

## Shell Models on Recurrent Sequences: Fibonacci, Padovan and Other Series

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Shell models are a particular class of toy models in turbulence research, that reduce the turbulence field into a set of variables defined on shells in  $k$ -space, which preserve the inviscid invariants of the Navier-Stokes system, their dynamics successfully mimicking some nontrivial aspects of the turbulent cascade such as power-law spectra and intermittency.

The standard archetype of shell model is the GOY model, with a logarithmic spacing  $g = 2$ . Considering this as a 1D decimation of the wave-number domain, for example in the spirit of spiral chain models<sup>1</sup>, or its generalization into multiple dimensions in the form of logarithmically discretized models<sup>2</sup> nested polyhedra<sup>3</sup> models or log-lattice models<sup>4</sup>, the shell spacing imposed may be the golden-ratio, or other special numbers. However interpreting such models as decimations of a Fourier space grid is complicated, since the wave-numbers that are computed on such logarithmic grids do not fall on integer values.

In order to remedy this, we can use recurrent sequences. It is possible to construct models where shell wave-numbers are integers and asymptotically self-similar by construction, hence enabling the connection between a regular grid and a logarithmic lattice. In order to test this approach, we propose a new class of shell models<sup>5</sup>, where the shell variables are defined on a recurrent sequence of integer wave-numbers such as the Fibonacci series, and other variations. Considering the simplest model, which involves only local interactions, the interaction coefficients can be generalized in such a way that the inviscid invariants, such as energy and helicity, can be conserved even though there is no exact self-similarity. It is shown (Fig. 1 & 2) that these models basically have identical features with standard shell models. In addition to conventional shell models with local interactions that result in forward cascade, a particular helical shell model<sup>6</sup>, with long range interactions is considered on a similarly recurrent sequence of wave numbers, corresponding to the Fibonacci series, and found to result in the usual inverse cascade.

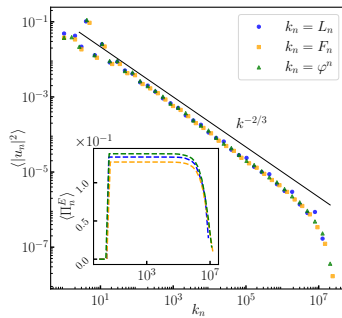


Figure 1: Energy spectrum and flux (inset plot) for the shell model implemented on recurrent sequences and their corresponding self-similar sequences, showing how the proposed model displays the same features of standard shell models without the requirement of exact self-similarity.

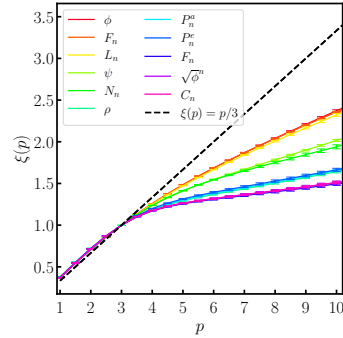


Figure 2: Intermittency corrections for the various recurrent series considered, showing comparable level of intermittency for sequences corresponding to the same asymptotically self-similar intershell ratio. Smaller inter-shell ratios are associated with an increase of the intermittency corrections.

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<sup>1</sup>Gürcan et al., *Phys. Rev. E* **100**, 043113 (2019).

<sup>2</sup>Gürcan et al., *Phys. Rev. E* **94**, 033106 (2016).

<sup>3</sup>Gürcan, *Phys. Rev. E* **95**, 063102 (2017).

<sup>4</sup>Campolina and Mailybaev, *Nonlinearity* **34**, 4684 (2021).

<sup>5</sup>Manfredini and Gürcan, *arXiv (submitted to Phys. Rev. E)*, arxiv.org/abs/2411.12750 (2024)

<sup>6</sup>De Pietro et al., *Phys. Rev. E* **92**, 043021 (2015).