Semi-Coskewness Asset Pricing Model

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Abstract

Skewness, the third moment of the return distribution, has been demonstrated to be an effective factor in explaining individual asset expected returns, along with mean and variance. In this study, we propose a semi-coskewness model to predict future cross-sectional return variations in the US stock market. Inspired by Bollerslev, Patton, and Quaedvlieg's (2022) realised semibeta model, the semi-coskewness model decomposes the coskewness into four components based on the signs of individual stock returns and market returns. This four-component decomposition allows us to examine the relationship between coskewness and asset return under various market conditions. Our empirical evidence demonstrates the consistent predictive power of the semi-coskewness model. Furthermore, the findings of our study indicate that investors demonstrate an asymmetric response to coskewness subject to market conditions, thereby aligning with the behavioural finance theories.

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1 Introduction

The Capital Asset Pricing Model (CAPM) is based on the assumption that rational investors make investment decisions under mean-variance optimisation, indicating that investors seek to minimise variance, irrespective of its direction (Lintner, 1965; Sharpe, 1964). Alternative studies, including Roy's safety-first criterion (1952) and Markowitz's mean-semivariance theory (1959), demonstrate that investors exhibit an asymmetric response to upside and downside variance. These studies constitute the foundation of subsequent research in the field of downside risk. Beyond capturing investors' asymmetric risk preference effectively, the mean-semivariance framework also effectually models non-normal asset returns.

The competing theories on mean-variance and mean-semivariance provide divergent responses to the question: How do investors respond to the risk they face in making a particular investment? Inspired by the recent studies on downside risk and the importance of higher moments measurements in addressing this issue, we propose and test a semi-coskewness model, which is a four-component decomposition of coskewness. We provide empirical evidence on the predictive power of individual asset semi-coskewnesses with respect to its future cross-sectional returns. Furthermore, comparisons are made between coskewness and downside and upside coskewness (a two-component decomposition) in the US stock market. The empirical results obtained reveal that investors exhibit asymmetric responses to coskewness, subject to market conditions, which cannot be explained by classical financial theory.

Coskewness, otherwise known as systematic skewness, has proven to be an effective risk factor for explaining the cross-sectional asset return variation, and it also serves as a supplementary factor to the beta in the CAPM, thereby forming a three-moment CAPM (Dittmar, 2002; Harvey and Siddique, 2000; Kraus and Litzenberger, 1976). Harvey and Siddique (2000) state: "We define coskewness as the component of an asset's skewness related to the market portfolio's skewness". The underlying formula is similar to the CAPM beta measurement, with the substitution of r_m (market return) for r_m^2 (squared market return) in the numerator of the equation[2]. In essence, the coskewness measures how an individual asset reacts to extreme market movements, as the squared market return amplifies the market movement. The direction of market movement, whether it be an upside or downside trend, is irrelevant to the calculation of coskewness, as the squared market return will shroud the information in the direction of market movement. However, extant downside risk studies already suggest that investors exhibit asymmetric response to the direction of variance. Consequently, investors may also exhibit asymmetric response to the direction of coskewness.

An asset exhibiting positive coskewness will result in the asset portfolio becoming more right skewed. Consequently, the portfolio is deemed to be less risky and requiring lower return, making the asset favourable to the investor, regardless of the market return being positive or negative. The expected utility theory provides an explanation for investors' preference to the right skewed asset, as investors are rational and their decisions should maximise their utility, and irrelevant to market conditions. However,

[2] There are other methods of measuring coskewness, but we choose to follow Ang, Chen, and Xing (2006) and Bollerslev, Patton, and Quaedvlieg (2022). The construction is similar to the CAPM beta and more suitable for our study. CAPM beta can be seen as a measurement of how an individual asset contributes to the variance of the market portfolio, whereas coskewness measures how an individual asset contributes to the skewness of the market portfolio.

subsequent prospect theory (Kahneman and Tversky, 1979) and disposition effect (Shefrin and Statman, 1985) both suggest that investors' cognitive biases affect their own decision making. As suggested by prospect theory, investors place greater weight on losses than on gains when making decisions. This suggests that assets with positive skewness are more attractive to investors in downward markets than in up ward markets, due to the loss aversion.

Nevertheless, there is little empirical evidence regarding the divergent treatment of coskewness in upside and downside markets. To shed more light, this study aims to utilise the two-component and four-component coskewness decomposition models to fill the research gap. The concept of two-component and four-component coskewness decomposition is derived from corresponding downside risk studies that further decompose the CAPM beta. The two-component coskewness decomposition is similar to the downside and upside beta decomposition by Ang, Chen, and Xing (henceforth ACX) (2006), while the four-component decomposition is similar to the realised semibeta decomposition by Bollerslev, Patton, and Quaedvlieg (henceforth BPQ) (2022).

The two-component method decomposes the coskewness into two components: the downside coskewness (CskN) and upside coskewness (CskP), depending on the sign of the market return. Specifically, CskN measures how an individual asset co-moves with the extreme market when the market return is negative, and CskP measures when the market returns are positive. The decomposition allows the information about market movement directions to be included in the coskewness measurements, which will assist in examining whether an investor's decision making is influenced by the market movements.

Similarly, the CAPM beta can be decomposed into downside and upside betas, where the downside beta measures the downside risk. In the downside beta study conducted by ACX (2006), the authors provide empirical evidence that US investors receive a downside risk premium in the US stock market. However, subsequent studies have provided mixed results regarding the existence of such a downside risk premium in both US and international stock markets (Lettau, Maggiori, and Weber, 2014; Atilgan, Demirtas, and Gunaydin, 2020; Levi and Welch, 2021). With respect to the downside and upside coskewness, Galagedera and Brooks (2007) test the downside coskewness model in emerging markets and suggest that it outperforms the downside beta model. However, the performance of their model in the US market is still left unexplored.

The four-component decomposition of coskewness further extends the two-component decomposition so that is depends concurrently on the signs of both individual assets and market return. Allowing information about both individual asset and market movement will assist us in further examining investors' reactions to coskewness under various scenarios. For example, the CskN could be further decomposed into two semi-coskewness—CskPN and CskNN, with the former capturing the association when the individual asset exhibits a positive return while the market return is negative, and the latter capturing association when both the individual asset and the market are negative returns. A similar logic underpins the decomposition of CskP into semi-coskewness CskPP and CskNP.

BPQ (2022) in their study of realised semibeta model, show that finer four-component CAPM beta decomposition provides additional information to that provided by the two-component decomposition measurement. The authors demonstrate that a focus on the sign of market returns only, as in the downside beta model, fails to provide a complete picture of return information. Consequently, investors should focus on the signs of individual stock returns and market returns simultaneously. The results prove that, in comparison with the two-component decomposition of the CAPM beta (downside and upside betas), the four-component decomposition reveals more information that could be used to explain the risk–return relationship in the US stock market.

In the empirical results presented by BPQ (2022), the coskewness factor still reports a significant coefficient when the four semibetas are included, suggesting that return information is not fully captured by the sophisticated semibeta model. The risk premiums for coskewness and downside risk come from different sources (Li, Li, and Su, 2024). To express the information conveyed by coskewness, this study tests whether the two- and four-component decompositions that have succeeded in decomposing CAPM beta will reveal investors' asymmetric reactions to coskewness. We test the predictive power of downside and upside coskewness models and the semi-coskewness model, in the US stock market.

To the best of our knowledge, this study is the first one to propose and examine the semi-coskewness model in the US stock market. The study makes several contributions, which are outlined below. Firstly, it fills the research gap in the extant asset pricing literature by extending the higher return moments and downside risk studies. Secondly, it provides empirical evidence that investors have asymmetric responses to coskewness subject to the market movements. Thirdly, the results demonstrate that compared to other well-known risk factors, the semi-coskewness measurements have the advantage of using the basic return information but still achieve robust cross-sectional predictive power. Finally, while the present study is pioneering in the US stock markets, there is considerable potential for its application to international multi-asset-class markets.

The remainder of the study is structured as follows. Section 2 provides a brief review of related skewness studies. Section 3 provides a detailed description of the US stock market data from the LSEG DataStream used in the study. Section 4 provides a detailed methodology for constructing various coskewness models and their independent variables. Section 5 presents the Fama-MacBeth regression results for the baseline regression and robustness tests. Section 6 concludes the study.

2 A Brief review of related studies

Asset skewness captures the asymmetry in the distribution of returns. Investors naturally prefer positive or right-skewed assets because they have a higher probability of positive payoffs. On the other hand, investors demand higher compensation to bear negative or left-skewed assets, as they are perceived to be riskier. Similar to the idiosyncratic risk and the systematic risk conveyed by asset, the skewness of individual assets can also be divided into two parts: idiosyncratic skewness and systematic skewness (or coskewness) (Conrad, Dittmar, and Ghysels, 2013; Langlois, 2020). The idiosyncratic part measures the skewness of individual asset returns that cannot be diversified.

In contrast with the idiosyncratic skewness, the systematic part of the skewness of individual assets measures the joint distribution of individual and market portfolio returns (Patton, 2004). The systematic skewness serves as the overarching basis for the three-moment CAPM. In addition, the three-moment CAPM relaxes the oversimplified assumptions that return follows a normal distribution and that the representative investor has a quadratic utility function. Smith (2007) further notes that a non-increasing absolute risk aversion assumption is more appropriate for the three-moment CAPM.

