LES-based data assimilation in turbulent flow using weakly constrained 4D-Var

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We propose a data assimilation algorithm that combines large-eddy simulation (LES) with a weakly constrained four-dimensional variational (4D-Var) approach for reconstructing turbulence in a neutral atmospheric boundary layer. Unlike the conventional strong 4D-Var formulation (see, e.g., Ref.[1]), this method treats the state equations as a weak constraint, necessitating the estimation of state-space modeling errors. This leads to an optimization problem defined over both space and time, with model errors incorporated as a space-–time forcing term in the evolution equations. The 4D-Var framework is derived from a Bayesian maximum a posteriori formulation, where prior information regularizes the optimization problem. Leveraging turbulence theory, we approximate the prior statistics of the error sources, focusing on state-space modeling errors arising from the weak formulation. These errors are primarily attributed to inaccuracies in the subgrid-scale representation within the LES. To solve the 4D-Var optimization problem, we precondition it using the Hessians of the regularization terms and employ a classical L-BFGS method with line search. The proposed approach is tested using virtual lidar measurements derived from a high-resolution reference simulation of a pressure-driven boundary layer. These measurements are used in the 4D-Var framework, where a coarse-grid LES model, operating on a reduced computational domain, is used for the reconstruction. Across the test cases, the algorithm demonstrates super-linear convergence and achieves up to a 40% improvement in reconstruction accuracy compared to the strong 4D-Var formulation.
Figure 1 provides an overview of an assimilation test case using a circular scanning patteren of the lidar sensor.

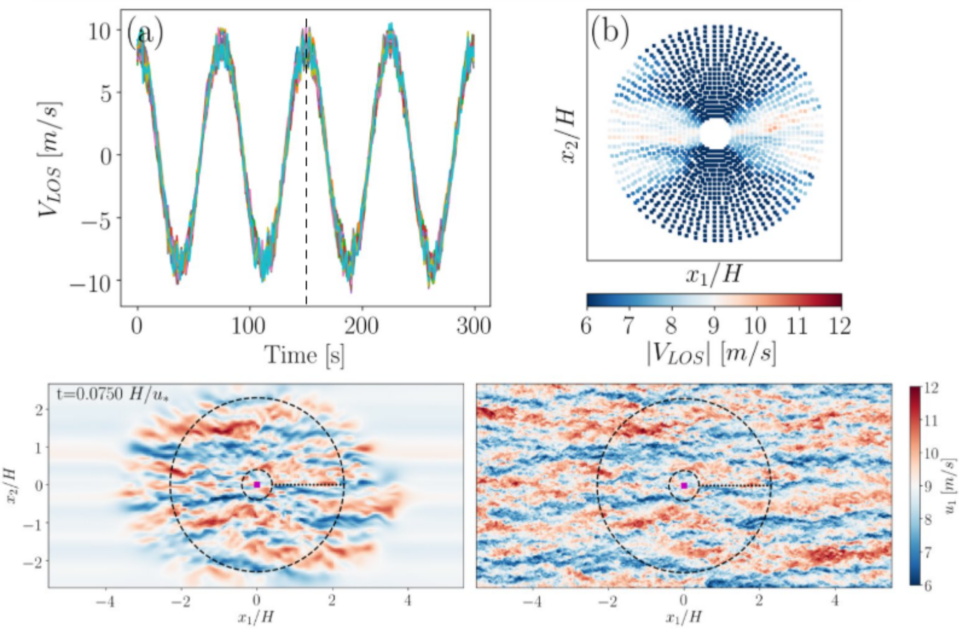

Figure 1: (a) Lidar raw speed signal over a 300 s measurement horizon. (b) Visualization of the lidar measurements after projecting on the reconstruction grid.(c) and (d) show the reconstructed and reference streamwise velocity component at $x_3 = 0.1H$ when $t = T_m/2$. The dashed lines represent the boundaries of the scanning area.

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[1]Alreweny et al., *J. Fluid Mech.* **981**, A28 (2024)