

Word Problems? No Problem! Effective Strategies for Teaching Math Word-Problem Solving



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Say hello.

Describe your role as an educator.

Describe how your support word-problem solving.



How Students Solve Word Problems



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



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71

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Donna



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96 paper fold

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$$\begin{array}{r} 96 \\ + 25 \\ \hline \end{array}$$

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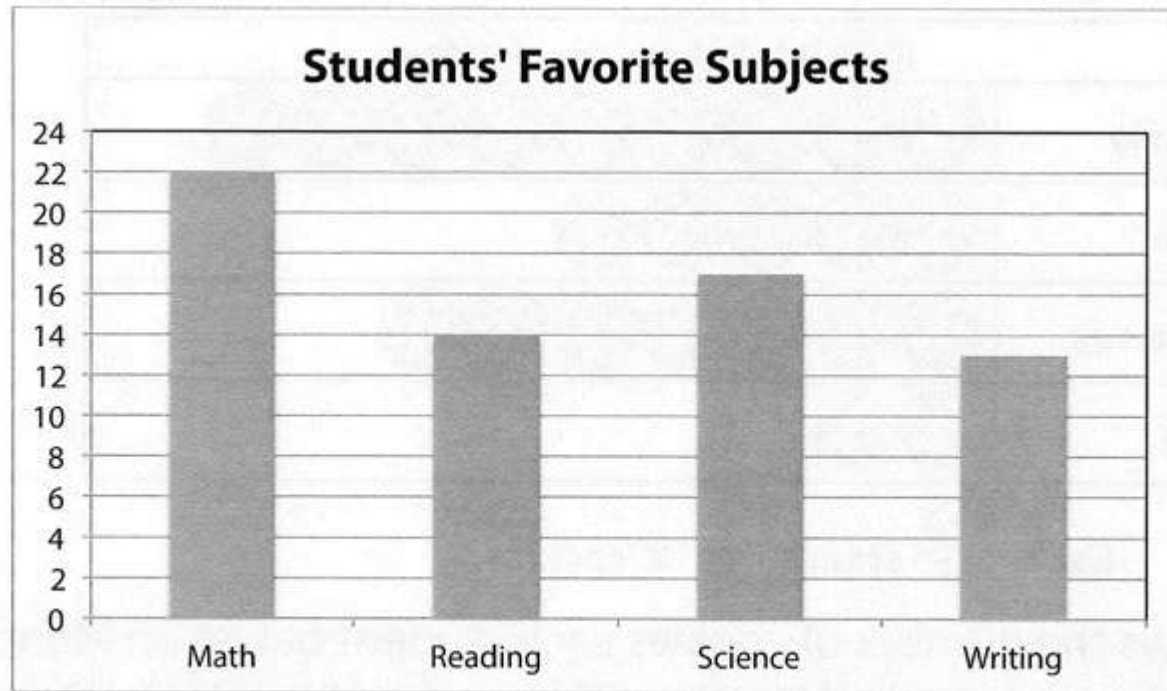
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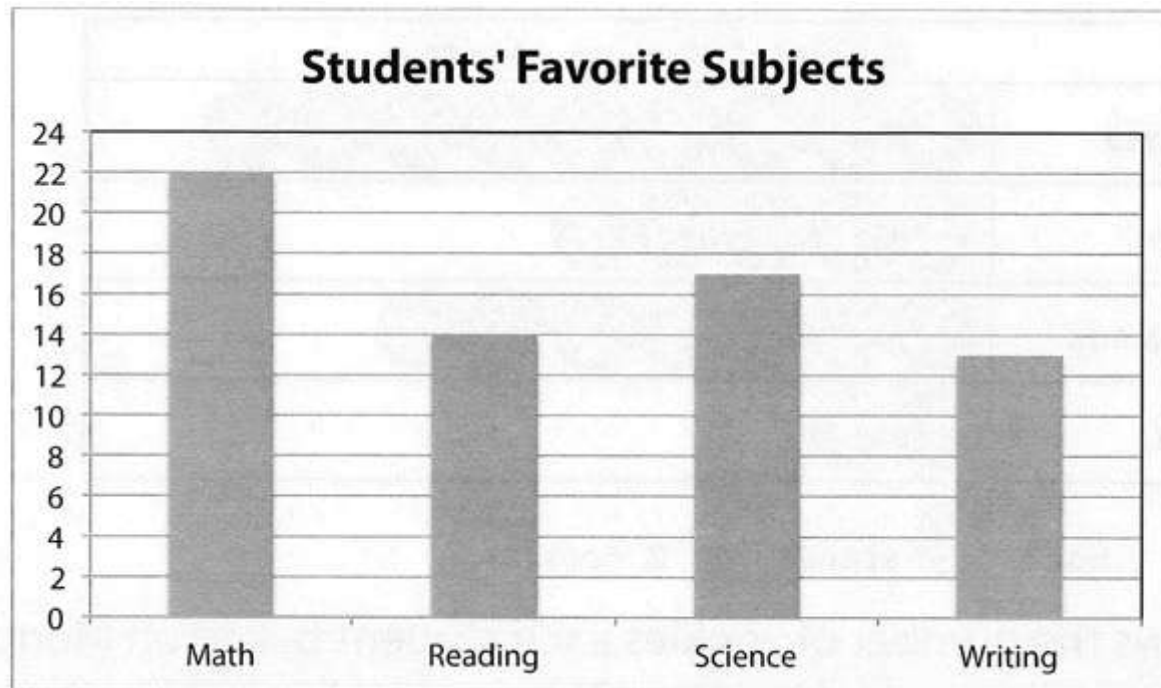
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?