Early literature on skewness suggests that the idiosyncratic skewness of an asset can be fully diversified, thus, should not be rewarded. However, subsequent studies have shown that under diversification exists, and that idiosyncratic skewness can explain cross-sectional return variation (Barberis and Huang, 2008; Boyer, Mitton, and Vorkink, 2010; Conrad, Dittmar, and Ghysels, 2013; Langlois, 2020). The systematic skewness (or coskewness) measures how an individual asset contributes to the skewness of the market portfolio when added to the portfolio, should be rewarded. The greater the coskewness, the greater the increase in the skewness of individual assets to the portfolio as a whole, indicating lower risk and should be rewarded with a lower return (Harvey and Siddique, 2000; Yang, Zhou, and Wang, 2010).

In addition to the risk-based explanation of skewness, several studies also explain skewness on the basis of behavioural finance. Barberis and Huang (2008) argue that the preference for skewness is related to the investor's gambling nature and misperceptions about the probability of future payoffs. Hur and Singh (2017) suggest that the role of skewness in asset pricing can be attributed to investor attention, prospect theory, and mental accounting.

International evidence also supports the ability of skewness to explain cross-sectional return variation across stock markets (Dong, Dai et al., 2022; Dong, Kot et al., 2022). In addition, Galagedera and Brooks' (2007) study downside coskewness models around 27 emerging stock markets. Coskewness is also studied on other asset classes, including bonds, mutual funds and futures markets (Back, Crane, and Crotty, 2018; Chiang, 2016; Christie-David and Chaudhry, 2001; Moreno and Rodríguez, 2009; Yang, Zhou, and Wang, 2010).

Although the existing coskewness literature has covered many areas, the majority of studies still consider coskewness as a whole rather than as decomposed parts of the whole. Inspired by the downside risk studies, our study explores a revised asset pricing model that incorporates decomposed coskewness.

3 Data

We focus on the US stock market, which is the world's leading financial market. To avoid survivorship bias, the sample includes stock data of the constituent stocks that are listed and delisted. They are retrieved from both the NYSE and NASDAQ from the LSEG DataStream between 1 January 1980 and 30 April 2024[3].

[3] A variety of data types are retrieved, but there are missing variables for the whole sample period.

We use the S&P 500 index as a proxy for the value-weighted market portfolio, and the risk-free rate is proxied by the US 10-year bond yield. The data include the total return index, closing price, trading volume, daily and monthly market capitalisation and market-to-book value. Following the extant literature, we first remove all stocks in a given month if the monthly stock prices are less than $5 in order to single out these thinly traded, illiquid stocks, as these stock prices may largely deviate from their intrinsic value (BPQ, 2022; Dittmar, 2002). Second, we follow Ince and Porter (2006) and Karolyi, Lee, and van Dijk (2012) to address any potential issues associated with data in DataStream. Firstly, we remove the total return index on a given day if its value is less than 0.1. Secondly, we treat the day as a non-trading day if more than 90% of the stocks on the exchange report a return of 0. After detailed screening, our sample contains 9,728 individual stocks with 1,012,017 monthly observations. March 1980 has the smallest observation with 194 firms, while March 2022 has the largest with 3,184 firms.

4 Methodology

Kraus and Litzenberger (1976) and Harvey and Siddique (2000) show that coskewness, when combined with the CAPM beta, negatively predicts future cross-sectional stock returns. Their results suggest that the usefulness of coskewness to further explain variations in cross-sectional returns. Our coskewness estimations align with those of ACX (2006) and BPQ (2022), which is conceptually similar to Harvey

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Consequently, the sample is restricted to the period between 1 January 1980 and 30 April 2024.

and Siddique's (2000) conditional coskewness measurement.

The realised variance framework directly measures the underlying moments and has the advantage of taking the variance as observed. We estimate the conditional coskewness of individual stocks on an overlapping 12-month basis. The current month's coskewness is calculated from the daily realised returns of individual stocks and market portfolios over the past 12 months. It is imperative to note that a minimum of 125 valid daily stock and market returns over the 12-month period is required for the estimation process to be conducted. Since the coskewness estimate is directly related to the tail distribution of the market portfolio, we deliberately choose the 12-month window. The existing literature suggests different estimation intervals ranging from daily to 60 months (ACX, 2006; Amaya et al., 2015; Dong, Kot et al., 2022). The high-frequency interval measurement captures the time-varying nature of coskewness, but at the cost of fewer observations of the tail market portfolio distribution. Conversely, the low-frequency measure tends to sacrifice the time-varying nature of coskewness. The choice of a 12-month interval strikes a balance between capturing the time-varying nature of coskewness and sufficient tail observations, and has also been used in the downside beta and semibeta estimations in prior studies (ACX, 2006; BPQ, 2022).

The coskewness measure is as follows:

$$Csk_{i,t} = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})(r_{m,t-j,d}-\bar{r}_{m,t-j})^2}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})^2}\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{m,t-j,d}-\bar{r}_{m,t-j})^2} \quad (1)$$

where $Csk_{i,t}$ denotes the coskewness for individual stock i in month t, $r_{i,t-j,d}$ denotes the daily log return (measured as the difference between the natural logarithm of two consecutive daily total return index values) of stock i on the d^{th} trading day of month $t-j$; $t-j$ denotes the 12 months rolling window starting from the $t-11$ month; $N_{i,t}$ denotes the number of trading days in the past 12 months for stock i in month t; $D_{i,t-j}$ denotes the number of trading days of stock i in month $t-j$; and $r_{m,t-j,d}$ denotes the daily log market return on the d^{th} trading day of month $t-j$. $\bar{r}_{i,t-j}$ and $\bar{r}_{m,t-j}$ denote the month $t-j$ average return on the individual stock and market, respectively.

In addition, we also estimate the realised CAPM beta following BPQ (2022) as follows:

$$\beta_{i,t} = \frac{\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}r_{m,t-j,d})}{\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}} r_{m,t-j,d}^2} \tag{2}$$

where $\beta_{i,t}$ denotes the realised CAPM beta for individual stock i in month t.

Inspired by ACX's (2006) downside beta model, the two-component decomposition method decomposes the coskewness into downside and upside coskewness. The downside and upside coskewness factors are defined as follows:

$$Csk_{i,t}^N = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})\left(r_{m,t-j,d}^- - \bar{r}_{m,t-j}\right)^2}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})^2}\,\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}^- - \bar{r}_{m,t-j}\right)^2}$$

$$Csk_{i,t}^P = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})\left(r_{m,t-j,d}^+ - \bar{r}_{m,t-j}\right)^2}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}(r_{i,t-j,d}-\bar{r}_{i,t-j})^2}\,\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}^+ - \bar{r}_{m,t-j}\right)^2} \tag{3}$$

where $r_{i,t-j,d}^+ =$ max $(r_{i,t-j,d}, 0)$ and $r_{i,t-j,d}^-$ = min $(r_{i,t-j,d}, 0)$, $r_{m,t-j,d}^+ =$ max $(r_{m,t-j,d}, 0)$ and $r_{m,t-j,d}^-$ = min $(r_{m,t-j,d}, 0)$.

In spirit of BPQ's (2022) realised semibeta model, we propose a semi-coskewness model that decomposes the coskewness into four components. The semi-coskewness model is defined as follows:

$$Csk_{i,t}^{PP} = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}^{+}-\bar{r}_{i,t-j}\right)\left(r_{m,t-j,d}^{+}-\bar{r}_{m,t-j}\right)^{2}}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}-\bar{r}_{i,t-j}\right)^{2}}\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}-\bar{r}_{m,t-j}\right)^{2}}$$

$$Csk_{i,t}^{PN} = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}^{+}-\bar{r}_{i,t-j}\right)\left(r_{m,t-j,d}^{-}-\bar{r}_{m,t-j}\right)^{2}}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}-\bar{r}_{i,t-j}\right)^{2}}\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}-\bar{r}_{m,t-j}\right)^{2}}$$

$$Csk_{i,t}^{NP} = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}^{-}-\bar{r}_{i,t-j}\right)\left(r_{m,t-j,d}^{+}-\bar{r}_{m,t-j}\right)^{2}}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}-\bar{r}_{i,t-j}\right)^{2}}\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}-\bar{r}_{m,t-j}\right)^{2}}$$

$$Csk_{i,t}^{NN} = \frac{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}^{-}-\bar{r}_{i,t-j}\right)\left(r_{m,t-j,d}^{-}-\bar{r}_{m,t-j}\right)^{2}}{\sqrt{\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{i,t-j,d}-\bar{r}_{i,t-j}\right)^{2}}\frac{1}{N_{i,t}}\sum_{j=0}^{11}\sum_{d=1}^{D_{i,t-j}}\left(r_{m,t-j,d}-\bar{r}_{m,t-j}\right)^{2}} \quad (4)$$

The semi-coskewness model decomposes the coskewness based on the signs of both individual stock and market returns. Hence, $Csk_{i,t}^{PP}$ ($Csk_{i,t}^{NP}$) measures a fraction of $Csk_{i,t}$ conditional on positive (negative) individual stock return with positive market return; $Csk_{i,t}^{PN}$ ($Csk_{i,t}^{NN}$) measures a fraction of $Csk_{i,t}$ conditional on positive (negative) individual stock return with negative market return.

Compared to the semi-coskewness factors, the CskN and CskP factors focus only on the sign of market returns but not the sign of individual stock returns. According to the factor construction, the semi-coskewness, CskN, and CskP factors should have the following relationship[4]:

$$Csk_{i,t}^{PP} + Csk_{i,t}^{NP} \approx Csk_{i,t}^{P}$$

$$Csk_{i,t}^{PN} + Csk_{i,t}^{NN} \approx Csk_{i,t}^{N} \quad (5)$$

[4] Although the denominators of CskPP and CskNP are identical (equation 4), but they are slightly different from CskP (equation 3). Therefore, the sum between CskPP and CskNP is only approximately equal to CskP. A similar relationship exists between CskPN, CskNN, and CskN.

