Harmonic Analysis & Differential Equations Conference: In Honour of Jill Pipher

Macquarie University, Sydney, Australia

23–29 June 2024

I. Speakers & Talk Titles

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Tony Carbery	Weighted inequalities for the Fourier extension operator
William Chen	Discrepancy theory for half-infinite geodesics in non-integrable dynamical systems
José Manuel Conde-Alonso	Non-doubling measures and dyadic models: sparse domination, weights and commutators
Michael Cowling	Flag Hardy spaces
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Ji Li	Fefferman–Stein type inequality in multiparameter settings and applications
Linhan Li	The Poisson-Neumann problem and its application to the Neumann problem
Svitlana Mayboroda	Free boundary problems for the partially reflected Brownian motion: dimension of the Robin harmonic measure
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Yumeng Ou	Bilinear spherical maximal functions with general dilation sets
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Neil Trudinger	On the maximum principle for general elliptic equations
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Qingquan Deng	The maximal estimates and point-wise convergence for Schrödinger operators with potentials
Sean Douglas	Normed inequalities for fractional derivatives
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Tianpeng Gou	Non-commutative Kirillov theory and coadjoint orbits
Walton Green	Wavelet representation and Sobolev regularity of quasiregular maps
Qingdong Guo	Balayage of Carleson measures and BMO($\mathbb{R}_+, dm_\lambda$)
Elizabeth Hale	Fractional Leibniz rules in quasi-Banach function spaces
Dangyang He	Endpoint estimates for Riesz transform on manifolds with ends
Junyan Huang	Applied and computational harmonic analysis: DCNN application
Jin Bong Lee	L^p bounds for maximal operators given by Fourier multipliers with dilation of fractional dimensions
Zane Li	Mixed norm Fourier decoupling for paraboloids
Chong-Wei Liang	Representation theorem and endpoint estimates for lacunary maximal function on homogeneous groups
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Blanca Radillo-Murguia	Probabilistic construction of a Kakeya-type set
Jaakko Sinko	Two-weight $L^p \to L^q$ boundedness of commutators in the setting $p > q$
Alex Stokolos	Sharp weak-type estimates for maximal operators associated to rare bases
Amelia Stokolosa	Inverses of product kernels and flag kernels on graded Lie groups
Brandon Sweeting	Multiplier weak-type inequalities for the maximal operator and singular integrals
Liangchuan Wu	Bi-parameter Hoeffding type inequalities in almost-orthogonal settings and applications

III. Talks: Titles & Abstracts

1. **Theresa Anderson (Carnegie Mellon University) – Zoom talk**

   Title: Harmonic analysis in arithmetic statistics – a journey inspired by Jill

   Abstract: I'll give a brief glimpse, geared toward analysts, of how analysis has been used to count polynomials. This union of analysis and algebra – and indeed my mathematical journey – has been largely made possible by Jill's support and encouragement.

2. **Ariel Barton (University of Arkansas)**

   Title: The Dirichlet problem with boundary data of fractional smoothness in domains with Ahlfors regular boundary

   Abstract: In recent years, numerous authors have investigated the Dirichlet problem for second order elliptic differential equations, with boundary data in Lebesgue or Sobolev spaces, in domains with very rough boundaries. In particular, some authors have considered the Dirichlet problem in domains with higher codimensional boundary. In this talk we will extrapolate their results to the Dirichlet problem with boundary data in Besov spaces. This is joint work with Svitlana Mayboroda and Alberto Pacati.

3. **Dmitriy Bilyk (University of Minnesota)**

   Title: Harmonic analysis methods in geometric discrepancy theory

   Abstract: Geometric discrepancy theory deals with measuring irregularities of distribution of finite point configurations with respect to various families of geometric shapes and harmonic analysis methods play a crucial role in this area.

   In the first part of the talk, we shall discuss the behavior of the discrepancy of N-point sets in the unit square with respect to rectangles. It is well known that there is a drastic difference between the discrepancy with respect to axis-parallel rectangles (logarithmic behavior in N) and rectangles allowed to rotate in arbitrary directions (powers of N). We shall discuss this difference and will address the situation when the rectangles are allowed to be rotated in some restricted set $\Omega \subset [0, 2\pi]$ of directions. While a precise relation between the geometry of Ω and the discrepancy bounds is not yet completely understood, we shall provide some partial results in this direction. In particular, we shall present some upper bound based on "good" rotations of the square lattice which are obtained by measure-theoretic and diophantine methods such as constructing certain special Cantor-like sets (this part is based on earlier joint work with J. Pipher, X. Ma, and C. Spencer), as well as lower bounds based on the average decay of the Fourier transform (joint with M. Mastrianni).

   In the second part, we turn to the discrepancy on the sphere. A classical result of J. Beck guarantees that the optimal discrepancy of an N-point set in $\mathbb{S}^d$ over all spherical caps with unrestricted radii is of the order $N^{-\frac{1}{2}-\frac{1}{2d}}$. However, it was completely unclear what happens if one fixes the radius. We provide a partial answer to this question by

describing a set of radii for which the above bound continues to hold. To this end we introduce "gegenbadly approximable numbers (an analog of badly approximable numbers in diophantine approximations) for which the values of Gegenbauer polynomials stay away from zero in a certain quantitative sense. We also discuss other versions of this question in various settings (e.g. balls in the unit cube or torus), an interesting role played by the case $d \equiv 1 \mod 4$, as well as the so-called 'freak theorem' about continuous functions which have mean zero over all spherical caps of a given radius (joint with M. Mastrianni and S. Steinerberger).

4. **Tony Carbery (University of Edinburgh)**

Title: Weighted inequalities for the Fourier extension operator

Abstract: We give an overview of some of the weighted inequalities for the extension operator for the Fourier transform that have arisen in the last few years in connection with the so-called Mizohata–Takeuchi conjecture.

