roughness, the pressure drop thereby increases by a factor of

$$\alpha = \frac{\Delta p^{(rough)}}{\Delta p^{(smooth)}} = \frac{(1/L)\int_0^{0.9123L} w_h^{-3}\,dx}{7/\left(\pi w_w^3\right)} \tag{3}$$

where w_h is the local hydraulic aperture. Estimations of α are summarized in Section 3 for some 'typical' field conditions.

In order to use Eq. (3), we need a relation between the fracture length L and the near-well fracture aperture w_w. For a KGD fracture, it is given by (Valkó and Economides 1995)

$$w_w = 3.22\left(\frac{\mu q L^2}{E'H}\right)^{1/4} \tag{4}$$

Furthermore, we need a relation between the hydraulic aperture and the mean aperture, to be used in Eq. (3). Several dozen relations between the hydraulic aperture w_h and the mean aperture w of natural fractures have been proposed in the literature for the past four decades. A recent review of these 'modified cubic laws' can be found in (He et al. 2021). In this study, we use three relations, proposed by Patir and Cheng (1978), Zimmerman and Bodvarsson (1996) and Amadei and Illangasekare (1994). Common to these three relations is that they were obtained by considering flow in a fracture plane rather than along a linear profile. Therefore, they automatically take into account the in-plane tortuosity of the flow.

The first relation is due to Patir and Cheng (1978) and is given by

$$w_h = w\left[1 - 0.9\exp\left(-0.56w/\sigma\right)\right]^{1/3} \tag{5}$$

Eq. (5) results in the following hydraulic aperture as a function of the distance along the hydraulic fracture x, to be used in Eq. (3):

$$w_h = w_w\left(1 - x^2/L^2\right)^{1/2}\left[1 - 0.9\exp\left(-0.56w_w\sqrt{1 - x^2/L^2}/\sigma\right)\right]^{1/3} \tag{6}$$

The second one is due to Zimmerman and Bodvarsson (1996) and is given by

$$w_h = w\left[1 + 9\left(\sigma/w\right)^2\right]^{-1/6} \tag{7}$$

Eq. (7) results in the following hydraulic aperture as a function of the distance along the hydraulic fracture x, to be used in Eq. (3):

$$w_h = w_w\left(1 - x^2/L^2\right)^{1/2}\left[1 + \frac{9\sigma^2}{w_w^2\left(1 - x^2/L^2\right)}\right]^{-1/6} \tag{8}$$

The third one is due to Amadei and Illangasekare (1994) and is given by

$$w_h = w\left[\frac{1}{1 + 0.6\left(w/\sigma\right)^{-1.2}}\right]^{1/3} \tag{9}$$

Eq. (9) results in the following hydraulic aperture as a function of the distance along the hydraulic fracture x, to be used in Eq. (3):

$$w_h = w_w\left(1 - x^2/L^2\right)^{1/2}\left[\frac{1}{1 + 0.6\left(w_w\sqrt{1 - x^2/L^2}/\sigma\right)^{-1.2}}\right]^{1/3} \tag{10}$$

3 Results

Consider as an example a hydraulic fracturing job with the following parameters: the fracture height H = 50 m, the injection rate into one fracture wing q = 0.05 m^3/s, fluid viscosity μ = 0.2 Pa·s, plane-strain Young's modulus E' = 50 GPa. In order for the KGD model to be valid, the fracture length has to be within 25 m. The fracture aperture at the fracture mouth increases with the fracture length as shown in Fig. 2.

Results of simulations with scenario (i) are presented in Figure 3. In this scenario, the aperture RMS is equal to 0.1 mm and is independent of the fracture length. The effect of the roughness on the fracturing pressure decreases rapidly as the fracture grows. This is expected since the relative contribution of roughness decreases as the aperture increases with L (Fig. 2) and the fracture flow approaches the parallel-plate flow. The effect of roughness is appreciable only at the first few meters of the fracture propagation in this case.

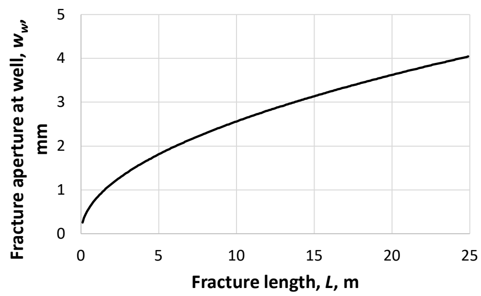


Fig. 2 Plot of Eq. (4) with the data used in the simulations in Section 3.