Harvey and Siddique (2000) demonstrate that the coskewness of assets has a negative relationship with their returns. If the rational investor expectations are valid, all the two-component and four-component coskewness measurements should exhibit negative relationships with their returns. Conversely, when considering the behavioural finance theory, the negative relationship might not always hold when investors trading in rising or falling stock markets. Keep other things unchanged, an asset with positive coskewness in a portfolio would reduce the risk of the portfolio by increasing the portfolio’s right skewness, regardless of the market situation.

The main independent variables in this study are coskewness, downside and upside coskewness, as well as semi-coskewnesses. In addition, we introduce some mainstream cross-sectional factors in the existing literature to serve as the control variables. The size factor (SIZE) is constructed following Fama and French (1993); we use the month-end logarithm of individual stock market capitalisation. Following Fama and French (1992), we use the end-of-month Market-to-Book ratio divided by one to measure the Book-to-Market (BM). Momentum (MOM) and Reversal (REV) are constructed following Jegadeesh and Titman (1993) and Jegadeesh (1990), respectively. The momentum factor is measured by the compounded return between the past 12 months and the past 2 months, while the past month’s return measures the reversal factor. The idiosyncratic volatility (IVOL) factor is constructed according to Ang et al. (2006), as follows:

$$r_{i,t,d} - r_t^f = \alpha_i + \beta_i(r_{m,t} - r_t^f) + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \epsilon_{i,t} \tag{6}$$

where r_t^f denotes the risk-free rate at time t, and SMB and HML denote the size and value factors are taken from Ken French's website[5]. Monthly idiosyncratic volatility is measured as the standard deviation of the current month's daily residuals. The realised variance (RV) factor is constructed following Andersen et al. (2001) and is calculated as the sum of the squares of the current month's daily returns. The illiquidity factor (ILLIQ) follows Amihud (2002):

$$ILLIQ_{i,t} = \frac{1}{m}\sum_{d=1}^{D_{i,t}}\left(\frac{r_{i,t}}{volume_{i,t} * price_{i,t}}\right) \tag{7}$$

where $volume_{i,t}$ denotes the daily trading volume for individual stock i in month t; $price_{i,t}$ denotes the daily trading volume for individual stock i in month t; and m denotes the number of trading days in a month. The illiquidity factor is calculated based on the previous month's daily data.

5 Empirical results

5.1 Summary statistics

Panels A and B of Table 1 show the monthly summary statistics and the correlation matrix of the variables calculated on the basis of the constituent stocks in the NYSE and NASDAQ. The screened sample covers the period between January 1980 and April 2024. Specifically, following BPQ (2022) to address price discreteness, we remove stocks in a given month if their monthly stock prices are less than \$5. Furthermore, to mitigate the impact of outliers, we winsorise the main independent variables, setting the boundaries at 1% and 99% each month, following ACX (2006). The dependent

[5] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

variable is the monthly realised log return r_i. The main independent variables are Beta (realised CAPM beta), Csk (coskewness), CskN (downside coskewness), CskP (upside coskewness), CskPP, CskPN, CskNP and CskNN (four semi-coskewnesses). All variables reported in the table are time-series averaged and then cross-sectional, we report the mean, standard deviation, median, minimum, maximum, skewness and kurtosis. The average number of monthly observations is 1,903.

[Table 1 inserted here]

Panel A of Table 1 shows that the mean of monthly individual stock returns (r_i) is 0.002, while the standard deviation is 0.114, and slightly skewed to the left. The realised CAPM beta (Beta) has a mean of 0.84, with a minimum of -0.429 and a maximum of 2.881. It is important to note that winsorisation can cause the mean of the realised CAPM beta to deviate from 1. The coskewness (Csk) has a mean of -0.164 with a standard deviation of 0.159. Nevertheless, it is symmetric as the skewness is equal to 0. The downside coskewness is the most volatile among all the main independent variables, with a standard deviation of 0.522, along with an upside coskewness of 0.460. In terms of the four semi-coskewnesses, the group of individual stock and market returns with consistent signs (CskPP and CskNN) shows a similar pattern to the group with reversed signs (CskPN and CskNP), and similar patterns are also observed in studies by BPQ (2022) and Li, Li, and Su (2024).

Panel B of Table 1 presents the correlation matrix between different variables. The dependent variable r_i exhibits a low magnitude of correlation with all independent variables, with a highest correlation of -0.025 with the realised CAPM beta. The realised CAPM beta demonstrates a low correlation (-0.090) with the coskewness, suggesting that the coskewness conveys different information from that the realised CAPM beta captures (Harvey and Siddique, 2000). More interestingly, the realised CAPM beta has moderate correlations (around 0.5) with any of the coskewness measures, including downside, upside and semi-coskewnesses. Based on the equations construction, the information captured by CskP (CskN) should be approximately equal to CskPP plus CskNP (CskNN plus CskPN), and the correlation values support this relationship. CskP shows a high correlation of 0.946 with CskPP; and CskN shows a correlation of 0.944 with CskNN. In comparison, the correlations within the four semi-coskewnesses are at a moderate level of around 0.5-0.6 in absolute value, possibly indicating that the semi-coskewness captures different information.

[Figure 1 inserted here]

As demonstrated in Figure 1, the number of firms in each month of the screened sample period exhibits a remarkable increase, from 229 firms in January 1980 to 2,690 in April 2024. The number of firms fluctuates, but gradually increases over time. The minimum observations of 194 occur in March 1980, compared with the maximum of 3,184 firms in March 2022. There are some declines in the number of firm observations over the sample period. The declines are predominantly associated with well-known financial crises, such as the dotcom boom (2000), the global financial crisis (2008), and COVID-19 (2022).

5.2 Fama–MacBeth Regression

Following the existing cross-sectional asset pricing studies (ACX, 2006; BPQ, 2022; Fama and MacBeth, 1973), we use the conventional Fama-MacBeth ordinary least squares regressions to examine if different coskewness models can predict future stock returns cross-sectionally in the US stock market.

The Fama-MacBeth regression is with a two-stage process. In the first stage, we run the conventional cross-sectional regression in each month t. This results in $T-1$ (where T is the total number of months) number of λs. The baseline predictive regression equations are constructed as follows:

$$r_{i,t+1} = \lambda_{0,t+1} + \lambda_{t+1}\beta_{i,t} + \lambda_{t+1}^{PP}Csk_{i,t}^{PP} + \lambda_{t+1}^{PN}Csk_{i,t}^{PN} + \lambda_{t+1}^{NP}Csk_{i,t}^{NP} + \lambda_{t+1}^{NN}Csk_{i,t}^{NN} + \epsilon_{i,t+1} \quad (8)$$

where $r_{i,t+1}$ denotes the monthly individual stock log returns for stock i in month $t+1$ and serves as the dependent variable; $\lambda_{0,t+1}$ denotes the intercept in month $t+1$; λ_{t+1} denotes the coefficient estimate for realised CAPM beta for stock i in month $t+1$; λ_{t+1}^{PP}, λ_{t+1}^{PN}, λ_{t+1}^{NP}, and λ_{t+1}^{NN} denote the coefficients for four semi-coskewnesses; and $\epsilon_{i,t+1}$ denotes the residual errors. In addition to the baseline equations, we use the additional control variables such as SIZE, BM, MOM, REV, RV, IVOL and ILLIQ. The addition of control variables help to test whether they subsume the explanatory power of the main independent variables.

In the second step, we take the time series average of λ over the $T-1$ months in the sample. We report the Newey-West *t*-statistic values with 10 lags in the following tables, following BPQ (2022), to account for the heteroskedasticity and autocorrelation issue, as we use the overlapping measurement of different coskewness models.

[Table 2 inserted here]

Table 2 presents the baseline Fama–MacBeth regression results for the constituent stocks in the NYSE and NASDAQ between January 1980 and April 2024. Following Kraus and Litzenberger (1976) and Harvey and Siddique (2000), we consider coskewness as a complementary risk factor to the CAPM beta, as opposed to relying on coskewness alone. Column (1) of Table 2 shows that the realised CAPM beta cross-sectionally predicts negative returns in the following month in the US stock market at the 1% significance level. This result persists across all 8 columns.

This intriguing negative result contradicts what classical finance theory suggests, as the results imply that investors who hold riskier assets (higher betas) are not rewarded with higher returns. Indeed, the phenomenon in which stocks with lower CAPM betas tend to have higher returns than stocks with higher betas is well-known in the literature as the low beta anomaly (Haugen and Heins, 1975). This low beta anomaly, which has been observed in both developed and developing countries, is persistent over time (Blitz and van Vliet, 2007; Blitz, Pang, and van Vliet, 2013; Fama and French, 1992). The negative coefficient for CAPM beta is a special condition for the low beta anomaly. Han, Li, and Li (2020) find that a downward sloping SML in the Chinese market, our results confirm the existence of the low beta anomaly in the US stock market.

The existing studies on the low beta anomaly offer several possible explanations of the above phenomenon. On the one hand, many studies conjecture that in certain market, the security market line (SML) deviates from what it should be. Black (1972) suggests that the SML tends to be flatter when faced with leverage constraints. Naive individuals and sophisticated institutional investors are likely to face leverage constraints due to either their inability or legal requirements (Frazzini and Pedersen, 2014). Another explanation, related to the agency effect, is that the performance of portfolio managers is usually measured against specific benchmarks. Managers are willing to pay more for high beta stocks than low beta stock in order to compete with the benchmark, thereby reducing stock returns (Blitz, 2014).

On the other hand, many studies claim that the low beta anomaly can be explained by behavioural finance theories, as investors are not as rational as the simplistic assumptions underlying the CAPM suggest. Bali et al. (2017) propose that lottery stock-seeking investors drive up the price of high beta stocks, thereby lowering their returns.