5. **William Chen (Macquarie University)**

Title: Discrepancy theory for half-infinite geodesics in non-integrable dynamical systems

Abstract: There are two versions of the classical Kronecker–Weyl equidistribution theorem. Let $d \geq 1$ be a fixed integer. The discrete version forms part of classical discrepancy theory and concerns the distribution of an infinite sequence $\mathbf{s}_j = \mathbf{s}_0 + j\mathbf{v}_0$, $j = 0, 1, 2, 3, \ldots$, on the unit torus $[0,1)^d$, with starting point $\mathbf{s}_0 \in [0,1)^d$ and increment $\mathbf{v}_0 \in [0,1)^d$. The continuous version concerns the distribution of a half-infinite geodesic $\mathcal{L}(t) = \mathbf{s} + \mathbf{v}t$, $t \geq 0$, on the unit torus $[0,1)^{d+1}$, with starting point $\mathbf{s} \in [0,1)^{d+1}$ and increment $\mathbf{v} \in [0,1)^{d+1}$. We are interested in the special case $d = 1$ and extending the time-quantitative analogue of the continuous version from the unit torus $[0,1)^2$ to a finite polysquare translation surface, or a finite square-tiled translation surface.

6. **José Manuel Conde-Alonso (Universidad Autónoma de Madrid)**

Title: Non-doubling measures and dyadic models: sparse domination, weights and commutators

Abstract: Dyadic models of Calderón-Zygmund operators are interesting not only because of their applications to the study of their continuous counterparts, but also because they give us a direct way to apply probabilistic techniques in harmonic analysis. In this talk, we study Haar shifts defined with respect to filtrations more general than the one given by the dyadic one on the Euclidean space, with special attention to natural dyadic models of the Hilbert transform. We will discuss sparse domination, weighted inequalities and analogies and differences between the continuous and dyadic world for general measures/filtrations.

This is based on joint works with Tainara Borges, Jill Pipher and Nathan Wagner (Brown University).

7. **Michael Cowling (University of New South Wales) – Zoom talk**

Title: Flag Hardy spaces

Abstract: This is an account of two features of recent work with P. Chen, J. Li, M.Y. Lee, and A. Ottazzi. The flag structure on the Heisenberg group $\mathbb{H}^n$ is usually viewed as a projected version of a product structure on $\mathbb{H}^n \times \mathbb{R}$. To develop a useful Hardy space theory in this context, we should therefore need to identify an appropriate notion of atom (perhaps the natural quotient of a product of atoms on $\mathbb{H}^n$ and on $\mathbb{R}$), and an appropriate version of Journé's Lemma. This talk will discuss the "right" notion of atom (which turns out to be tricker than expected) and the "right" version of Journé's lemma (which turns out to use ideas of Jill Pipher).

8. **Martin Dindoš (University of Edinburgh)**

Title: A relationship between the Dirichlet and Regularity problems for parabolic equations

Abstract: Recall a well-known result of Shen that has established that if $\mathcal{L} = \operatorname{div}(A\nabla\cdot)$ is an elliptic operator on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ then there is a "near duality" between the solvability of the Regularity problem for $\mathcal{L}$ with boundary data $\nabla_T f \in L^p(\partial\Omega)$ and solvability of the Dirichlet problem for the adjoint operator $\mathcal{L}^*$ with boundary data $f \in L^{p'}(\partial\Omega)$ where $1/p + 1/p' = 1$. This duality holds, provided the Regularity problem for $\mathcal{L}$ is solvable for at least one $q \in (1, \infty)$.

In the talk I shall present a joint result with E. Sätterquist where we have managed to prove an analogous result in the parabolic case for an operator of the form $\partial_t - \mathcal{L}$ on parabolic cylinders $\mathcal{O} = \Omega \times \mathbb{R}$. One of the key obstacles while tackling this problem was the definition of an end-point atomic Hardy-Sobolev space for the Regularity problem and establishing its interpolation properties with Sobolev spaces.

9. **Yihong Du (University of New England)**

Title: On the KPP equation with nonlocal diffusion and free boundaries

Abstract: A new phenomenon in nonlocal diffusion models is that accelerated propagation may happen, that is, the propagation speed could be infinite, which never occurs in the corresponding local diffusion model with compactly supported initial data. In this talk, we will first briefly review the history of the Kolmogorov-Petrovsky-Piskunov (KPP) model used to describe the propagation of biological/chemical species, and then look at some very recent results on the KPP equation with nonlocal diffusion and free boundaries. For several natural classes of kernel functions appearing in the nonlocal diffusion term, we will show how the exact rate of acceleration can be determined. The talk is based on joint works with Dr Wenjie Ni.

10. **Xuan Duong (Macquarie University)**

Title: Recent progress on BMO spaces associated to operators: the Garnett-Jones decomposition

Abstract: In this talk, we explain the theory of BMO spaces associated to operators and its applications to estimates of singular integrals. We then give a recent progress on the Garnett-Jones decomposition of these BMO spaces. This is joint work with P. Chen, J. Li, L. Song and L. Yan.

11. **John Garnett (University of California, Los Angeles) – Zoom talk**

Title: H^1–BMO duality revisited

Abstract: Among domains in $\mathbb{R}^{d+1}$ satisfying a capacity density condition and an interior corkscrew condition we give conditions, some sufficient and some necessary, for the H^1–BMO duality theorem to hold for harmonic functions in two formulations.

