

Non-linear transient growth and transition to turbulence in pulsatile pipe flow

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Pulsatile fluid flows through straight pipes undergo a sudden transition to turbulence that is extremely difficult to predict. The difficulty stems here from the linear Floquet stability of the laminar flow up to large Reynolds numbers, in agreement with common experimental observations of sub-critical transition¹. This makes the instability problem fully non-linear and thus dependent on the shape and amplitude of the flow disturbances, in addition to the Reynolds number (Re), the Womersley number (Wo) and the pulsation amplitude (A).

Linear transient growth analysis revealed that, depending on these parameter values, a new optimal perturbation emerges in the deceleration phase². In the linear regime, this perturbation has a helical shape and experiences a growth rate increase of multiple orders of magnitude, compared to the optimal perturbation in steady pipe flow.

An open problem concerns the effect of non-linearities on the optimal growth and transition to turbulence in pulsatile pipe flow. This problem can be tackled by optimizing over the space of all admissible disturbances to the laminar flow with an adjoint method³. In this presentation, we will present a numerical adjoint optimization code based on a GPU implementation of the pseudo-spectral Navier—Stokes solver *nspipe*⁴. Our code incorporates an automatic checkpointing strategy that maximizes the computational efficiency in a given computer architecture and enables the possibility to study the effect of multiple problem parameters at a reasonable cost⁵. Our results show that standing wave perturbations are most efficient in destabilizing the flow at intermediate Womersley numbers. We will discuss the effect of non-linearities on the shape and growth of optimal disturbances during different phases of the period and the role of the helical disturbance in the transition to turbulence in pulsatile pipe flow.

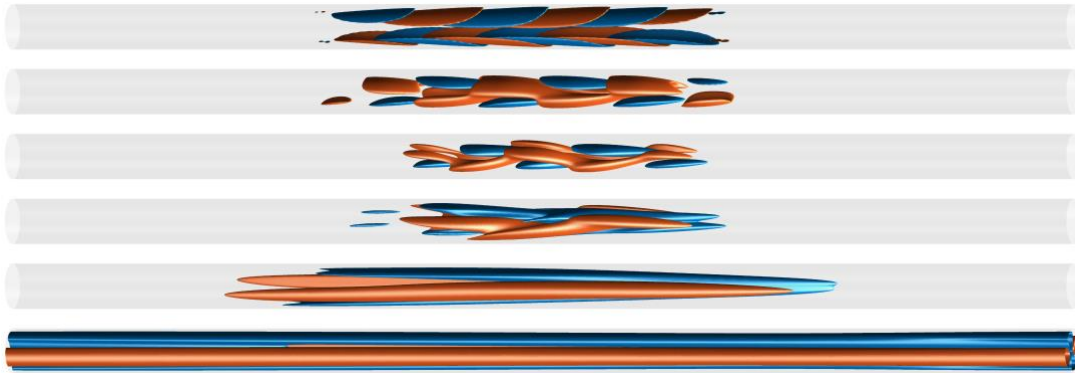


Figure 1: Iso-contours of the axial velocity $\pm u_z$ of the (non-linear) optimal standing wave perturbation $\mathbf{u}'(t = 0)$ in the deceleration phase of pulsatile pipe flow at $(Re, Wo, A) = (2200, 5.6, 0.85)$.

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¹Xu et al., *Proc. Natl. Acad. Sci.* **117**, 11233–11239 (2020)

²Xu et al., *Journal of Fluid Mechanics* **907** (2023)

³Kerswell et al., *Annual Review of Fluid Mechanics* **50**, 319–345 (2018)

⁴Morón et al., *Phys. Rev. Fluids* **9**, 024601 (2024)

⁵Griewank et al., *ACM Transactions on Mathematical Software* **26**, 19–45 (2010)