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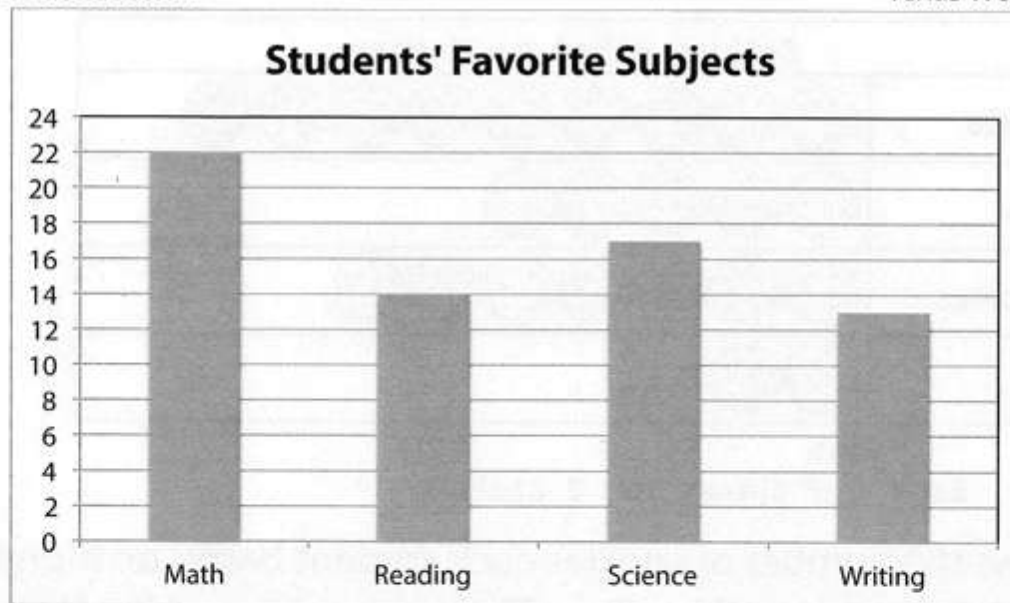