12. **Tainara Gobetti Borges (Brown University) – short talk**

Title: Sparse bounds for some maximal operators associated to bilinear averages

Abstract: Let S be a $(2d-1)$-dimensional compact smooth surface in $\mathbb{R}^{2d}$ such that k of the $(2d-1)$ principal curvatures do not vanish. Let μ_S be the normalized natural surface measure on S and define the (scale t) bilinear averaging operator

$$\mathcal{A}_{S,t}(f,g)(x) := \int_S f(x-ty)g(x-tz)\,d\mu_S(y,z).$$

We will discuss sparse bounds for the bilinear maximal operator

$$\mathcal{M}_S(f,g)(x) := \sup_{t>0} |\mathcal{A}_{S,t}(f,g)(x)|$$

and some generalizations of it. For the case of the unit sphere in $\mathbb{R}^{2d}$, in collaboration with B. Foster, Y. Ou, J. Pipher, and Z. Zhou, we proved sparse bounds for $\mathcal{M}_{S^{2d-1}}$ for all $d \geq 2$, while for the lacunary version of it we proved sparse bounds for any $d \geq 1$.

In a joint work with B. Foster and Y. Ou, we studied sparse bounds for more general surfaces S and more general dilation sets, where instead of taking the supremum $t > 0$ one can take the supremum over $t \in \tilde{E}$ where $\tilde{E}$ is the set of all scales obtained by dyadic dilation of fixed set of scales $E \subseteq [1,2]$. These sparse bounds have weighted inequalities corollaries, and in particular, unweighted Lebesgue bounds.

13. **Steve Hofmann (University of Missouri)**

Title: The Dirichlet problem for elliptic equations with a singular drift term

Abstract: We establish solvability of the L^p Dirichlet problem, for some $p < \infty$, for elliptic equations of the form

$$Lu = -\text{div } (A\nabla u) + \mathbf{b} \cdot \nabla u \,,$$

in a 1-sided chord arc domain $\Omega \subset \mathbb{R}^{n+1}$ (i.e., a uniform domain with an Ahlfors-David regular boundary), provided that the drift term satisfies $\text{dist}(X, \partial\Omega)|\mathbf{b}(X)| \lesssim 1$, as well as the Carleson measure condition

$$\iint_{\Omega \cap B(x,r)} |\mathbf{b}(Y)|^2 \text{dist}(Y, \partial\Omega)\, dY \lesssim r^n \,.$$

In previous work with J. L. Lewis, we had treated the analogous problem for parabolic equations in the half-space (and hence, via a pullback mechanism, in certain parabolic Lipschitz graph domains), and had presented a claimed, simpler proof in the elliptic case, based on an erroneous proof of doubling of elliptic measure for operators with drift terms. In fact, it appears that doubling remains an open problem (except in the small constant case), so the putative elliptic argument in that work was incorrect.

The proof presented in the present work provides a purely elliptic argument, still simpler than that in the parabolic case, which does not require doubling (again, except in the small constant case). Moreover, the present argument allows the treatment of much more general domains, whose boundaries need not be given locally as graphs.

14. **Carlos Kenig (University of Chicago)**

Title: Unique continuation and boundary unique continuation, old and new

Abstract: We will recall the notion of unique continuation of solutions of elliptic equations from the work of Hadamard and Carleman in the early 20th century, with further contributions by many of the top analysts on the 20th century. We will then discuss recent progress, in connection with problems in geometry motivated by the study of nodal sets, singular sets and critical sets. We will then turn to corresponding problems up to the boundary and explain old and recent progress on the connection between boundary unique continuation and regularity of the boundary.

15. **Loredana Lanzani (Universitá di Bologna) – Zoom talk**

Title: A numerical method for the solution of boundary value problems on convex planar domains

Abstract: The Unified Transform Method (UTM) was pioneered in the early '90s by A. S. Fokas and I. M. Gel'fand in their study of the numerical solution of boundary value problems for elliptic PDEs and for a large class of nonlinear PDEs. The UTM provides a connection between the Fourier Transform method for linear PDEs (FT) and its nonlinear counterpart, namely the Inverse Spectral method – also known as Non Linear Fourier Transform method (NLFT).

At the heart of the matter is a new derivation of the FT method for linear equations in one and two (space) variables that follows the same conceptual steps needed to implement the NLFT method for a class of nonlinear evolution equations, thus pointing to a unified approach to the numerical solution of linear and nonlinear PDEs.

From the very beginning, the UTM has attracted a great deal of interest in the applied mathematics community. A multitude of versions of the original method have since been developed, each dealing with a specific family of equations. Here we focus on a 2003 result of A.S. Fokas and A.A. Kapaev pertaining to the study of boundary value problems for the Laplacian on convex polygons: their original approach relied on a variety of tools (spectral analysis of a parameter-dependent ODE; Riemann-Hilbert techniques, etc.) but it was later observed by D. Crowdy that the method can be recast within a complex function-theoretic framework which, in turn, expands the applicability to so-called circular domains (domains bounded by arcs of circles, with line segments being a special case).

We extend the original approach of Fokas and Kapaev for polygons, to arbitrary convex domains. It turns out that ellipses (which are not circular in the sense of Crowdy) are of particular relevance in applications to engineering because the most popular heat exchangers (namely the shell-and-tube exchangers) have elliptical cross section.

In this talk I will describe a complex function-theory based new algorithm for convex domains, and will highlight the numerical challenges that arise when implementing it.

This is joint work with J. Hulse (Syracuse University), S. Llewellyn Smith (UCSD & Scripps Institute of Oceanography) and Elena Luca (The Cyprus Institute).

16. **Ji Li (Macquarie University) – short talk**

Title: Fefferman–Stein type inequality in multiparameter settings and applications

Abstract: Let $u(x,t)$ be a harmonic function in $\mathbb{R}^n \times (0,\infty)$. The non-tangential maximal function $u^*(x) = \sup_{|x-y|<t} |u(y,t)|$ and area integral $S(u)(x)^2 = \int_{|x-y|<t} |\nabla u(y,t)|^2 t^{1-n} dydt$ are two fundamental tools in the theory of singular integrals and the related function spaces. Fefferman and Stein first showed that $\|u^*\|_{L^p(\mathbb{R}^n)} \approx \|S(u)\|_{L^p(\mathbb{R}^n)}$, $0 < p \leq 1$, when $u(x,t) \to 0$ as $t \to \infty$. The key objects in their proof are the following inequality

$$|\{x \in \mathbb{R}^n : S(u)(x) > \lambda\}| \lesssim |\{x \in \mathbb{R}^n : u^*(x) > \lambda\}| + \frac{1}{\lambda^2} \int_0^\lambda s|\{x \in \mathbb{R}^n : u^*(x) > s\}|ds$$

and the corresponding inequality of the same type but with u^* and $S(u)$ interchanged.

