

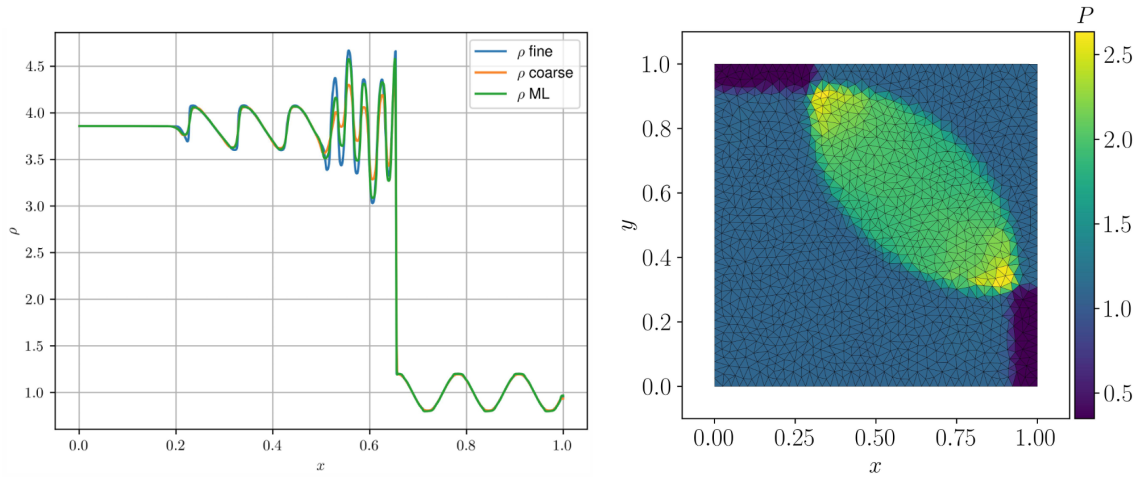
A data-driven learned discretization approach in finite volume schemes for compressible fluid dynamics

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The recent development of Machine Learning (ML) and Deep Learning methods, coupled with recent advances in GPU-based computing defined new promising techniques for the numerical resolution of PDEs entirely solved with ML, as well as tuning existing algorithms for learning corrections or discretizations¹.

In this work, we combine finite volume numerical schemes and neural networks to learn the discretization of the spatial derivatives of partial differential equations (PDEs) in order to better resolve the small spatial scales. We use approximate solutions of the 1D and 2D Euler equations obtained on a fine Cartesian grid for the reference database in order to learn an optimal spatial discretization on a coarse grid, even with discontinuities in the solution. This is often described as super-resolution. We post-process the outputs of the neural network to guarantee consistency of the space discretization, and penalty functions are added for regularization. The method has been generalized to on unstructured grids using Graph Neural Networks (GNNs) and taking boundary conditions into account. Examples are shown in Figure 1.

Different types of boundary conditions are included in the database and learned by the neural network. For the latter, numerical experiments on solutions with shocks in one and two space dimensions are proposed to evaluate the performances of the present approach. The method outperforms reference finite volume scheme used as reference in the learning step, allowing us to integrate twice as precisely on a same grid.



(a) Shu-Osher test case, 4096 cells for fine integration, 1024 cells for ML and coarse solutions, $T = 0.156$.

(b) ML solution on 2746 triangular cells for Test case n°4. $T = 0.25$.

Figure 1

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¹Y. Bar-Sinai, S. Hoyer, J. Hickey, and M. P. Brenner, “Learning data-driven discretizations for partial differential equations,” Proceedings of the National Academy of Sciences, vol. 116, no. 31, pp. 15 344–15 349, 2019.