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60

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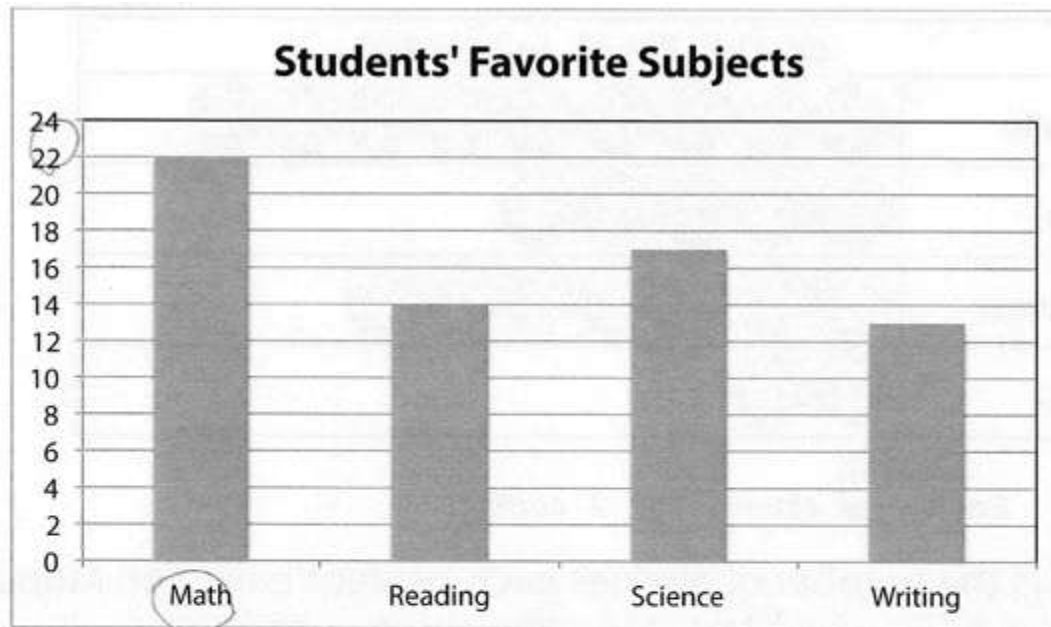
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The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

They choose 8% more than
reading

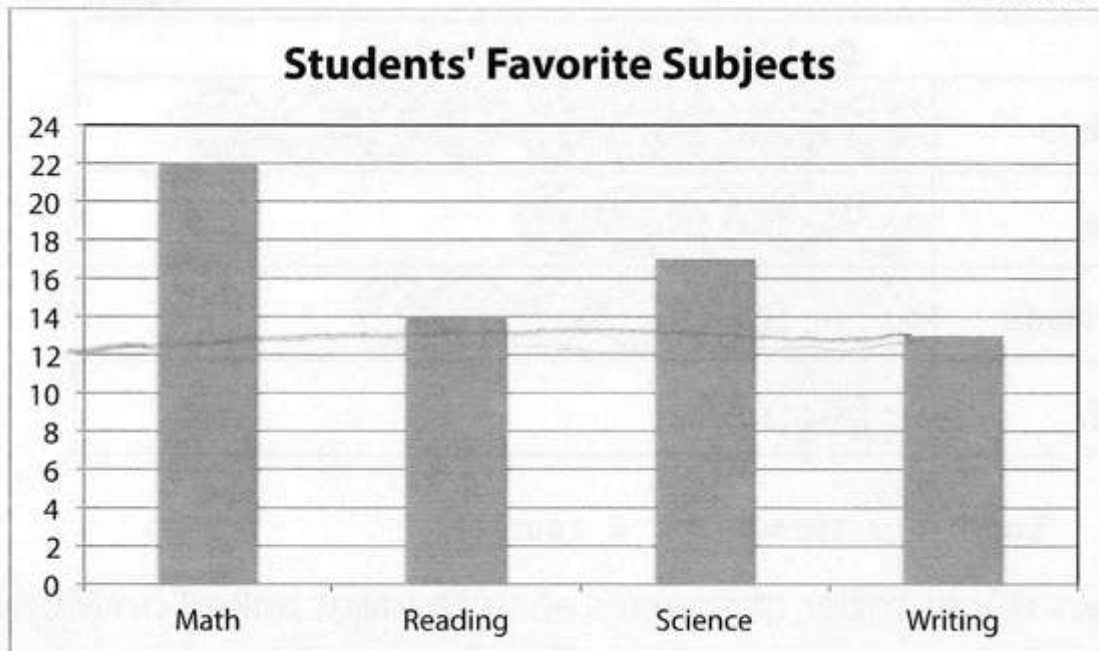
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22
Math

J.