We establish such an inequality in certain multiparameter settings, including the Shilov boundaries of tensor product domains in $\mathbb{C}^{2n}$, and the Heisenberg groups $\mathbb{H}^n$ with flag structure. Our technique bypasses the use of Fourier, and it is in fact independent of group structure. Direct applications include the maximal function characterisation of product/flag Hardy space, and the weak type endpoint estimate for product/flag Calderón–Zygmund operators.

17. **Linhan Li (University of Edinburgh)**

Title: The Poisson-Neumann problem and its application to the Neumann problem

Abstract: Recent years have seen much progress in boundary value problems for elliptic operators in non-smooth settings. In particular, we now have a good understanding of solvability of the L^p Dirichlet problem and many of its characterizations. There have also been big breakthroughs recently on the Regularity problem. However, little progress has been made on the Neumann problem since the works of Kenig and Pipher in the mid 90s. In a new joint work with Joseph Feneuil, we introduce the Poisson-Neumann problem and its variants, with the hope that it can serve as a stepping stone to eventually solving the Neumann problem. In the talk, I will discuss some characterizations of the Poisson-Neumann problem and its weaker variants, their connections to the Neumann problem, and will show that an extrapolation result on the Neumann problem obtained by Kenig and Pipher can be improved with the help of the Poisson-Neumann problem.

18. **Svitlana Mayboroda (ETH Zürich and University of Minnesota)**

Title: Free boundary problems for the partially reflected Brownian motion: dimension of the Robin harmonic measure

Abstract: We construct the harmonic measure associated to partially reflective Brownian motion (that is, Robin boundary data) and demonstrate that its behavior is surprisingly different from the Dirichlet analogue. In particular, contrary to the traditional setting and to some predictions in physics literature, it is absolutely continuous with respect to the Hausdorff measure for a large class of operators and domains.

19. **Andrea Nahmod (University of Massachusetts Amherst)**

Title: Probabilistic scaling, propagation of randomness and invariant Gibbs measures

Abstract: In this talk, we will start by describing how classical tools from probability offer a robust framework to understand the dynamics of waves via appropriate ensembles on phase space rather than particular microscopic dynamical trajectories. We will continue by explaining the fundamental shift in paradigm that arises from the "correct" scaling in this context and how it opened the door to unveil the random structures of nonlinear waves that live on high frequencies and fine scales as they propagate. We will then discuss how these ideas broke the logjam in the study of the Gibbs measures associated to nonlinear Schrödinger equations in the context of equilibrium statistical mechanics and of the hyperbolic Φ^4_3 model in the context of constructive quantum field theory.

We will end with some open challenges about the long-time propagation of randomness and out-of-equilibrium dynamics.

20. **Yumeng Ou (University of Pennsylvania)**

Title: Bilinear spherical maximal functions with general dilation sets

Abstract: I'll discuss in this talk some recent results on the boundedness properties of various bilinear averaging operators and their maximal functions associated with general fractal dilations sets. Some model examples here include the single scale bilinear spherical averaging operator and the triangle averaging operator, together with their associated full and lacunary maximal functions. We are interested in their L^p improving estimates, Sobolev smoothing estimates, Lebesgue space bounds, as well as sparse bounds. The estimates depend on both the structure of the operator itself and the dimension of the dilation set.

21. **Cristina Pereyra (University of New Mexico) – Zoom talk**

Title: Resolvent of the dyadic paraproduct, t-Haar multipliers, and weighted inequalities

Abstract: We discuss weighted inequalities for t-Haar multipliers, $T^t_{w,\sigma}$. We seek necessary and sufficient conditions on triples of weights (u, v, w) so that the operators are uniformly (on the choice of signs σ) bounded from $L^2(u)$ into $L^2(v)$. The t-Haar multipliers are analogous to pseudo-differential operators where the trigonometric functions have been replaced by the Haar functions. The symbols we study are of the form $\sigma_t(x, I) = \sigma_I(w(x)/\langle w\rangle_I)^t$ where $t \in \mathbb{R}$, w is a weight, $\langle w\rangle_I$ denotes the integral average of w on the dyadic interval I, and $\sigma_I = \pm 1$. We recover known necessary and sufficient conditions of Nazarov, Treil, and Volberg for two-weight inequalities for the martingale transform (the case $w \equiv 1$), and the known necessary and sufficient conditions of Katz and Pereyra for the unweighted case ($u = v \equiv 1$). A close cousin to one of these operators, denoted P_w, is the resolvent of the dyadic paraproduct π_b, where b is in BMO, and the weight w is related to b via the R. Fefferman, Kenig and Pipher dyadic multiplicative relation introduced in their celebrated 1991 Annals paper.

This is joint work with Daewon Chung, Weiyan Huang, Jean Carlos Moraes, and Brett Wick.

22. **Stefanie Petermichl (Universität Würzburg) – Zoom talk**

Title: Some news on matrix weights

Abstract: We'll discuss a few recent developments in matrix weighted estimates of singular operators. An operator (such as the Hilbert transform) acts component wise on vector functions. When computing its weighted L^2 norm, the weight perturbs both in magnitude (such as in the scalar case) as well as through rotating the entries of the component vectors. The non-commutative aspect of the matrix weights makes the theory more interesting but also substantially more difficult. These questions stem from so called mixing problems in probability as well as aspects of the theory of Toeplitz operators. Certain results we know from the scalar case remain true while others do not. In this lecture we will give a few examples of what works and what doesn't. In particular, we will recall some early, very clever observations of Jill and her co-authors within the theory of weights and show their matrix analog.

