

Optimal transient growth in turbulent Rayleigh–Bénard convections

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Rayleigh–Bénard (RB) convection, a fundamental model for natural convection, is widely studied due to its relevance in geophysics, climatology, and astrophysics¹. In RB convection, a fluid layer is heated from below and cooled from above, creating buoyancy-driven turbulence. This phenomenon is observed in natural systems like atmospheric circulation, ocean currents, and planetary interiors. Recent research has focused on turbulent superstructures in high-Rayleigh-number RB convection^{2,3}, where large-scale flow patterns, such as coherent rolls and cells, emerge. These superstructures, spanning several times the height of the convection cell, exhibit a distinct mode of organization separate from smaller turbulent fluctuations². Studies suggest that these superstructures persist over long timescales, providing insights into the long-term organization of turbulent convection. However, the mechanism behind the formation of superstructures remains unclear.

Furthermore, the theory of transient growth successfully explains the formation mechanisms of flow structures in other flows, such as streaks and large-scale motions in wall turbulence⁴. This theory suggests that although some eigensolutions of the problem are damped for certain wavenumbers, initial perturbations can experience temporary growth due to non-modal effects before eventually decaying. This temporary growth may further trigger nonlinear effects, potentially linking it to the formation of flow structures. In the present work, we attempt to provide a possible mechanism for the formation of superstructures through the theory of transient growth.

Here, we examine the linearized Navier-Stokes equations in Rayleigh–Bénard convection with turbulent mean temperature profiles and turbulent thermal diffusivity. The mean temperature profiles and turbulent thermal diffusivity are obtained from statistical data generated by direct numerical simulations using the AFiD code. The Rayleigh number for the statistical flow field is 10^7 , the Prandtl number is 1, and the aspect ratio is 24. The linear stability analysis code is based on the open-source Dedalus code. We found that all the eigensolutions of the problem are damped; however, initial perturbations with a horizontal wavenumber around 1 can experience temporary growth before eventually decaying. The mode of optimal growth manifests as large-scale impacting and emitting temperature fluctuations, accompanied by rolls corresponding to velocity fluctuations, resembling superstructures in RB convections.

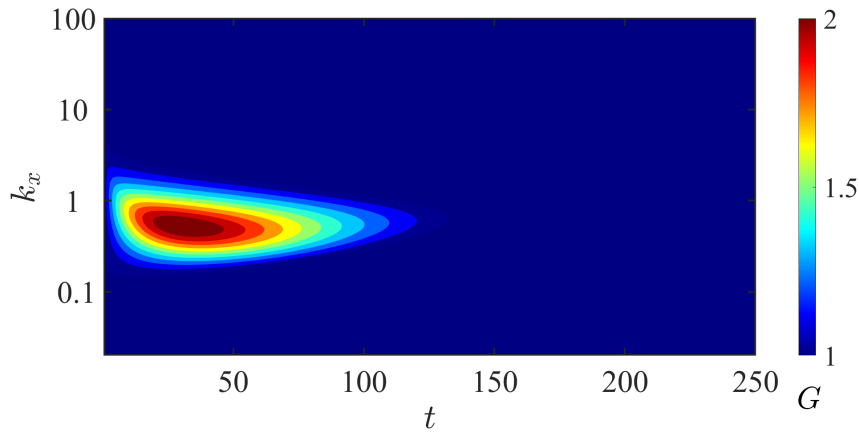


Figure 1: Time evolution of the maximum energy amplification G of the perturbations at different horizontal wavenumbers k_x .

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¹ Ahlers et al., *Reviews of modern physics*, 81(2), 503-537 (2009).

² Stevens et al., *Physical review fluids*, 3(4), 041501 (2018).

³ Blass et al., *Journal of Fluid Mechanics*, 906, A26 (2021).

⁴ Del Alamo., Jimenez., *Journal of Fluid Mechanics*, 559, 205-213 (2006).