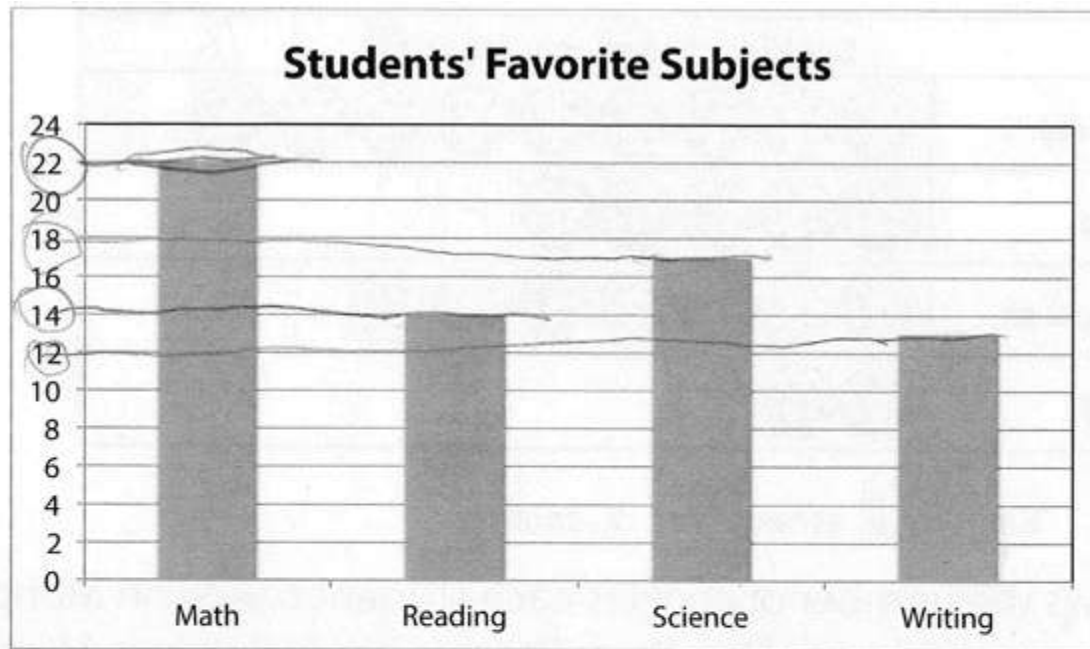
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(22) math

(13) writing



J.