23. **Rodolfo Torres (University of California, Riverside)**

Title: Extrapolation of compactness for certain pseudodifferential operators

Abstract: The extrapolation result of Rubio de Francia has become a powerful tool to extend the weighted boundedness of an operator from a particular weighted Lebesgue space into others. This classical theorem has been extended to many contexts over the years and found many useful applications and, more recently, versions to extrapolate compactness have been studied by several authors too. We will provide a simple alternative version of such extrapolation of compactness results and present a novel application to a class of pseudodifferential operators, establishing their compactness on weighted Lebesgue spaces. This is joint work with María Jesus Carro and Javier Soria.

24. **Neil Trudinger (The Australian National University)**

Title: On the maximum principle for general elliptic equations

Abstract: This talk concerns maximum principles and related estimates for linear second order elliptic partial differential operators in n-dimensional Euclidean space, where the ellipticity is determined through the principal coefficient matrices $\mathcal{A}$ lying in sub-cones of the positive cone, which are dual cones of the Gårding k-cones, for $k = 1, \ldots, n$. The results improve previous work with H.-J. Kuo through sharp L^p dependence on the drift coefficients b. We will also consider the applications to local estimates, including extensions of the Krylov-Safonov Hölder and Harnack estimates as well as the Pucci conjecture.

25. **Nathan Wagner (Brown University) – short talk**

Title: Weighted estimates for the Bergman projection on planar domains

Abstract: We investigate weighted Lebesgue space estimates for the Bergman projection on a simply connected planar domain via the domain's Riemann map. We extend the bounds which follow from a standard change-of-variable argument in two ways. First, we provide a regularity condition on the Riemann map, which turns out to be necessary in the case of uniform domains, in order to obtain the full range of weighted estimates for the Bergman projection for weights in a Bekolle-Bonami-type class. Second, by slightly strengthening our condition on the Riemann map, we obtain the weighted weak-type (1,1) estimate as well. Our proofs draw on techniques from both conformal mapping and dyadic harmonic analysis. Moreover, we highlight important connections with quasiconformal mappings and the Beltrami equation. This talk is based on joint work with Walton Green from Washington University.

26. **Lesley Ward (University of South Australia) – short talk**

Title: Unboundedness of some weighted Zygmund and flag maximal functions on weighted Lebesgue spaces

Abstract: We present examples of measures μ showing that three weighted strong maximal functions are unbounded on weighted Lebesgue spaces. These maximal functions include the weighted strong Zygmund maximal function $\mathcal{M}_{\mathfrak{z},\mu}$ on $\mathbb{R}^3$ and the weighted strong flag maximal function $\mathcal{M}_{\text{flag},\mu}$ on $\mathbb{R}^2$. The examples are in the spirit of R. Fefferman's example, and exploit a construction due to Barron, Conde-Alonso, Ou, and Rey. We also show that the dyadic Zygmund and dyadic flag maximal operators cannot be bounded by multiparameter sparse operators associated with the corresponding dyadic grid.

This is joint work with Guillermo Flores and Ji Li.

27. **Brett Wick (Washington University in St. Louis)**

Title: Wavelet representation of singular integral operators

Abstract: In this talk, we'll discuss a novel approach to the representation of singular integral operators of Calderón-Zygmund type in terms of continuous model operators. The representation is realized as a finite sum of averages of wavelet projections of either cancellative or noncancellative type, which are themselves Calderón-Zygmund operators. Both properties are out of reach for the established dyadic-probabilistic technique. Unlike their dyadic counterparts, our representation reflects the additional kernel smoothness of the operator being analyzed. Our representation formulas lead naturally to a new family of T1 theorems on weighted Sobolev spaces whose smoothness index is naturally related to kernel smoothness. In the one parameter case, we obtain the Sobolev space analogue of the A_2 theorem; that is, sharp dependence of the Sobolev norm of T on the weight characteristic is obtained in the full range of exponents. As an additional application, it is possible to provide a proof of the commutator theorems of Calderón-Zygmund operators with BMO functions. Portions of this talk are joint with Francesco Di Plinio, Walton Green, and Tasos Fragkos.

IV. Posters: Titles & Abstracts

1. **Zijun Chen (Monash University)**

   Title: Complex-valued solutions of the mKdV equation in general Fourier–Lebesgue spaces

   Abstract: We consider the Cauchy problem for the modified Korteweg-de Vries equation of *complex-valued* solutions on the real line. We introduce a general Fourier-Lebesgue space $\widehat{M}^s_{r,q}(\mathbb{R})$ that unifies modulation spaces and Fourier-Lebesgue spaces, and then obtain the sharp local well-posedness result in this space by perturbation methods. Our results improve the previous one in [**?**].

   This is joint work with Zihua Guo and Chunyan Huang.

References

[1] A. Grünrock and L. Vega, Local well-posedness for the modified KdV equation in almost critical $\widehat{H}^r_s$ spaces. Trans. Amer. Math. Soc. 361 (2009): 5681-5694.

2. **Chuhee Cho (Seoul National University)**

   Title: Bourgain's counterexample in the sequential convergence problem for the Schrödinger equation

   Abstract: We study the problem of pointwise convegence for the Schrödinger operator on $\mathbb{R}^n$ along time sequences. We show that the sharp counterexample to the sequential Schrödinger maximal estimate given recently by Li, Wang and Yan based in the construction by Lucà and Rogers can also be achieved with the construction of Bourgain, and we extend it to the fractal setting.