Column (2) of Table 2 reports that coskewness has a negatively predictive ability for stock returns in the subsequent month cross-sectionally, which is similar to the finding of Harvey and Siddique (2000). Columns (3) to (5) focus on testing the predictive power of downside and upside coskewnesses together and with additional control variables. Similarly, columns (6) to (8) focus on the semi-coskewness model. The downside coskewness reports persistently negative coefficients through columns (3) to (5), while upside coskewness shows positive coefficients. For the control variables, BM, MOM and ILLIQ have positive coefficients, while RV and IVOL have negative coefficients at the 1% significant level. The coefficient estimate for momentum (MOM) is consistent with the finding that the predictive power of coskewness can be related to momentum (Harvey and Siddique, 2000).

Focusing on the semi-coskewness model, CskPP consistently reports positive coefficients, while CskPN and CskNN consistently report negative coefficients, all at the 1% significant level. CskP is roughly a combination of CskPP and CskNP; since the coefficient estimate of CskNP is insignificant, the predictive power of CskP is rooted in CskPP. For CskN, it is roughly CskPN plus CskNN, and both semi-coskewnesses have negative coefficients. The Fama–MacBeth regression results of downside and upside coskewness and semi-coskewness are consistent. The negative coefficient for coskewness is consistent with the negative coefficient that has been observed in downside coskewness, as well as CskPN and CskNN from semi-coskewness.

The signs of the coefficients of downside and upside coskewness and semi-coskewness are consistent. In addition, the empirical evidence tend to support the behaviour finance theories over the classical ones, as evidenced by the different signs of the coefficients for different coskewness factors. The finding of a positive coefficient associated with upside coskewness and the negative coefficient associated downside coskewness, suggesting that investors respond differently to asset coskewness. The semi-coskewness model reveals that the positive coefficient from upside coskewness mainly comes from CskPP (when both individual stock and the market portfolio exhibit positive returns). A number of behavioural explanations exist for the abnormal positive relationship observed between CskPP and returns. One such explanation could be that investors sell winner stock too early, as suggested by the disposition effect, which has the effect of depressing the asset price and thus increasing its return.

In addition, the control variables in columns (6) to (8) show similar results to those in columns (3) to (5). The average R^2 gradually increases as more variables are added to the regression, which also indicates that the semi-coskewness model has a higher explanatory power.

Although the correlations between the four semi-coskewnesses are only moderate, to check whether these correlations would affect our Fama–MacBeth regression results, we test each semi-coskewness with additional control variables similar to columns (6) to (8) in Table 2. The results are reported in Appendix Table A1, which shows that all four semi-coskewnesses have significant coefficients at the 1% level, indicating that the baseline results in Table 2 are robust.

5.3 Hypothesis testing

The Fama–MacBeth regression results show that the predictive power of the downside and upside coskewness models and the semi-coskewness model are fairly close. Recall also that CskP and CskPP have a correlation of 0.946 and CskN and CskNN have a correlation of 0.944, both close to 1. A critical question is, whether the information conveyed by the three coskewness models, i.e., the coskewness, downside and upside, and semi-coskewness models is the same or not. Therefore, we propose two hypotheses to answer the above question.

The first hypothesis tests whether the information captured by the coskewness is the same as that captured by the semi-coskewness model. Our null hypothesis is constructed as follows:

$$H_0^{Csk}: \lambda^{NN} = \lambda^{PN} = \lambda^{PP} = \lambda^{NP} \tag{9}$$

if we cannot reject H_0^{Csk}, which means the λs are the same for all the semi-coskewnesses, then the realised semi-coskewness can be reduced to the form of the coskewness. As a result, the semi-coskewness model does not convey any additional information to that of the coskewness, and we can rely on the coskewness model.

The second hypothesis compares the upside and downside coskewness models with the semi-coskewness model. The second hypothesis is formulated as follows:

$$H_0^{ACX}: \lambda^{NN} = \lambda^{PN} \cap \lambda^{PP} = \lambda^{NP} \tag{10}$$

the second hypothesis is derived from the equations that $CskP \approx CskPP + CskNP$ and $CskN \approx CskNN + CskPN$. Assuming that H_0^{ACX} holds, the semi-coskewness model carries the same information as the upside and downside coskewness models, and can be reduced to the upside and downside coskewness models, and there is no need for the more complicated semi-coskewness model.

[Table 3 inserted here]

Table 3 reports the Chi-square test results for the two hypotheses mentioned above. We can reject the null hypothesis H_0^{Csk} at the 1% level with 3 degrees of freedom, and to a lesser extent, reject H_0^{ACX} at the 5% level with 2 degrees of freedom. The rejection of both hypotheses indicates that although the correlations between the aforementioned coskewness factors are high, the information conveyed by them is different, and the semi-coskewness model captures the additional information.

5.4 Robustness tests

To confirm that the baseline results are not merely one occurrence in a specific sample, we provide a set of comprehensive robustness tests in this section. The robustness tests include dividing the full sample into subsamples based on different market values and time periods.

[Table 4 inserted here]

Table 4 shows the subsample analysis based on monthly market value cut-off points. Following the SMB factor constructed by Fama and French (1993), we include the top 30% of stocks sorted by market value each month as large-cap stocks, which are investable stocks with high liquidity. Meanwhile, the other subsample includes the large cap (top 30%) and mid cap (mid 40%) stocks, thus excluding the small cap (bottom 30%) stocks that may have low liquidity concern.

Columns (1) to (6) of Table 4 report the Fama–MacBeth regression results for the top 30% (4,635 firms with 438,716 monthly observations) and 70% (8,740 firms with 880,600 monthly observations) subsamples for the downside and upside coskewness models, while columns (7) to (12) report the results for the semi-coskewness model. Both the downside and upside coskewness factors report similar regression results to the baseline results. For the semi-coskewness model, CskNP also reports insignificant results similar to the baseline. As a result, the subsample analysis constructed on the basis of market value does not alter the baseline results.

Instead of dividing the stocks into three broad tiers of large-mid-small-cap, we also look at the top ranking stocks on a monthly basis, based on their market capitalisation. These stocks tend to be associated with the best-known companies and are attractive to large institutional investors. They are also valuable to fund managers as these stocks meet investment covenant requirements and can be included in portfolios.

[Table 5 inserted here]

Table 5 shows the regression results for the top 500 (1,606 firms with 162,838 monthly observations) and top 1,000 (3,337 firms with 322,274 monthly observations) stocks, which have been ranked according to their individual market capitalisation on the US stock market on a monthly basis. The ranking is based on the sample in our study. Therefore, it may differ from the ranking in the real world. However, our choice solves the problem that certain stocks may not be available in our sample and it can provide more consistent monthly ranking measures. The layout of Table 5 is identical to that of Table 4, but we observe different results compared to those found in Table 4 and the baseline results. For the downside and upside coskewness models in columns (1) to (6), the coefficients of CskN are only significant at the 10% level in columns (3) and (6), while the coefficients of CskP are intact. BM, MOM and RV are still significant compared to the baseline results, while IVOL and ILLIQ are not. A possible explanation for the change in ILLIQ coefficients may be that top-ranking firms naturally have sufficient liquidity.

The regression results for the semi-coskewness model also differ from the baseline result. CskNN does not show consistent significant coefficients. In comparison, CskPP and CskPN retain their predictive power. Given that CskN reports significant levels of 10% in columns (3) and (6) and the high correlation between CskN and CskNN, it is not surprising that CskNN also reports insignificant coefficients in both the top 500 and top 1,000 ranking subsamples. However, the regression results in Table 4 for the top

30% do not show similar results, suggesting that these large companies may have different characteristics from the rest of the companies.

In addition to dividing the sample by market capitalisation, we also divide the sample by different time periods to ensure that the baseline results do not suffer from the time-specific problem.

[Table 6 inserted here]

Table 6 presents the Fama–MacBeth regression results for two types of subsample division by time period. Columns (1) to (8) of Table 6 divide the sample into two subsample periods with cut-off point in year 2000, each period about 20 years long. This allows us to see how different business cycles would affect the predictive power of the three coskewness models. Equally importantly, in columns (9) to (12), we divide the sample into pre-Covid and during-Covid periods and set February 2020 as the Covid breakthrough point. The Covid pandemic arguably had the most catastrophic impact globally, and the aftermath still affects us in many ways.

The downside coskewness model delivers consistent results compared to the baseline results, but the upside coskewness model fails in the 1980-1999 period. The coefficients of the control variables differ from the baseline results, with BM and ILLIQ showing relatively stable predictive power over two periods. For the semi-coskewness model, CskPP and CskNN show relatively consistent results compared to the other two. It is interesting to note that CskPP still shows consistent coefficients, although CskP does not. The significant control variables between columns (5) to (8) are BM and ILLIQ, which is similar to columns (1) to (4). The coefficients of MOM report a significant level of 10% after 2000, which can explain the momentum crash that happened during the 2008 global financial crisis.

For the pre-covid and during-covid periods reported in columns (9) to (12), the pre-covid results are identical to those of the baseline. Conversely, the results for the during-covid period are different, primarily because the during-covid subsample contains only a small fraction of observations, given that we use a 12-month overlapping estimation of factors.

[Table 7 inserted here]

We then test the predictive power of three coskewness models during economic recessions, as shown in Table 7. During a recession, the stock market typically experiences a flight-to-liquidity phenomenon, which causes the stock price to be depressed relative to normal market conditions. Examining relevant coskewness models in such distressed situations would greatly enhance the understanding of such models. Following the National Bureau of Economic Research's (NBER) definition of recessions, we include all the recessions that occurred in the US over the entire sample period, which gives us a different perspective in testing the predictive power.

