

Finitely many determining functionals, data assimilation and a novel regularity criterion for the 3D Navier-Stokes equations

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We present a novel regularity criterion for the 3D Navier-Stokes equations (NSE) based on finitely many modal, nodal or volume element observations of the velocity field. The proof is based on a data assimilation algorithm utilizing a Newtonian relaxation scheme (nudging) motivated by feedback-control. The observations, which may be either modal, nodal or volume elements, are obtained from a weak solution of the 3D NSE and are collected almost everywhere in time over a finite grid. The regularity criterion we propose follows from our data assimilation algorithm and is hence intimately connected to the notion of determining functionals (modes, nodes and volume elements). Our regularity criterion is fundamentally different from any preexisting regularity criterion as it is based on finitely many observations (modes, nodes and volume elements). We further prove that the regularity criterion we propose is both a necessary and sufficient condition for regularity. Thus, our result can be viewed as a natural generalization of the notion of determining modes, nodes and volume elements as well as the asymptotic tracking property of the nudging algorithm for the 2D NSE to the 3D setting.



Figure 1: Caption of this figure.

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