A Hierarchical Bayesian Approach for Adaptive Model-Assisted Probability of Detection Over Multiple Conditions and/or Specimen Types

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Abstract

In the non-destructive testing industry, probability of detection (POD) studies can be prohibitively expensive because of specimen and testing costs. Model-assisted probability of detection (MAPOD) allows the use of simulation to reduce the number of specimens, but current MAPOD practices require precision models with high enough accuracy that predictions can be treated as equivalent to real experimental data. Unfortunately, many models are too idealized to be used directly, even if the model has strong informative capacity. Our adaptive POD approach develops trust in the model according to how well the model predictions follow the trends observed in experimental data. The approach extends the traditional MAPOD concept of a simulation-to-experiment transfer function, now acknowledging its uncertainty and variability. When you have multiple test conditions or specimen types, normally you would need entirely new and separate POD studies for each condition or specimen type. If you have a model that can meaningfully predict NDE outcomes as a function of test condition and specimen characteristics, and some limited amount of experimental data, then you can evaluate transfer functions across the population of test conditions and specimen types. With limited experimental data, these transfer functions will be highly uncertain. However, treating the transfer function characteristic of each member of that population as an uncertain sample of a common distribution, we can draw strength across test conditions and specimen types based on confidence in the model.

In our demonstration, the transfer function between simulation and experiment for a specimen type tested under particular conditions is described by a pair of scaling factors. This pair is treated as a sample from a distribution of scaling factor pairs. If the model is highly predictive, then all the scaling factor pairs will be approximately the same, and the distribution will be narrow. If the model is not predictive, then the distribution of scaling factor pairs will be very wide. We set up a statistical model where the width of the distribution is an unknown parameter. Then, based on the experimental data and model output we evaluate this width, along with the distributions of the various scaling factors and other parameters using a Markov Chain Monte Carlo (MCMC) sampler on a hierarchical Bayesian statistical model. The outcome of the evaluation gives us histograms for the width of the scaling factor pair distribution as well as histograms for each of the scaling factors themselves. It also gives the ability to estimate POD and confidence for each specimen type under the corresponding test conditions. If the scaling factor pair distribution is narrow, then simulation and experiment are strongly correlated, and the use of the model significantly reduces the amount of experimental data required. If the scaling factor pair distribution is wide, then the simulation is not predictive and is ignored. The approach naturally measures and adapts to the quality of the model.