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Modelling stratified turbulence with probability density functions

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Turbulent transport and mixing in density stratified fluids are found in a wide range of natural and human-made environments, such as the ocean, reservoirs, buildings and the atmosphere. In the ocean, for example, diapycnal mixing plays a significant role in sustaining its large-scale circulation. In buildings, on the other hand, stratification determines the amount of heating and cooling that is required, in addition to the fate of airborne pathogens. Modelling stratified turbulence in these areas remains challenging, due to incomplete understanding of the underlying physical processes and the wide range of different forcing conditions that one finds across applications [1]. In the Reynolds-averaged governing equations, it is the buoyancy flux is that is often the focus of investigation and parameterisation.

We explore the possibility of viewing the problem probabilistically, as a means of understanding and modelling transport and mixing in a wide range of applications. There are at least three reasons to think that this might be a good idea [2]. First, in the Fokker-Planck equation that governs the evolution of the joint probability distribution of the vertical velocity and buoyancy, the buoyancy flux is a known quantity. Secondly, the concept of 'available potential energy', which is applied widely to understand the energetics behind stratified turbulence, can constructed using the probability distribution of the buoyancy field and is therefore intrinsic. Thirdly, statements of the problem in terms of probability distributions lend themselves naturally to the incorporation of uncertainty from measurements and forcing conditions.

We apply existing closure schemes for local probability density methods[3] to model the (negative) diffusion of probability over control volumns due to diapycnal mixing and viscous dissipation. The resulting governing equations for the joint probability of vertical velocity and buoyancy are nonlinear Fokker-Planck equations. We will compare predictions from the models with results from direct simulations of forced stratified turbulence. Time permitting, we will discuss extensions to the basic joint modelling of vertical velocity and buoyancy through the incorporation of additional observables and the simulation of stochastic differential equations to establish 'particle' trajectories.

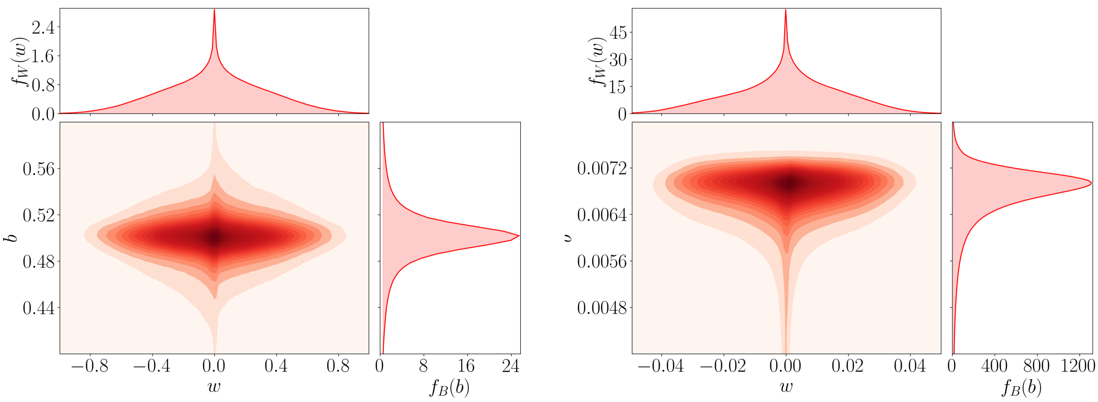

Figure 1: Joint density f_{WB} of the vertical velocity W and buoyancy B and its marginal densities f_W, f_B from a simulations of 2D Rayleigh-Bénard (left) and 2D internally heated convection (right) at $Ra = 10^9$ and $Ra = 10^{11}$ respectively.

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