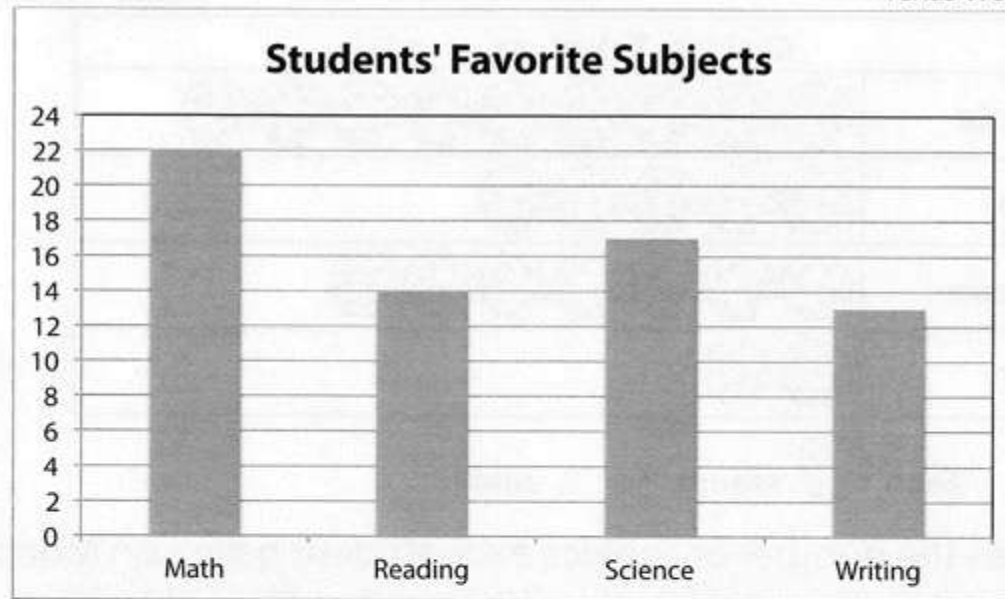


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22
Math

14
Writing

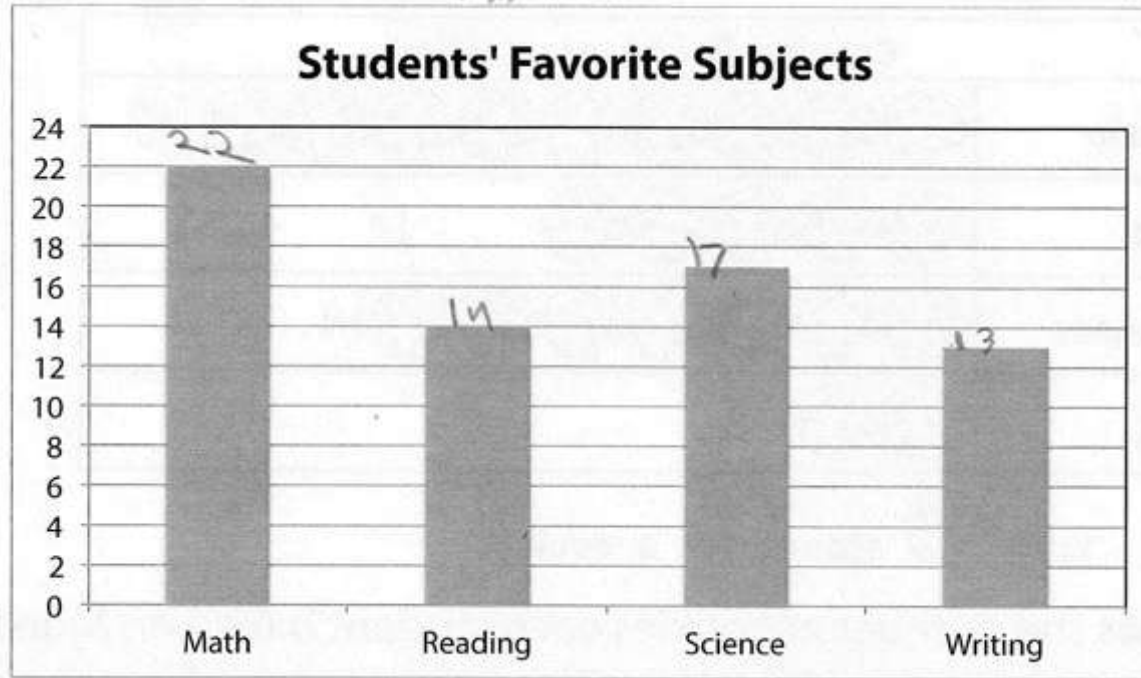
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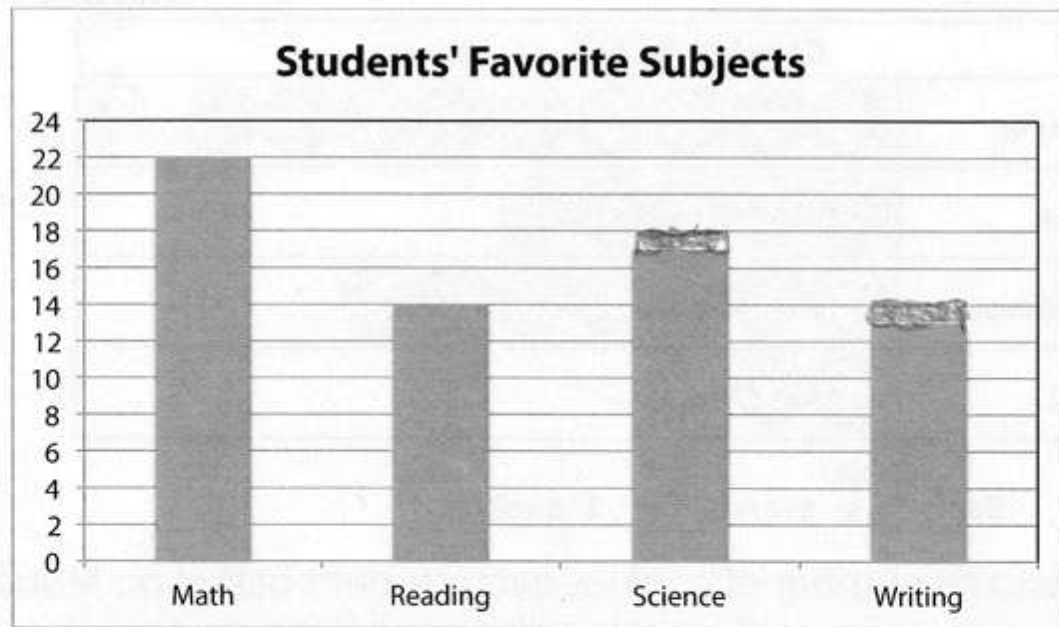
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love more math d'can es
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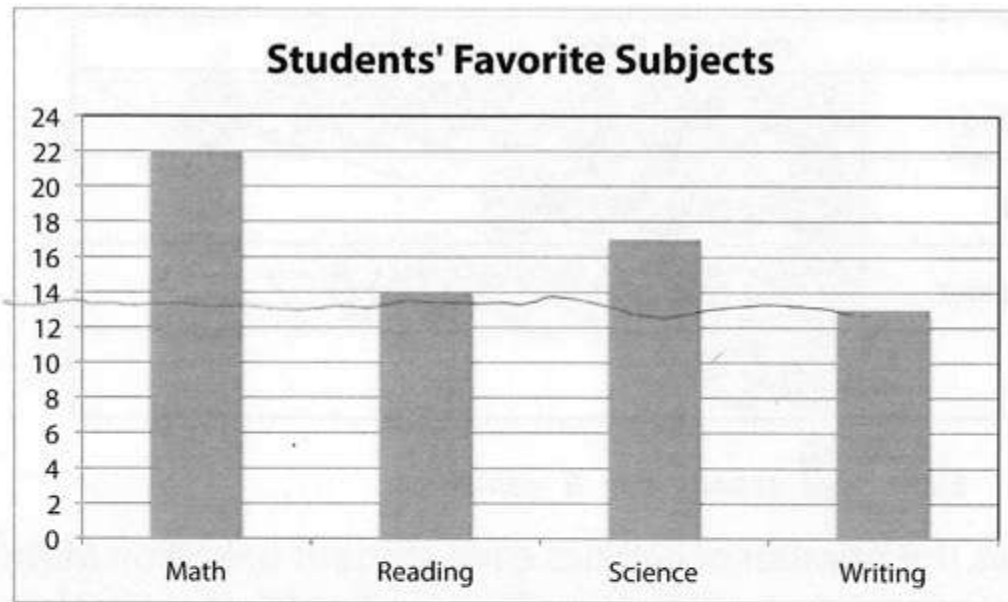
$$\begin{array}{r} 22 \\ + 13 \\ \hline 35 \end{array}$$