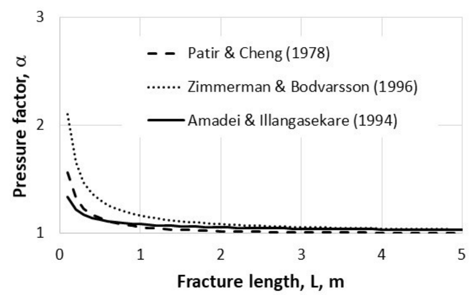
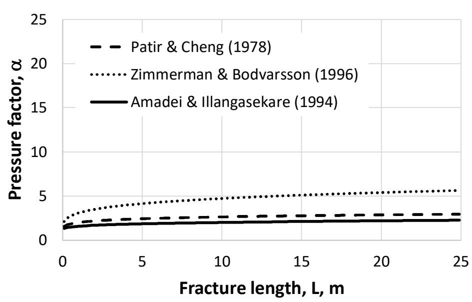
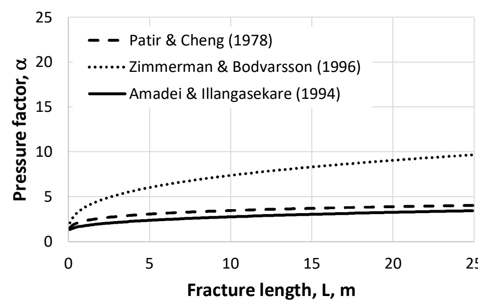


Fig. 3 Pressure factor as a function of fracture length in scenario (i) – aperture RMS = 0.1 mm independent of fracture length.



a



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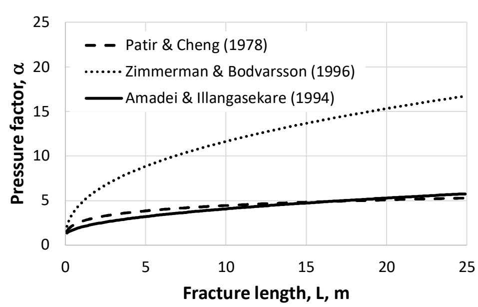

c

Fig. 4 Pressure factor as a function of fracture length in scenario (ii) for $\gamma = 0.7$ (a), $\gamma = 0.8$ (b) and $\gamma = 0.9$ (c).

Results of simulations with scenario (ii) are presented in Figure 4. Three sets of simulations were performed, with the scaling exponent γ equal to 0.7, 0.8 and 0.9. The value $\gamma = 0.8$ is frequently used in fracture-mechanical studies since it was once proposed as a universal exponent for fracture surfaces in aluminium (Bouchaud et al. 1990). In rocks, γ can vary considerably but the chosen three values are within the range of the values commonly reported for rocks.

In all three cases presented in Figure 4 the pressure factor α increases with the fracture length. The degree of this increase depends strongly on γ. To illustrate the effect of γ, the results obtained with the Patir and Cheng (1978) modified cubic law with different γ-values are collected in Figure 5. Figure 5 suggests that a slight inaccuracy in the assumed value of γ may lead to considerable error in the prediction of fracturing pressure if, indeed, the aperture RMS scales with L as assumed in scenario (ii).

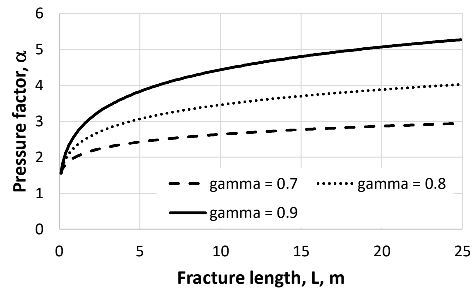

Fig. 5 Pressure factor as a function of fracture length in scenario (ii) for γ = 0.7, 0.8 and 0.9 obtained with the modified cubic law due to Patir and Cheng (1978).

4 Discussion: Limitations of the current analysis

In Section 2, it was assumed that the effect of fracture roughness on the fracturing pressure in the KGD model can be accounted for by simply adjusting the fracture conductivity at a given fracture length, and then correcting the pressure accordingly. This results in an elevated pressure. In reality, this elevated pressure would cause an increased fracture aperture in the entire fracture because pressure and deformation are coupled. This, in turn, would increase the fracture conductivity, trying to bring the pressure back down. The result would most likely be a smaller increase in the fracturing pressure than the estimates in Section 2 suggest, and a wider fracture, with implications for the injected volume and for the placement of proppant inside the fracture. Our results should therefore be viewed as an upper bound for the roughness-induced effect on the pressure.

Another limitation of our analysis in Sections 2 and 3 is the implicit assumption of low Reynolds number. In reality, in parts of a hydraulic fracture the flow can be turbulent (Dontsov 2016), which would necessitate modifications to the modified cubic laws in Eqs. (5), (7) and (9). The estimates made in Section 3 would need to be adjusted accordingly.

It should be emphasized that the strong effect of roughness on the fracturing pressure seen in Figure 4 represents an extreme case, when the fracture surfaces are uncorrelated, resulting in a self-affine aperture field without characteristic length (cutoff). In reality, the effects should be smaller. Also, the KGD model used in this study has well-known limitations, in particular pressure that decreases with the fracture length. It was used here to make only a crude estimate of the possible effect of roughness on the pressure. A more detailed and accurate study is still outstanding.

5 Conclusions

First estimates obtained in this study indicate that fracture roughness may have a significant effect on hydraulic fracturing pressure if there is shear displacement of the fracture faces (e.g. due to heterogeneity of the rock, heterogeneity of the stress field, presence of natural fractures). If the RMS variation of the fracture aperture does not depend on the fracture length, this effect is limited to the first few meters of the fracture propagation. If the root mean square (RMS) variation of the aperture scales with the fracture length in a way similar to natural fractures, the effect is stronger and increases as the fracture propagates. The effect works to increase the fracturing pressure. The effect of roughness on the fracturing pressure might be difficult to predict in this case since it is strongly affected by the exponent of the scaling law.

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