The upside coskewness factor delivers persistently positive coefficients, except in column (3) at the 5% significance level, but remarkably outperforms the downside coskewness factor. Columns (7) to (12) of Table 7 further explain that the predictive power of the upside coskewness model is rooted in the CskPP and not in the CskNP. In contrast, the downside coskewness factor performs well only in the non-recession subsample (columns (4) to (6)) and can be explained by CskPN and CskNN. The only significant control variable is BM, which is reported in all columns. The results for the non-recession period are similar to the baseline results.

Compared to the results in Tables 4 and 5, which focus on the market capitalisation split, the regression results based on different periods in Tables 6 and 7 are more unstable, suggesting that the conditional coskewness may have a time-varying nature. However, we observe a persistent predictive power for CskPP.

The baseline results focus on the use of coskewness measures in month t to predict cross-sectional returns in month $t+1$. In addition, the long-term predictive power is also crucial, as it provides insights into long-term returns.

[Table 8 inserted here]

Panels A and B of Table 8 present the predictive regression results for the future 3, 6, 9 and 12 months based on the downside and upside coskewness and semi-coskewness models, respectively. CskN and CskP produce consistent results in all columns of Panel A of Table 8, similar to the baseline result. In comparison, the semi-coskewness model also reports robust results similar to those of the baseline. Furthermore, we observe a slight increase in the average R^2 as the forecast period extends into the future. This finding suggests that all the coskewness models possess stable long-term predictability.

6 Conclusion

The present study investigates the predictive power of the proposed semi-coskewness model based on NYSE and NASDAQ constituent stocks in the US market. Our semi-coskewness model is inspired by Bollerslev, Patton, and Quaedvlieg's (2022) seminal realised semibeta model. The semi-coskewness model decomposes the information conveyed by coskewness into four components, based on the signs of individual stock and market returns.

The regression results and extensive robustness tests demonstrate that CskPP persistently and positively predicts future cross-sectional stock returns, and conventional asset pricing studies risk factors do not subsume the predictive power. To a lesser extent, CskPN predicts future stock returns positively and CskNN predicts future returns negatively. Conversely, CskNP did not pass the robustness tests.

In addition to the semi-coskewness model, the present study also investigates the downside and upside coskewness models. The downside and upside coskewness models also show generally persistent results, with a few exceptions in the subsample analysis. However, hypothesis testing strongly rejects that the information conveyed by the semi-coskewness model is similar to that conveyed by the downside and upside coskewness models.

Our study provides a unique angle to understand the risk–return relationship in the US stock market through a semi-coskewness model. This understanding can be extended to the international stock markets and other asset classes. More importantly, the empirical evidence indicates that investors exhibit an asymmetrical response to coskewness conditional on market movement directions, which is consistent with the behavioural finance theories. Future studies may explore the underlying reasons for this discrepancy.

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Tables and Figures:

Table 1 Summary statistics and correlations

Panel A reports the monthly summary statistics including individual stock returns and main independent variables. All the independent variables are calculated based on the daily total return index from LSEG DataStream from 1/1/1980 to 30/4/2024. We derive independent variables from overlapping 12-month periods using daily frequency data. In addition, all the figures shown in the table are cross-sectional averaged and then time series averaged. The average number of monthly observations for all variables is 1,903.

Panel A: Summary statistics

Variable	Mean	Std Dev	Median	Min	Max	Skewness	Kurtosis
r_i	0.002	0.114	0.005	-0.963	0.699	-0.674	12.758
Beta	0.840	0.464	0.791	-0.429	2.881	0.595	0.733
Csk	-0.164	0.159	-0.162	-0.577	0.255	0.000	-0.364
CskP	0.835	0.460	0.850	-0.382	1.916	-0.061	-0.410
CskN	-1.178	0.522	-1.202	-2.342	0.272	0.196	-0.380
CskPP	0.366	0.122	0.364	0.081	0.676	0.118	-0.403
CskPN	0.072	0.043	0.062	0.012	0.277	1.444	3.713
CskNP	-0.083	0.046	-0.073	-0.269	-0.014	-1.063	1.367
CskNN	-0.517	0.175	-0.515	-0.949	-0.102	-0.009	-0.429

Panel B: Correlations

Panel B reports correlations of monthly individual stock returns with the main independent variables. Moreover, all the figures shown in the table are cross-sectional averaged and then time series averaged.

	r_i	Beta	Csk	CskP	CskN	CskPP	CskPN	CskNP	CskNN
r_i	1	-0.025	-0.009	0.014	-0.016	0.017	-0.008	0.009	-0.016
Beta		1	-0.090	0.571	-0.589	0.515	-0.537	0.527	-0.544
Csk			1	0.177	0.415	0.214	0.247	0.128	0.454
CskP				1	-0.713	0.946	-0.560	0.793	-0.710
CskN					1	-0.704	0.744	-0.569	0.944
CskPP						1	-0.534	0.599	-0.664
CskPN							1	-0.501	0.559
CskNP								1	-0.555
CskNN									1

Figure 1

The line chart illustrates the number of firms in the screened US stock market (NYSE and NASDAQ) in each month. The sample starts from January 1980 to April 2024, with 9,728 firms and 1,012,017 monthly observations. March 1980 has the smallest number of observations with 194 firms, while March 2022 has the largest with 3,184 firms.