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 24 \\ + 14 \\ \hline 36 \end{array}$$

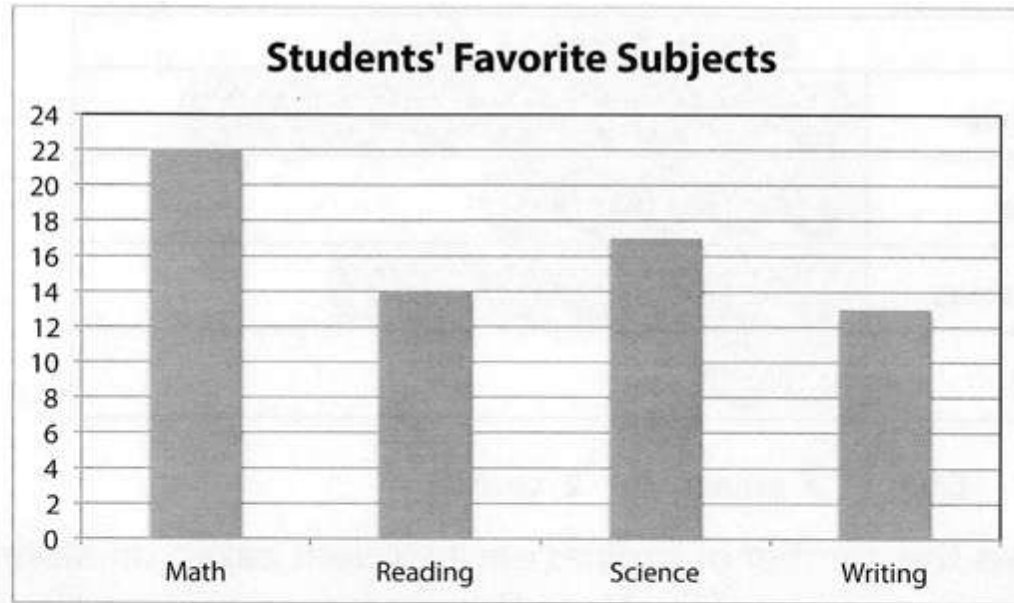
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$$\begin{array}{r} 24 \\ + 13 \\ \hline 37 \end{array}$$

