Bloch Sphere Representation and Logic Gate Analysis for Spin Behavior in Multi-Subband Systems

Tatsuki Tojo[1] and Kyozaburo Takeda[2]

[1,2]*Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan*

[1]tojo@qms.cache.waseda.ac.jp, [2]takeda@waseda.jp

When a Rashba external field is applied to a multi-subband system, such as a hole system in a semiconductor two-dimensional quantum well (2DQW), spin-orbit interaction (SOI) via inter-subband interaction (ISI) causes multiple degenerate points (DPs) in the Brillouin zone [2]. In the vicinity of DPs, the degeneracy generates indeterminateness of the wave function as well as non-adiabatic process, leading to a topological phase. In this study, we discuss how the ISI affects spin behavior through the topological parameters. For this purpose, we illustrate the spin on the Bloch sphere (BS), and attempt to understand the behavior by operating logic gates upon the BS parameters of zenith (θ_{B}) and azimuthal (ϕ_{B}) angles. In practical calculation, we focus on heavy holes (HHs) in Si 2DQW.

We show the BS θ_{B} and ϕ_{B} for HH in Fig. 1(a) and (b), respectively. The zenith angle θ_{B} varies periodically between 0 and π with forming a triangular wave (Fig. 1(a)). Contrary, the azimuthal angle ϕ_{B} changes from positive to negative, and causes a characteristic π jump when the HH passes the BS poles (Fig. 1(b)). The two-state and rotating-wave approximation elucidates that the slopes of θ_{B} and ϕ_{B} correspond to effective non-Abelian and Abelian Berry connections, respectively.

Next, we calculate the spin expectation value $\langle \boldsymbol{s} \rangle$ and represent its vector by the decomposition into the zenith and azimuthal angles (Θ, Φ). We further estimate the corresponding angles (Θ', Φ') by using the BS parameters via the following logic-gate operations, $\mathbb{R}_z$ and $\mathbb{R}_x$:

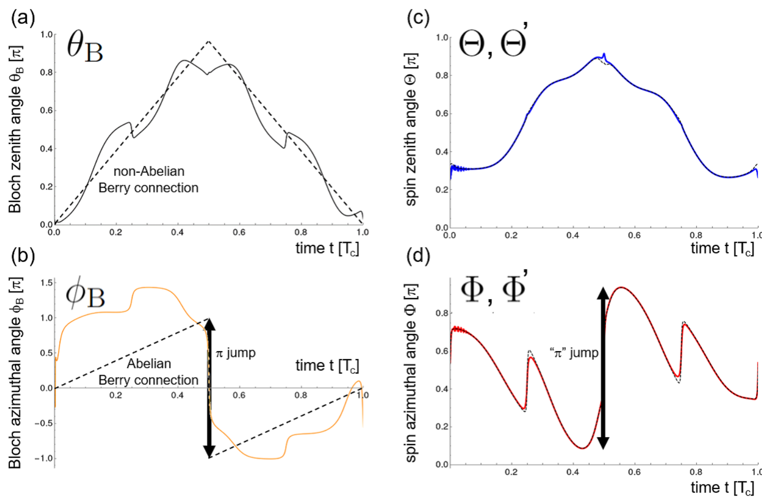

Fig. 1. BS zenith angle θ_{B} (a) and azimuthal angle ϕ_{B} (b), and spin vector zenith angles Θ and Θ' (c) and azimuthal ones Φ and Φ' (d) for HH in Si 2DQW. Θ mostly coincides with Θ'; Φ with Φ'.

$$\begin{pmatrix} \Theta \\ \Phi \end{pmatrix} \leftarrow [\mathbb{R}_z(\gamma) \circ \mathbb{R}_x(\beta) \circ \mathbb{R}_z(\alpha)] \begin{pmatrix} \theta_{\mathrm{B}} \\ \phi_{\mathrm{B}} \end{pmatrix}, \tag{1}$$

where α, β, γ are the logic-gate arguments.

In Fig. 1(c) and (d), we compare the evolution of the spin expectation vector (Θ and Φ; solid line) with that of the logic-gate approximated one (Θ' and Φ'; broken line by Eq. (1)). A similar triangular wave is found in spin behavior of Θ. A deformed but basically linear dependence having the "π"-jump structure is also found in that of Φ. The logic-gate approximation elucidates that $\Theta \simeq \theta_{\mathrm{B}}$ and $\Phi \simeq \phi_{\mathrm{B}} + \Delta\phi_{\mathrm{D}} + \chi(\alpha, \gamma)$ due to $|\beta| \ll \pi/2$. Thus, the spin vector component Φ *could* includes the topological phase (difference ϕ_{B}), with a correction by the dynamical phase (difference $\Delta\phi_{\mathrm{D}}$) and the logic-gate arguments α, β, γ.

References

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