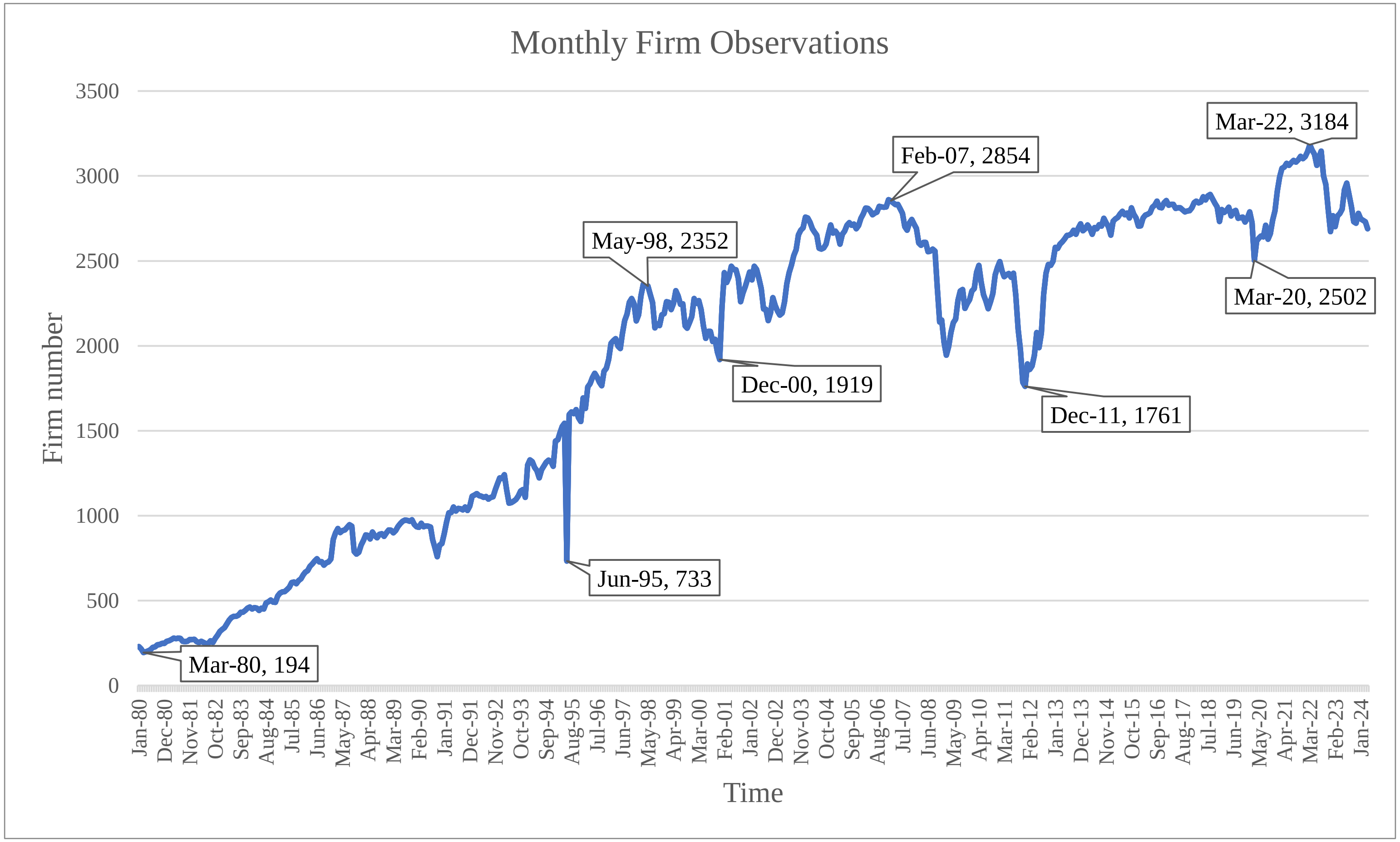

Table 2 Baseline Fama–MacBeth multivariate OLS regression

This table reports the results of the Fama–MacBeth multivariate OLS regression. The dependent variables are the monthly log returns of individual stock, calculated from the daily total return index at month $t+1$. The independent variables are measured monthly and winsorised at 1% and 99% at month *t*. All the data are taken from the LSEG DataStream databases for the sample period from 1/1/1980 to 31/4/2024. The sample contains 1,012,017 monthly observations and 9,728 stocks. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Beta	-0.0067***	-0.0079***	-0.0162***	-0.0156***	-0.0074***	-0.0165***	-0.0159***	-0.0075***
	(-2.81)	(-3.31)	(-4.51)	(-4.63)	(-2.59)	(-4.53)	(-4.66)	(-2.60)
Csk		-0.0072*						
		(-1.71)						
CskN			-0.0085***	-0.0080***	-0.0051***			
			(-5.90)	(-6.90)	(-5.11)			
CskP			0.0087***	0.0084***	0.0047***			
			(4.26)	(4.45)	(2.99)			
CskPP						0.0293***	0.0281***	0.0164***
						(5.22)	(5.52)	(3.91)
CskPN						-0.0384***	-0.0386***	-0.0268***
						(-4.95)	(-4.96)	(-3.65)
CskNP						0.0268***	0.0214**	0.0115
						(2.73)	(2.39)	(1.40)
CskNN						-0.0183***	-0.0175***	-0.0106***
						(-3.91)	(-4.70)	(-3.24)
SIZE				0.0005	0.0014**		0.0006	0.0014**
				(1.43)	(2.39)		(1.47)	(2.52)
BM				0.0079***	0.0072***		0.0079***	0.0073***
				(8.12)	(8.09)		(8.20)	(8.15)
MOM				0.0070***	0.0065***		0.0070***	0.0065***
				(5.40)	(5.15)		(5.41)	(5.15)
REV					-0.0024			-0.0026
					(-0.84)			(-0.87)
RV					-0.1221***			-0.1196***
					(-2.80)			(-2.76)
IVOL					-0.2155***			-0.2128***
					(-3.76)			(-3.69)
ILLIQ					0.0011***			0.0012***
					(3.25)			(3.36)
Alpha	0.0076***	0.0061***	0.0005	-0.0091**	-0.0003	0.0032	-0.0069*	0.0010
	(4.06)	(3.10)	(0.18)	(-2.42)	(-0.10)	(1.07)	(-1.74)	(0.31)
Avg. R^2	3.12%	3.49%	4.83%	6.48%	7.65%	4.98%	6.62%	7.76%

Table 3 Hypothesis testing

This table reports the results of hypothesis testing. The Chi-square statistics, p-value, and degrees of freedom are reported in the table.

Hypothesis	statistic	p-value	DF
$H_0^{Csk}: \lambda^{NN} = \lambda^{PN} = \lambda^{PP} = \lambda^{NP}$	41.92	0.00	3
$H_0^{ACX}: \lambda^{NN} = \lambda^{PN} \cap \lambda^{PP} = \lambda^{NP}$	7.34	0.03	2

Table 4 Fama–MacBeth OLS regression with market capitalisation restrictions

This table reports the results of the multivariate Fama–MacBeth OLS regression based on the different market capitalisation cut-off points. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	Top 30%			Top 70%			Top 30%			Top 70%		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0127***	-0.0128***	-0.0076**	-0.0146***	-0.0141***	-0.0075***	-0.0126***	-0.0128***	-0.0077***	-0.0147***	-0.0143***	-0.0076***
	(-3.78)	(-3.99)	(-2.57)	(-4.37)	(-4.45)	(-2.73)	(-3.75)	(-3.95)	(-2.58)	(-4.39)	(-4.50)	(-2.72)
CskN	-0.0059***	-0.0056***	-0.0039***	-0.0072***	-0.0071***	-0.0049***						
	(-5.17)	(-5.04)	(-3.80)	(-6.31)	(-6.54)	(-5.02)						
CskP	0.0071***	0.0071***	0.0053***	0.0075***	0.0073***	0.0047***						
	(3.97)	(4.12)	(3.40)	(4.00)	(4.17)	(3.06)						
CskPP							0.0242***	0.0248***	0.0189***	0.0255***	0.0250***	0.0166***
							(4.79)	(5.08)	(4.29)	(4.84)	(5.11)	(3.98)
CskPN							-0.0279***	-0.0292***	-0.0231***	-0.0349***	-0.0347***	-0.0244***
							(-3.73)	(-3.79)	(-2.96)	(-4.73)	(-4.51)	(-3.33)
CskNP							0.0203*	0.0149	0.0112	0.0223**	0.0186**	0.0109
							(1.89)	(1.43)	(1.15)	(2.38)	(2.13)	(1.35)
CskNN							-0.0126***	-0.0120***	-0.0070**	-0.0153***	-0.0158***	-0.0105***
							(-3.21)	(-3.24)	(-2.02)	(-3.85)	(-4.38)	(-3.25)
SIZE		0.0000	0.0000		0.0003	0.0007		0.0001	0.0001		0.0003	0.0008
		(0.05)	(0.03)		(0.86)	(1.48)		(0.17)	(0.10)		(0.87)	(1.58)
BM		0.0086***	0.0083***		0.0101***	0.0095***		0.0086***	0.0083***		0.0101***	0.0096***
		(5.84)	(5.92)		(8.80)	(8.85)		(5.88)	(5.98)		(8.90)	(8.95)
MOM		0.0063***	0.0064***		0.0062***	0.0059***		0.0063***	0.0063***		0.0062***	0.0059***
		(3.90)	(3.92)		(4.57)	(4.44)		(3.92)	(3.90)		(4.62)	(4.47)
REV			0.0058			-0.0008			0.0060			-0.0010
			(1.36)			(-0.27)			(1.45)			(-0.31)
RV			-0.1299*			-0.0888*			-0.1265*			-0.0855*
			(-1.92)			(-1.89)			(-1.89)			(-1.84)
IVOL			-0.0970			-0.1808***			-0.0850			-0.1781***
			(-1.17)			(-3.22)			(-1.03)			(-3.15)
ILLIQ			0.0002			0.0007**			0.0002			0.0007**
			(0.47)			(2.09)			(0.40)			(2.19)
Alpha	0.0029	-0.0037	-0.0004	0.0018	-0.0076**	-0.0013	0.0046*	-0.0030	0.0002	0.0041	-0.0060	-0.0005
	(1.42)	(-1.08)	(-0.13)	(0.79)	(-2.37)	(-0.45)	(1.73)	(-0.78)	(0.04)	(1.44)	(-1.59)	(-0.14)
Avg. R^2	6.82%	9.44%	10.89%	5.05%	6.98%	8.06%	7.13%	9.70%	11.11%	5.23%	7.13%	8.18%
Obs.		438,716			880,600			438,716			880,600	
Stocks		4,635			8,740			4,635			8,740	

Table 5 Fama–MacBeth OLS regression with stock ranking

This table reports the results of the multivariate Fama–MacBeth OLS regression based on the stock ranking. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	Top 500			Top 1000			Top 500			Top 1000		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0087***	-0.0088***	-0.0051*	-0.0115***	-0.0088***	-0.0051*	-0.0086***	-0.0088***	-0.0053*	-0.0113***	-0.0117***	-0.0067**
	(-2.64)	(-2.76)	(-1.69)	(-3.16)	(-2.76)	(-1.69)	(-2.63)	(-2.78)	(-1.76)	(-3.12)	(-3.34)	(-2.13)
CskN	-0.0028**	-0.0029***	-0.0020*	-0.0052***	-0.0029***	-0.0020*						
	(-2.54)	(-2.73)	(-1.89)	(-4.32)	(-2.73)	(-1.89)						
CskP	0.0068***	0.0072***	0.0057***	0.0070***	0.0072***	0.0057***						
	(4.28)	(4.40)	(4.19)	(4.33)	(4.40)	(4.19)						
CskPP							0.0247***	0.0260***	0.0216***	0.0248***	0.0258***	0.0195***
							(4.94)	(5.10)	(4.50)	(5.41)	(5.63)	(4.63)
CskPN							-0.0269***	-0.0318***	-0.0296***	-0.0301***	-0.0338***	-0.0295***
							(-2.70)	(-3.27)	(-2.96)	(-3.88)	(-4.36)	(-3.67)
CskNP							0.0108	0.0077	0.0054	0.0167	0.0126	0.0108
							(0.82)	(0.65)	(0.50)	(1.56)	(1.19)	(1.14)
CskNN							-0.0026	-0.0022	0.0004	-0.0098**	-0.0096**	-0.0041
							(-0.60)	(-0.54)	(0.11)	(-2.47)	(-2.49)	(-1.12)
SIZE		-0.0004	-0.0005		-0.0004	-0.0005		-0.0004	-0.0005		-0.0000	-0.0003
		(-0.96)	(-0.69)		(-0.96)	(-0.69)		(-0.92)	(-0.65)		(-0.06)	(-0.43)
BM		0.0063***	0.0066***		0.0063***	0.0066***		0.0063***	0.0066***		0.0081***	0.0082***
		(3.70)	(3.90)		(3.70)	(3.90)		(3.73)	(3.99)		(5.07)	(5.29)
MOM		0.0067***	0.0063***		0.0067***	0.0063***		0.0068***	0.0063***		0.0075***	0.0075***
		(3.81)	(3.45)		(3.81)	(3.45)		(3.92)	(3.52)		(4.88)	(4.76)
REV			0.0025			0.0025			0.0030***			0.0042
			(0.50)			(0.50)			(3.52)			(1.04)
RV			-0.2893**			-0.2893**			-0.2759**			-0.2254***
			(-2.18)			(-2.18)			(-2.08)			(-2.61)
IVOL			0.0919			0.0919			0.0866			-0.0201
			(0.78)			(0.78)			(0.72)			(-0.23)
ILLIQ			0.0000			0.0000			-0.0000			-0.0001
			(0.00)			(0.00)			(-0.02)			(-0.14)
Alpha	0.0028	0.0004	-0.0001	0.0026	0.0004	-0.0001	0.0047	0.0020	0.0017	0.0043	-0.0027	0.0003
	(1.39)	(0.09)	(-0.03)	(1.25)	(0.09)	(-0.03)	(1.49)	(0.46)	(0.37)	(1.47)	(-0.64)	(0.08)
Avg. R^2	8.33%	11.83%	13.95%	7.34%	11.83%	13.95%	8.89%	12.30%	14.34%	7.69%	10.55%	12.19%
Obs.		162,838			322,274			162,838			322,274	
Stocks		1,606			3,337			1,606			3,337	