3. **Adam Christopherson (Ohio State University)**

   Title: Weak-type regularity of the Bergman projection on generalized Hartogs triangles

   Abstract: This poster will provide a brief introduction to the Bergman projection, specifically in the setting of non-smooth domains in $\mathbb{C}^n$. Our results show that the Bergman projection satisfies a weak-type estimate at the upper endpoint of L^p boundedness but not at the lower endpoint of L^p boundedness on rational Hartogs triangles in $\mathbb{C}^2$, power-generalized Hartogs triangles in $\mathbb{C}^3$, as well as on other domains in $\mathbb{C}^n$. This work is joint with K.D. Koenig.

4. **Ana Čolović (Washington University in St. Louis)**

Title: Garsia norm and boundedness of Hankel operators in weighted Hardy spaces

Abstract: We discuss the boundedness of Hankel operators between a weighted Hardy space and a weighted L^2 space, with two different Muckenhoupt weights. In the Lebesgue measure setting, Hankel operator with a symbol f is bounded if and only if its symbol has a bounded Garsia norm, or equivalently, a bounded BMO norm. We generalize this result to the case of two weights, with the appropriate generalization of the Carleson embedding theorem.

5. **Qingquan Deng (Central China Normal University)**

Title: The maximal estimates and point-wise convergence for Schrödinger operators with potentials

Abstract: Denote by $\mathcal{H}$ the one-dimensional Schrödinger operator

$$\mathcal{H} = -\Delta + V,$$

where the potential V is either a real-valued function in weighted space $L^{1,\gamma}$, or a Dirac measure $q\delta_0$ with constant $q \in \mathbb{R}$. In this work, we focus on the point-wise convergence for Schrödinger groups $e^{-it\mathcal{H}}$.

By using the distorted Fourier transform associated to $\mathcal{H}$, the TT^* argument, as well as the theory of functions spaces related to $\mathcal{H}$, we proved that for $s \geq \frac{1}{4}$ and $f \in H^s(\mathbb{R})$,

$$\lim_{t\to 0} e^{-it\mathcal{H}} f(x) = f(x), \quad a.e.$$

Moreover, if $s \geq \frac{1}{4}$, $\lambda \geq 0$ and $\frac{1}{4} \leq s + \lambda < \frac{1}{2}$. Then for $f \in H^{s+\lambda}(\mathbb{R})$,

$$e^{-it\mathcal{H}} f(x) - f(x) = o(t^{\frac{\lambda}{2}}), \quad a.e. \text{ as } t \to 0.$$

This is joint work with Yuwei Jiang, Ping Li, and Yaoming Liu.

6. **Sean Douglas (University of Missouri)**

Title: Normed inequalities for fractional derivatives

Abstract: The objective of this poster is to present weighted norm estimates for the homogeneous and inhomogeneous fractional derivatives. The primary focus is on a detailed exploration of the weighted fractional Leibniz rule and the weighted fractional chain rule. This work seeks to augment the current body of knowledge by establishing appropriate inequalities for more generalized weight classes and extending the range of indices for which the inequalities are valid.

In particular the fractional Liebniz rule is extended to the product of m factors and the weight class is improved from the product of Muckenhoupt weight classes to the more natural multiple weight class denoted $A_{\vec{p}}$. Furthermore, a fractional chain rule is extended from Lebesgue spaces for $p > 1$ to weighted Triebel-Lizorkin spaces with $0 < p < \infty$, where an explicit relationship holds between the permissible Muckenhoupt weights, the smoothness index, and the integrability index.

7. **Zhijie Fan (Sun Yat-sen University)**

Title: Schatten–Lorentz characterization of Riesz transform commutator associated with Bessel operators

Abstract: Let Δ_λ be the Bessel operator on the upper half space $\mathbb{R}^{n+1}_+$ with $n \geq 0$ and $\lambda > 0$, and $R_{\lambda,j}$ be the $j-$th Bessel Riesz transform, $j = 1, \ldots, n+1$. We demonstrate that the Schatten–Lorentz norm ($S^{p,q}$, $1 < p < \infty$, $1 \leq q \leq \infty$) of the commutator $[b, R_{\lambda,j}]$ can be characterized in terms of the oscillation space norm of the symbol b. In particular, for the case $p = q$, the Schatten norm of $[b, R_{\lambda,j}]$ can be further characterized in terms of the Besov norm of the symbol. Moreover, the critical index is also studied, which is $p = n+1$, the lower dimension of the Bessel measure (but not the upper dimension). Our approach relies on martingale and dyadic analysis, which enables us to bypass the use of Fourier analysis effectively.

This is joint work with Michael Lacey, Ji Li, and Xiao Xiong.

8. **Tianpeng Gou (University of Technology Sydney)**

Title: Non-commutative Kirillov theory and coadjoint orbits

Abstract: Élie Cartan and Hermann Weyl's theorem of highest weight says that the integration of an irreducible highest weight representation $d\pi$ of the compact real form $\mathfrak{u}$ of a complex semi-simple Lie algebra $\mathfrak{g}$, is exactly an irreducible representation π of a simply connected compact Lie group U. However, in general, there is no explicit calculation for this correspondence.

Based upon the work of Raed Raffoul (2006), we combine the Kirillov orbit method, sum of adjoint orbits, convexity theorem of moment map, Neslon's formula for Weyl calculus, and transversality of invariant differential operators, to develop a non-commutative Kirillov formula, showing that the Fourier transform of the lift of the π of U, i.e., $\pi(\exp \mathfrak{u})$, is the matrix of directional derivatives of a compactly supported Ad-invariant measure of the image of the moment map of π, in the dual of its Lie algebra $\mathfrak{u}$, denoted by $\mathfrak{u}^*$, in fundamental weight directions, root directions, and Ad^*-invariant direction of $\mathfrak{u}^*$. We also extend this formula to spherical functions of symmetric spaces of compact type.

