## Outer-layer self-similarity in spatially developing turbulent boundary layers

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A direct numerical simulation (DNS) of a spatially developing flat-plate turbulent boundary layer in the low supersonic regime will be examined, with the goal of characterizing the changes to the mean flow field as the turbulent wake becomes self-similar. One challenge in acquiring cross-comparable turbulent boundary layer measurements, both numerically and experimentally, is the persistent influence of 'subjective' factors in the upstream flow, such as the method used to induce turbulent transition. Artifacts can propagate surprisingly far downstream, particularly in the wall-distant region of the boundary layer, making it difficult to draw canonical conclusions about the behavior of spatially developing boundary layers. The present DNS has used a digital filtering method at the domain inflow at a minimal Re, enabling e.g. >300 inflow boundary-layer thicknesses of streamwise relaxation space to be covered before  $Re_{\theta_i} \approx 1200$ , still squarely in the low Re regime, where  $Re_{\theta_i}$  is the incompressible-equivalent Reynolds number based on the momentum thickness. Fig. 1(*a*) shows the progression of the skin-friction coefficient  $c_f$ . At low Re,  $c_f$  is often empirically approximated by a power law, i.e.  $c_f \approx a \cdot Re_{\theta_i}^b$ , for example in [1], where *a* and *b* are constants. Analytically derived formulations become more applicable at sufficiently high Re, such as the 'Coles-Fernholz 2' correlation  $c_f \approx 2[\kappa^{-1} \ln(Re_{\theta}) + C]^{-2}$ , discussed in [2], where  $\kappa$  and *C* are constants. In



Figure 1: (a) Skin-friction coefficient  $c_f$ , normalized by the empirical fit for low Re from [1]. (b) Wake development metric  $\Delta/\delta_{99}$ , where  $\Delta$  is the Rotta-Clauser length scale and  $\delta_{99}$  is the 99% hydrodynamic boundary-layer thickness. Both quantities exhibit a distinct departure from their low-Re progression around  $4000 \leq Re_{\theta} \leq 5000$ .

the immediate vicinity of the crossing of the low- and high-*Re* correlations mentioned, there has been an effective 'blind spot' in the body of available data. DNS datasets were either limited in *Re* such that they did not capture the 'crossover' between the disparate  $c_f$  trends, or established inflow independence at *Re* in excess of the crossing. Experimental measurements which directly measure  $\tau_w$ , for example [3] [4], have largely focused on the high-*Re* regime where the wake profile has already become self-similar, as is visible in Fig. 1(*b*). The present DNS, whose reliable range spans  $1200 \leq Re_{\theta i} \leq 5600$ , covers the establishment of outer-layer self-similarity, a fundamentally important stage in the intermediate development of a turbulent boundary layer. Results indicate a relatively abrupt change in the progression of  $c_f$  vs ln(*Re*), coincident with the end of steady relative wake growth in the low-*Re* regime and the onset of self-similarity in the outer layer. Detailed analysis regarding the *Re*-dependence the mean flow field will be presented.

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<sup>1</sup>Smits et al., *Journal of Ship Research* **27**, 147-157 (1983)

- <sup>3</sup>Österlund, *Physics of Fluids* **12**, 1-4 (2000)
- <sup>4</sup>Nagib et al., IUTAM Symposium on One Hundred Years of Boundary Layer Research, 383-394 (2004)

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<sup>&</sup>lt;sup>2</sup>Nagib et al., Philosophical Transactions of the Royal Society A 365, 755-770 (2007)

<sup>&</sup>lt;sup>5</sup>Sillero et al., *Physics of Fluids* **25**, 105102 (2013)

<sup>&</sup>lt;sup>6</sup>Schlatter et al., Journal of Fluid Mechanics 659, 116-126 (2010)