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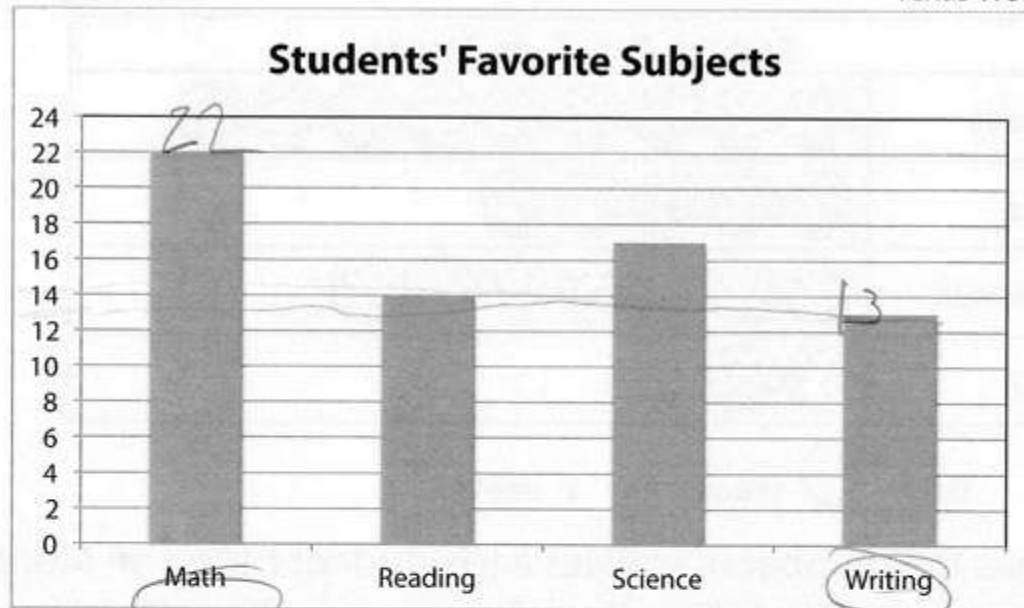


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$

34

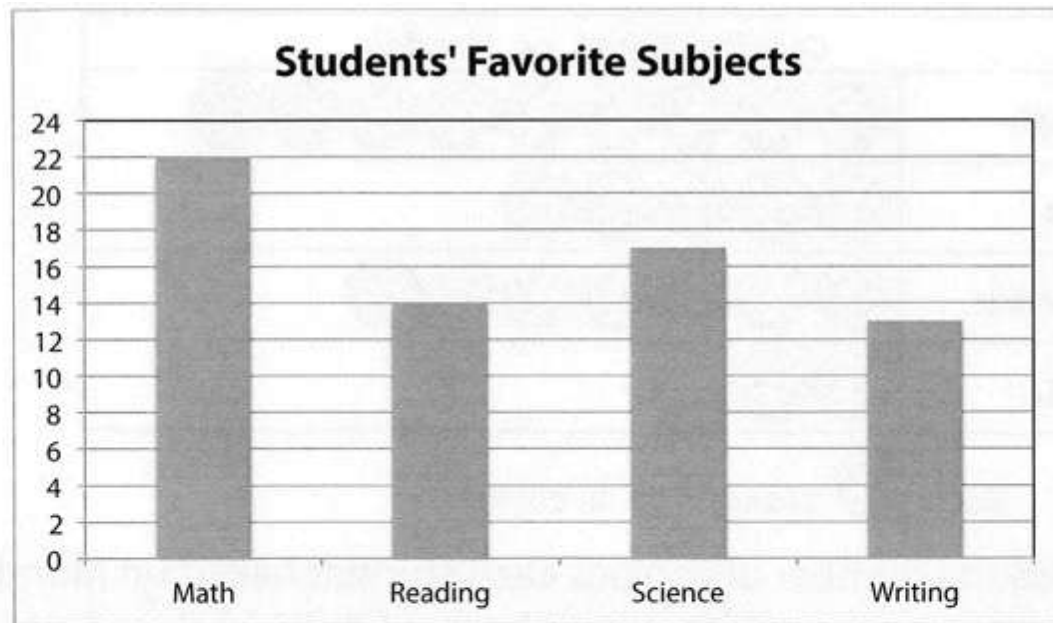
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 \hline
 35
 \end{array}$$