Table 6 Fama–MacBeth OLS regression with time periods

This table reports the results of the multivariate Fama–MacBeth OLS regression based on the different time periods. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	CskN + CskP				**Semi-Coskewness**				**CskN + CskP**		**Semi-Coskewness**	
	1980-1999		2000-2024		1980-1999		2000-2024		Pre-Covid	During-Covid	Pre-Covid	During-Covid
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0153***	-0.0049	-0.0170***	-0.0093**	-0.0150***	-0.0044	-0.0177***	-0.0100**	-0.0084***	0.0024	-0.0085***	0.0023
	(-2.96)	(-1.40)	(-3.29)	(-2.14)	(-2.87)	(-1.26)	(-3.39)	(-2.25)	(-2.85)	(0.26)	(-2.84)	(0.26)
CskN	-0.0068***	-0.0044***	-0.0098***	-0.0056***					-0.0049***	-0.0065*		
	(-3.31)	(-2.85)	(-4.97)	(-4.38)					(-4.79)	(-1.69)		
CskP	0.0065**	0.0041*	0.0105***	0.0053**					0.0048***	0.0040		
	(2.12)	(1.83)	(3.86)	(2.37)					(2.84)	(0.96)		
CskPP					0.0269***	0.0172***	0.0313***	0.0157***			0.0165***	0.0153
					(3.41)	(2.99)	(4.01)	(2.64)			(3.72)	(1.12)
CskPN					-0.0166	-0.0036	-0.0564***	-0.0458***			-0.0262***	-0.0318***
					(-1.35)	(-0.32)	(-6.29)	(-5.47)			(-3.24)	(-3.21)
CskNP					0.0102	0.0029	0.0405***	0.0233**			0.0130	-0.0030
					(0.73)	(0.27)	(3.00)	(1.99)			(1.49)	(-0.15)
CskNN					-0.0170**	-0.0140***	-0.0194***	-0.0077**			-0.0097***	-0.0186
					(-2.52)	(-2.66)	(-3.00)	(-2.00)			(-2.94)	(-1.47)
SIZE		0.0006		0.0020**		0.0006		0.0021***	0.0009*	0.0055**	0.0010*	0.0055**
		(0.73)		(2.56)		(0.84)		(2.62)	(1.74)	(2.04)	(1.91)	(1.99)
BM		0.0091***		0.0057***		0.0092***		0.0057***	0.0069***	0.0104***	0.0069***	0.0103***
		(5.98)		(5.97)		(6.06)		(6.01)	(7.55)	(3.41)	(7.61)	(3.39)
MOM		0.0115***		0.0025*		0.0114***		0.0025*	0.0064***	0.0080***	0.0064***	0.0079***
		(6.69)		(1.76)		(6.61)		(1.79)	(4.68)	(2.63)	(4.68)	(2.66)
REV		0.0008		-0.0051		0.0004		-0.0050	-0.0035	0.0074	-0.0036	0.0075
		(0.16)		(-1.53)		(0.08)		(-1.51)	(-1.10)	(1.49)	(-1.14)	(1.46)
RV		-0.2561***		-0.0119		-0.2506***		-0.0120	-0.1368***	0.0168	-0.1343***	0.0189
		(-2.94)		(-0.71)		(-2.88)		(-0.71)	(-2.85)	(1.11)	(-2.82)	(1.29)
IVOL		-0.0754		-0.3306***		-0.0726		-0.3281***	-0.1675***	-0.6683***	-0.1636***	-0.6773***
		(-0.80)		(-5.48)		(-0.77)		(-5.37)	(-2.94)	(-4.51)	(-2.86)	(-4.52)
ILLIQ		0.0011**		0.0012**		0.0011**		0.0012**	0.0010***	0.0023	0.0011***	0.0023
		(2.31)		(2.35)		(2.37)		(2.45)	(3.02)	(1.28)	(3.15)	(1.27)
Alpha	0.0059**	0.0035	-0.0040	-0.0034	0.0034	-0.0015	0.0030	0.0031	0.0022	-0.0239*	0.0036	-0.0237*
	(1.99)	(0.81)	(-1.10)	(-0.78)	(0.81)	(-0.31)	(0.72)	(0.70)	(0.75)	(-1.94)	(1.14)	(-1.91)
Avg. R^2	4.54%	7.79%	5.06%	7.55%	4.61%	7.83%	5.28%	7.71%	7.47%	9.42%	7.57%	9.59%
Obs.	251,301		760,716		251,301		760,716		864,848	147,169	864,848	147,169
Stocks	4,224		8,405		4,224		8,405		8,358	4,670	8,358	4,670

Table 7 Fama–MacBeth OLS regression with recessions

This table reports the results of the multivariate Fama–MacBeth OLS regression based on the recession and non-recession periods. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	Recessions			**Non-Recessions**			**Recessions**			**Non-Recessions**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0175*	-0.0192*	-0.0102	-0.0166***	-0.0159***	-0.0076***	-0.0185*	-0.0204*	-0.0111	-0.0169***	-0.0161***	-0.0077***
	(-1.67)	(-1.80)	(-1.01)	(-4.58)	(-4.63)	(-2.65)	(-1.72)	(-1.87)	(-1.07)	(-4.61)	(-4.67)	(-2.66)
CskN	-0.0042	-0.0075**	-0.0023	-0.0087***	-0.0081***	-0.0051***						
	(-1.27)	(-2.37)	(-0.77)	(-6.03)	(-6.90)	(-5.13)						
CskP	0.0108***	0.0118***	0.0074**	0.0088***	0.0083***	0.0048***						
	(2.80)	(3.22)	(2.29)	(4.23)	(4.42)	(3.30)						
CskPP							0.0401***	0.0416***	0.0276**	0.0291***	0.0276***	0.0161***
							(2.71)	(3.13)	(2.24)	(5.13)	(5.43)	(3.86)
CskPN							-0.0565**	-0.0677**	-0.0456*	-0.0385***	-0.0386***	-0.0269***
							(-2.08)	(-2.33)	(-1.77)	(-4.98)	(-4.89)	(-3.59)
CskNP							0.0367*	0.0360**	0.0249	0.0291***	0.0234***	0.0132
							(1.71)	(2.05)	(1.49)	(3.00)	(2.62)	(1.62)
CskNN							0.0016	-0.0084	0.0044	-0.0188***	-0.0176***	-0.0106***
							(0.15)	(-0.60)	(0.51)	(-3.97)	(-4.65)	(-3.21)
SIZE		-0.0008	-0.0036*		0.0006*	0.0015***		-0.0008	-0.0034*		0.0006*	-0.0016***
		(-0.64)	(-1.92)		(1.65)	(2.63)		(-0.60)	(-1.77)		(1.70)	(2.76)
BM		0.0043***	0.0035**		0.0080***	0.0073***		0.0044***	0.0036**		0.0080***	0.0073***
		(3.30)	(2.40)		(8.16)	(8.16)		(3.31)	(2.43)		(8.23)	(8.21)
MOM		0.0028	0.0012		0.0070***	0.0066***		0.0025	0.0010		0.0070***	0.0066***
		(0.44)	(0.20)		(5.32)	(5.12)		(0.40)	(0.16)		(5.32)	(5.11)
REV			0.0068			-0.0027			0.0068			-0.0029
			(0.40)			(-0.93)			(0.39)			(-0.97)
RV			-0.4531*			-0.1005***			-0.4436*			-0.0983***
			(-1.82)			(-2.75)			(-1.80)			(-2.69)
IVOL			0.0264			-0.2321***			0.0327			-0.2302***
			(0.12)			(-4.18)			(0.15)			(-4.14)
ILLIQ			-0.0020*			0.0012***			-0.0018**			0.0012***
			(-1.92)			(3.33)			(-1.83)			(3.43)
Alpha	-0.0043	-0.0068	0.0062	0.0003	-0.0095**	-0.0007	0.0041	0.0010	0.0125	0.0034	-0.0071*	0.0009
	(-0.47)	(-0.55)	(0.53)	(0.13)	(-2.54)	(-0.23)	(0.65)	(0.10)	(1.31)	(1.14)	(-1.76)	(0.28)
Avg. R^2	6.72%	9.03%	10.65%	4.75%	6.36%	7.53%	7.00%	9.22%	10.75%	4.90%	6.40%	7.64%
Obs.		88,183			923,834			88,183			923,834	
Stocks		6,180			9,727			6,180			9,727	

Table 8 Fama–MacBeth OLS regression with long-term horizons

This table reports the results of the multivariate Fama–MacBeth OLS regression based on long-term forecasts. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