9. **Walton Green (Washington University in St. Louis)**

Title: Wavelet representation and Sobolev regularity of quasiregular maps

Abstract: Extending the Sobolev theory of quasiconformal and quasiregular maps to subdomains of the complex plane motivates our investigation of Sobolev regularity of singular integral operators on domains. We introduce new paraproducts which lead to higher order T1-type testing conditions. A special case provides weighted Sobolev estimates for the compressed Beurling transform which imply quantitative Sobolev estimates for the Beltrami resolvent. This is joint work with Francesco Di Plinio and Brett D. Wick.

10. **Qingdong Guo (Xiamen University)**

Title: Balayage of Carleson measures and BMO$(\mathbb{R}_+, dm_\lambda)$

Abstract: Let $\lambda \in (0,\infty)$ and Δ_λ be the Bessel operator on $\mathbb{R}_+ := (0,\infty)$ defined by

$$\Delta_\lambda := -x^{-2\lambda}\frac{d}{dx}x^{2\lambda}\frac{d}{dx}.$$

The authors give a decomposition of the functions with bounded support in the space BMO$(\mathbb{R}_+, dm_\lambda)$ with $dm_\lambda(x) := x^{2\lambda}dx$, that is, for any $f \in \mathrm{BMO}(\mathbb{R}_+, dm_\lambda)$ with bounded support, then there exist $g \in L^\infty(\mathbb{R}_+, dm_\lambda)$ and an m_λ-Carleson measure μ on $\mathbb{R}_+ \times \mathbb{R}_+$ such that

$$f(y) = g(y) + S_{\mu,P^{[\lambda]}}(y), \quad a.e.\, y \in \mathbb{R}_+,$$

where

$$S_{\mu,P^{[\lambda]}}(y) := \iint_{\mathbb{R}_+\times\mathbb{R}_+} P_t^{[\lambda]}(y,x)d\mu(x,t), \quad x \in \mathbb{R}_+,$$

and $P_t^{[\lambda]}(y,x)$ is the Possion kernel associated with Δ_λ. Conversely, when μ is an m_λ-Carleson measure on $\mathbb{R}_+ \times \mathbb{R}_+$, the balayage $S_{\mu,P^{[\lambda]}}$ is in BMO$(\mathbb{R}_+, dm_\lambda)$.

This is joint work with Ji Li and Dongyong Yang.

11. **Elizabeth Hale (Kansas State University)**

Title: Fractional Leibniz rules in quasi-Banach function spaces

Abstract: Fractional Leibniz rules are inequalities whose structure resemble the product rule learned in calculus classes, giving estimates in the Lebesgue norm for fractional derivatives of a product of functions in terms of the Lebesgue norms of each function and its fractional derivative. Corresponding inequalities have also been obtained in many other function spaces using a variety of methods. We unify results across several settings by proving such estimates for Coifman-Meyer multiplier operators in the setting of Triebel-Lizorkin and Besov spaces based on quasi-Banach function spaces. Corollaries include results in the setting of rearrangement invariant quasi-Banach function spaces – specific examples of which include weighted Lebesgue, Orlicz, and Lorentz spaces – as well as in non-rearrangement invariant spaces such as weighted mixed Lebesgue spaces and Morrey spaces.

This is joint work with Virginia Naibo.

12. **Dangyang He (Macquarie University)**

Title: Endpoint estimates for Riesz transform on manifolds with ends

Abstract: We consider a class of non-doubling manifolds $\mathcal{M}$ consisting of finitely many "Euclidean" ends where the Euclidean dimensions are not necessarily all the same. In previous work, A. Hassell and A. Sikora proved that the Riesz transform on $\mathcal{M}$ is of weak type $(1,1)$ and bounded on L^p if and only if $1 < p < n_*$ where $n_* = \min_k n_k$. In this note, we supplement the picture by providing an endpoint estimate: the Riesz transform is bounded on the Lorentz space $L^{n_*,1}$ and unbounded from $L^{n_*,p} \to L^{n_*,q}$ for all $1 < p < \infty$ and $p \le q \le \infty$.

13. **Junyan Huang (Sun Yat-sen University)**

Title: Applied and computational harmonic analysis: DCNN application

Abstract: Deep learning based on deep convolutional neural networks (DCNN) is extremely efficient in solving classification problems in speech recognition, computer vision, and many other fields. But there is not enough theoretical understanding about this topic, especially the generalization ability of the induced CNN algorithms. In this paper, we approximate functions in the Sobolev space $W_p^r(\mathbb{S}^{d-1}), r > 0, 1 \leq p \leq \infty$ and the Holder space $C^{r,\alpha}(\mathbb{B}^d)$ with $0 < r + \alpha < \frac{d+2k+1}{2}$ by using $ReLU^k$ activation function in fully connected layer. Our result provides a theoretical side of the modelling and approximation ability of deep convolutional neural networks followed by downsampling and one fully connected layer or two. We showed that the output function has higher-order derivative and less free parameters which depend on the power of $ReLU^k$ than previous works. And we also proved that the convergence rate depends on the power of $ReLU^k$.

This is joint work with Jun Xian.

14. **Jin Bong Lee (Seoul National University)**

Title: L^p bounds for maximal operators given by Fourier multipliers with dilation of fractional dimensions

Abstract: We investigate L^p bounds of maximal Fourier multiplier operators with dilation of fractional dimensions. For the Fourier multipliers, we suggest a criterion related to dimensions of dilation sets which guarantees L^p bounds of the maximal operators for each $p \in (1, \infty)$. Our criterion covers Mikhlin-type multipliers, multipliers with limited decay, and multipliers with slow decay.

15. **Zane Li (North Carolina State University)**

Title: Mixed norm Fourier decoupling for paraboloids

Abstract: We discuss sharp mixed norm Fourier decoupling estimates for the paraboloid. One motivation of considering such an estimate is a conjectured mixed norm Strichartz estimate on the torus which essentially is an estimate about exponential sums. This is joint work with Shival Dasu, Hongki Jung, and José Madrid.

