

## Numerical Modeling of Ice Crystal Formation and its Diffusional Growth in Turbulent Mixed-phase Clouds Using Lagrangian Multiphase DNS

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Mixed-phase clouds are composed of supercooled liquid droplets together with ice crystals<sup>1</sup>, and prevail over the troposphere in all seasons, from polar to tropical regions<sup>2</sup>. They are linked to diverse meteorological conditions and cloud types<sup>3</sup>. Despite extensive measurements and laboratory experiments, the detailed microphysical interactions within mixed-phase clouds are remain uncertain. In particular, there is a lack of studies employing a Lagrangian framework to precisely capture the behavior of supercooled liquid droplets and ice crystals, especially in the cloud-top region, where turbulence-driven mixing and phase transitions play a critical role. A comprehensive numerical approach that resolves these processes at high fidelity is essential for improving our understanding of cloud microphysics and their implications for atmospheric dynamics (weather and climate). To gain a comprehensive understanding of the microphysical processes in mixed-phase clouds, particularly the influence of turbulent temperature and saturation fluctuations, we perform computational studies using a Direct Numerical Simulation (DNS) approach. Specifically, we investigate the condensational growth of supercooled liquid droplets, droplet freezing (heterogeneous ice nucleation), and subsequent diffusional growth of the formed ice crystals in a turbulent channel flow. In our study, the supercooled droplets include Snomax particles (ice nucleating proteins from *P. syringae* bacteria) which can act as ice nucleating particles. The simulations are conducted within an Eulerian-Lagrangian framework, where the turbulent flow field is coupled with two scalar fields representing temperature and water vapor mixing ratio. The developed Lagrangian particle tracking model enables precise representation of cloud droplet and ice crystal dynamics. Heterogeneous ice nucleation (immersion freezing) is modeled using the stochastic model of Hartmann et al. (2013)<sup>4</sup>, and our implementation has been validated against their results. Additionally, the underlying turbulent channel flow has been extensively validated against previous studies, providing a robust foundation for analyzing cloud microphysical processes under controlled conditions. Our simulations provide new insights into the microphysical interactions in mixed-phase clouds, emphasizing the role of turbulence in shaping cloud droplet and ice crystal dynamics.

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<sup>1</sup>Morrison et al., *Nature Geoscience* **5.1**, 11-17 (2012)

<sup>2</sup>Korolev and Milbrandt, *Geophysical Research Letters* **49.18**, e2022GL099578 (2022)

<sup>3</sup>Korolev et al., *Meteorological Monographs* **58**, 5-1 (2017)

<sup>4</sup>Hartmann et al., *Atmospheric Chemistry and Physics* **13.1**, 5751-5766 (2013)