Panel A

	3 months			6 months			9 months			12 months		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0465***	-0.0448***	-0.0206***	-0.0895***	-0.0839***	-0.0395***	-0.1313***	-0.1207***	-0.0581***	-0.1710***	-0.1563***	-0.0783***
	(-4.69)	(-4.84)	(-2.69)	(-5.18)	(-5.23)	(-2.92)	(-5.54)	(-5.54)	(-3.18)	(-6.15)	(-6.11)	(-3.62)
CskN	-0.0210***	-0.0197***	-0.0114***	-0.0402***	-0.0351***	-0.0213***	-0.0605***	-0.0511***	-0.0313***	-0.0785***	-0.0669***	-0.0425***
	(-5.55)	(-6.50)	(-4.53)	(-6.28)	(-6.39)	(-4.57)	(-7.06)	(-6.85)	(-4.99)	(-7.72)	(-7.64)	(-5.76)
CskP	0.0288***	0.0268***	0.0161***	0.0582***	0.0512***	0.0319***	0.0857***	0.0736***	0.0458***	0.1148***	0.0988***	0.0629***
	(4.85)	(5.05)	(3.80)	(5.87)	(5.48)	(4.17)	(6.24)	(5.84)	(4.53)	(6.84)	(6.41)	(5.15)
SIZE		0.0019*	0.0067***		0.0048***	0.0156***		0.0075***	0.0242***		0.0092***	0.0304***
		(1.76)	(4.25)		(2.63)	(5.65)		(3.13)	(6.13)		(3.15)	(5.83)
BM		0.0212***	0.0198***		0.0386***	0.0361***		0.0504***	0.0472***		0.0584***	0.0544***
		(7.57)	(7.54)		(7.31)	(7.32)		(6.82)	(6.85)		(6.48)	(6.47)
MOM		0.0179***	0.0174***		0.0284***	0.0275***		0.0310***	0.0307***		0.0300***	0.0301***
		(4.97)	(4.88)		(4.39)	(4.32)		(3.75)	(3.77)		(3.29)	(3.38)
REV			0.0160***			0.0364***			0.0618***			0.0742***
			(2.90)			(4.30)			(5.51)			(5.68)
RV			-0.1754**			-0.2242*			-0.1428			-0.0281
			(-2.01)			(-1.71)			(-0.94)			(-0.14)
IVOL			-0.7614***			-1.3919***			-2.1053***			-2.8109***
			(-5.38)			(-5.37)			(-6.40)			(-6.94)
ILLIQ			0.0052***			0.0111***			0.0168***			0.0212***
			(5.71)			(6.45)			(6.64)			(6.44)
Alpha	0.0021	-0.0257**	-0.0040	0.0016	-0.0540***	-0.0183	-0.0005	-0.0779***	-0.0285	-0.0037	-0.0955***	-0.0315
	(0.29)	(-2.37)	(-0.43)	(0.12)	(-2.74)	(-1.11)	(-0.02)	(-2.84)	(-1.22)	(-0.15)	(-2.73)	(-1.02)
Avg. R^2	5.61%	7.75%	9.15%	6.15%	8.59%	10.13%	6.80%	9.25%	10.90%	7.31%	9.66%	11.40%
Obs.		438,716			880,600			438,716			880,600	
Stocks		4,635			8,740			4,635			8,740	

Panel B

	3 months			6 months			9 months			12 months		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0470***	-0.0453***	-0.0211***	-0.0899***	-0.0844***	-0.0401***	-0.1320***	-0.1212***	-0.0586***	-0.1722***	-0.1570***	-0.0788***
	(-4.71)	(-4.86)	(-2.72)	(-5.18)	(-5.23)	(-2.92)	(-5.55)	(-5.53)	(-3.17)	(-6.20)	(-6.13)	(-3.62)
CskPP	0.0938***	0.0871***	0.0521***	0.1887***	0.1672***	0.1039***	0.2764***	0.2409***	0.1494***	0.3649***	0.3201***	0.2021***
	(5.63)	(5.87)	(4.65)	(6.54)	(6.22)	(5.06)	(6.65)	(6.39)	(5.38)	(6.91)	(6.62)	(5.71)
CskPN	-0.0937***	-0.0933***	-0.0626***	-0.1688***	-0.1571***	-0.1118***	-0.2598***	-0.2301***	-0.1642***	-0.3414***	-0.2955***	-0.2171***
	(-4.51)	(-4.53)	(-3.19)	(-4.91)	(-4.42)	(-3.31)	(-5.73)	(-4.84)	(-3.65)	(-6.62)	(-5.44)	(-4.31)
CskNP	0.0981***	0.0825***	0.0552**	0.1956***	0.1597***	0.1130***	0.02828***	0.2284***	0.1603***	0.3707***	0.3034***	0.2155***
	(3.48)	(3.19)	(2.38)	(3.93)	(3.54)	(2.78)	(4.09)	(3.66)	(2.89)	(4.49)	(4.10)	(3.30)
CskNN	-0.0406***	-0.0380***	-0.0194**	-0.0790***	-0.0669***	-0.0336**	-0.1233***	-0.1000***	-0.0520***	-0.1637***	-0.1350***	-0.0749***
	(-3.51)	(-4.24)	(-2.48)	(-4.06)	(-4.18)	(-2.32)	(-4.81)	(-4.76)	(-2.72)	(-5.48)	(-5.58)	(-3.31)
SIZE		0.0019*	0.0069***		0.0049***	0.0158***		0.0077***	0.0246***		0.0095***	0.0309***
		(1.82)	(4.37)		(2.72)	(5.75)		(3.24)	(6.27)		(3.25)	(5.97)
BM		0.0212***	0.0199***		0.0386***	0.0362***		0.0504***	0.0474***		0.0584***	0.0546***
		(7.68)	(7.63)		(7.42)	(7.40)		(6.94)	(6.94)		(6.61)	(6.57)
MOM		0.0179***	0.0173***		0.0284***	0.0275***		0.0310***	0.0307***		0.0298***	0.0301***
		(5.01)	(4.90)		(4.43)	(4.36)		(3.79)	(3.81)		(3.31)	(3.40)
REV			0.0156***			0.0358***			0.0612***			0.0731***
			(2.81)			(4.22)			(5.47)			(5.65)
RV			-0.1703*			-0.2168*			-0.1317			-0.0183
			(-1.93)			(-1.67)			(-0.89)			(-0.09)
IVOL			-0.7536***			-1.3860***			-2.1043***			-2.8086***
			(-5.27)			(-5.37)			(-6.45)			(-7.00)
ILLIQ			0.0052***			0.0111***			0.0169***			0.0213***
			(5.83)			(6.53)			(6.80)			(6.61)
Alpha	0.0108	-0.0183	0.0019	0.0169	-0.0415*	-0.0065	0.0220	-0.0603*	-0.0119	0.0250	-0.0737*	-0.0101
	(1.30)	(-1.58)	(0.19)	(1.03)	(-1.92)	(-0.35)	(0.89)	(-1.91)	(-0.44)	(0.78)	(-1.83)	(-0.28)
Avg. R^2	5.80%	7.92%	9.28%	6.36%	8.77%	10.27%	7.04%	9.44%	11.06%	7.53%	9.84%	11.53%
Obs.		438,716									438,716	
Stocks		4,635									4,635	

Online Appendix:

Table A1 Fama–MacBeth OLS regression with single semi-coskewness

This table reports the results of the multivariate Fama–MacBeth OLS regression. We use the Newey–West *t*-statistic with 10-lags and report the *t*-statistic value in the parentheses below the coefficient. *, **, *** represent significance level at 10%, 5%, and 1%, respectively.

	CskPP			CskPN			CskNP			CskNN		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta	-0.0127***	-0.0118***	-0.0048*	-0.0110***	-0.0104***	-0.0037*	-0.0111***	-0.0100***	-0.0033	-0.0124***	-0.0114***	-0.0043**
	(-4.03)	(-3.99)	(-1.94)	(-3.83)	(-3.87)	(-1.70)	(-3.72)	(-3.59)	(-1.42)	(-4.14)	(-4.19)	(-1.99)
CskPP	0.0507***	0.0394***	0.0219***									
	(5.71)	(5.78)	(4.29)									
CskPN				-0.0904***	-0.0620***	-0.0347***						
				(-5.67)	(-5.53)	(-3.82)						
CskNP							0.0923***	0.0559***	0.0268***			
							(5.04)	(4.26)	(2.61)			
CskNN										-0.0357***	-0.0280***	-0.0149***
										(-4.73)	(-5.54)	(-3.89)
SIZE		0.0013***	0.0017***		0.0020***	0.0021***		0.0020***	0.0022***		0.0014***	0.0019***
		(3.18)	(2.89)		(4.27)	(3.23)		(4.62)	(3.43)		(3.42)	(3.01)
BM		0.0081***	0.0073***		0.0085***	0.0075***		0.0084***	0.0074***		0.0083***	0.0074***
		(8.13)	(8.10)		(8.15)	(8.13)		(8.10)	(8.10)		(8.11)	(8.12)
MOM		0.0069***	0.0064***		0.0066***	0.0063***		0.0065***	0.0062***		0.0066***	0.0063***
		(5.29)	(5.06)		(5.14)	(5.03)		(5.11)	(4.98)		(5.10)	(5.00)
REV			-0.0028			-0.0031			-0.0035			-0.0034
			(-0.95)			(-1.07)			(-1.23)			(-1.20)
RV			-0.1098**			-0.1106***			-0.1112***			-0.1111**
			(-2.56)			(-2.48)			(-2.58)			(-2.52)
IVOL			-0.2597***			-0.2843***			-0.2837***			-0.2716***
			(-4.29)			(-4.37)			(-4.56)			(-4.32)
ILLIQ			0.0012***			0.0012***			0.0013***			0.0012***
			(3.25)			(3.23)			(3.32)			(3.25)
Alpha	-0.0043	-0.0161***	-0.0032	0.0176***	-0.0046	0.0039	0.0184***	-0.0048	0.0032	-0.0033	-0.0163***	-0.0029
	(-1.33)	(-3.66)	(-0.96)	(7.69)	(-1.22)	(1.21)	(7.00)	(-1.35)	(1.00)	(-1.07)	(-3.72)	(-0.90)
Avg. R^2	4.37%	6.21%	7.48%	3.66%	5.89%	7.34%	3.89%	5.99%	7.40%	4.10%	6.07%	7.41%

(END)