16. **Chong-Wei Liang (Macquarie University)**

Title: Representation theorem and endpoint estimates for lacunary maximal function on homogeneous groups

Abstract: The lacunary maximal function has an impressive influence in harmonic analysis. We establish a special representation formula on homogeneous groups, and then following the line of Andreas Seeger and James Wright, we prove an endpoint estimate for the lacunary maximal function on homogeneous groups.

This is joint work with Sanghyuk Lee and Ji Li.

17. **Ljupcho Petrov (Washington University in St. Louis)**

Title: Weighted estimate for one-sided singular integrals

Abstract: We present a proof of the one-sided A_2 theorem in dimension one, with a logarithmic loss. This theorem involves one-sided Calderón-Zygmund operators (CZOs), which generalize standard CZOs by having a kernel $K(x, y)$ that vanishes when $x < y$. Our proof extends the Pérez-Treil-Volberg result, reducing the norm estimate to testing on indicator functions. By combining this with the weak-(1,1) estimate of Riveros and Vidal and an extrapolation theorem, we achieve the one-sided A_2 estimate with a logarithmic loss.

This is joint work with A. Walton Green and Brett D. Wick.

18. **Blanca Radillo-Murguia (Baylor University)**

Title: Probabilistic construction of a Kakeya-type set

Abstract: Maximal operators are central objects in harmonic analysis, and their boundedness in different spaces of functions may be used to prove differentiation theorems. In this pursuit, Michael Bateman and Nets Katz accomplished a significant feat by providing the proof for a maximal operator associated to line segments in the directions Ω is unbounded on $L^p(\mathbb{R}^2)$ for all $p < \infty$ when this set of directions Ω admits Kakeya-type sets. Inspired by their reasoning, we will elaborate on the construction of the Kakeya-type sets. In this direction, we will introduce a necessary condition on the set of directions, which we refer to as to be η-separated, alongside employing a probabilistic argument to derive certain estimates.

This is joint work with Paul Hagelstein and Alex Stokolos.

19. **Jaakko Sinko (University of Helsinki)**

Title: Two-weight $L^p \to L^q$ boundedness of commutators in the setting $p > q$

Abstract: Let $1 < p, q < \infty$. Suppose $\mu \in A_p$ and $\lambda \in A_q$ are Muckenhoupt weights. Two-weight boundedness of commutators $[b, T]\colon L^p(\mu) \to L^q(\lambda)$, of a pointwise multiplier b and a Calderón-Zygmund operator T, was recently characterised in the setting $p < q$: the characterisation is given in terms of the symbol b. The analogous characterisation in the setting $p = q$ was known even earlier. The poster presents the characterisation in the missing exponent range $p > q$. The characterising condition, given on the sharp maximal function of b, differs significantly from the earlier known condition in the special case $p > q$ and $\mu = \lambda = 1$. A possible application of the main result is also discussed.

This is joint work with Timo Hänninen and Emiel Lorist.

20. **Alex Stokolos (Georgia Southern University)**

Title: Sharp weak-type estimates for maximal operators associated to rare bases

Abstract: I am presenting the sharp weak type estimates for geometric maximal operators associated to families of Cordoba-type, Soria-type, and Zygmund-type bases in $\mathbb{R}^n$. This is a joint presentation with Dmitriy Dmitrishin, Paul Hagelstein, and Giorgi Oniani.

21. **Amelia Stokolosa (University of Wisconsin–Madison)**

Title: Inverses of product kernels and flag kernels on graded Lie groups

Abstract: Consider the following problem studied by Christ and Geller in the 1980s. Let T be a left-invariant operator given by group convolution with a single-parameter homogeneous distribution K on a graded Lie group G; that is, $Tf = f*K$. Assume T is L^2 invertible. Is T^{-1} also given by convolution with a homogeneous distribution? Christ and Geller proved that the answer is yes. Extending the above problem to *almost homogeneous* distributions with respect to *multi-parameter dilations*, consider the operator $Tf = f*K$, where K is a product or a flag kernel on a graded Lie group G. Assume T is L^2 invertible. Is T^{-1} also given by group convolution with a product or a flag kernel accordingly? We prove that the answer is again yes. In the non-commutative setting, one cannot make use of the Fourier transform to answer this question. Instead, the key construction is an a priori estimate.

22. **Brandon Sweeting (University of Alabama)**

Title: Multiplier weak-type inequalities for the maximal operator and singular integrals

Abstract: We discuss a kind of weak-type inequality for the Hardy-Littlewood maximal operator and singular integrals that was first introduced by Muckenhoupt and Wheeden. This formulation treats the weight for the image space as a multiplier, rather than a measure, leading to fundamentally different behavior; in particular, as shown by Muckenhoupt and Wheeden, the class of weights characterizing such inequalities is strictly larger than A_p. This presentation concerns best constants for such inequalities, as well as recent work on the full characterization of those weights for which these inequalities hold for the Hardy-Littlewood maximal operator in dimension one.

This is joint work with David Cruz-Uribe.

23. **Liangchuan Wu (Anhui University)**

Title: Bi-parameter Hoeffding type inequalities in almost-orthogonal settings and applications

Abstract: In this poster, we establish bi-parameter Hoeffding inequalities on product spaces of homogeneous type. In contrast to the classical results in terms of the double dyadic martingale version and its one-parameter analog, we bypass the need for cancellation and orthogonal properties, instead assuming a natural almost-orthogonal condition on the involved atomic functions. To achieve this, we first study a one-parameter Hoeffding inequality by constructing an iterative algorithm on scales, and then extend to the double version by further iteration. Our arguments are intrinsically based on the almost-orthogonal property. As an application, we determine the sharp order of local integrability with a bounded double square function on the product Carnot-Carathéodory space, extending the classical Chang-Wilson-Wolff theorem (1985) and Pipher's bidisc work (1986).