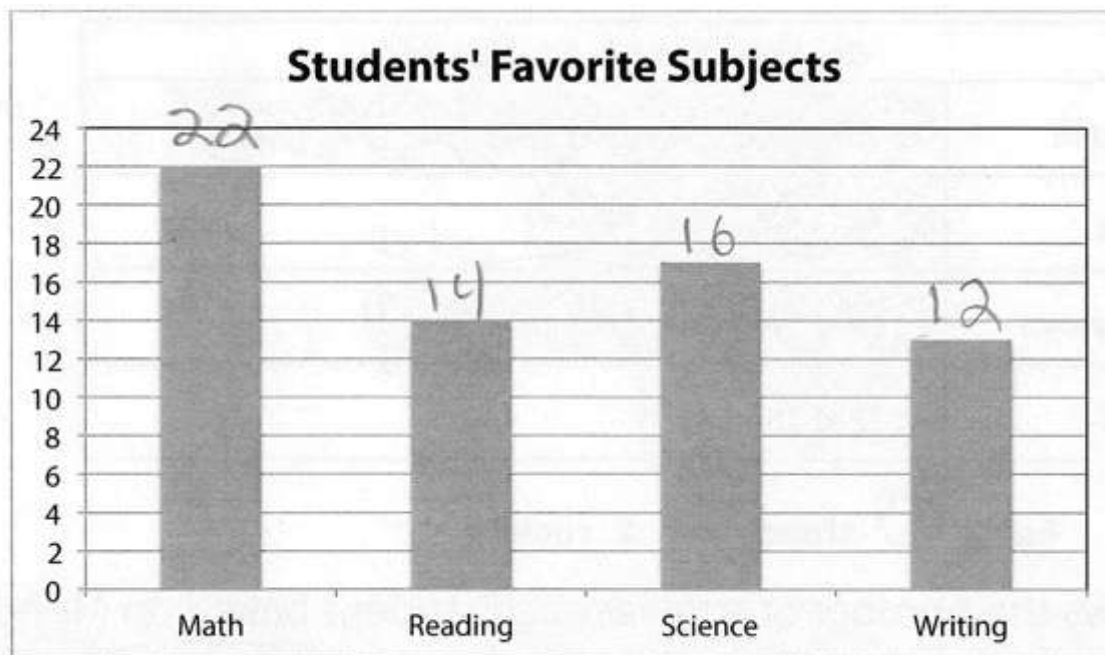
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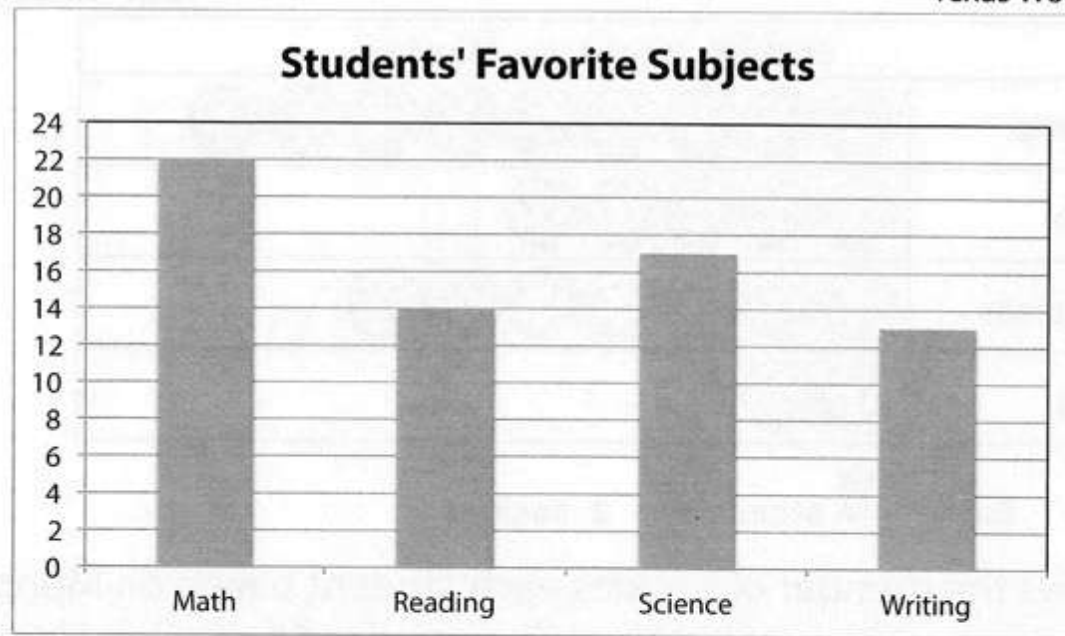
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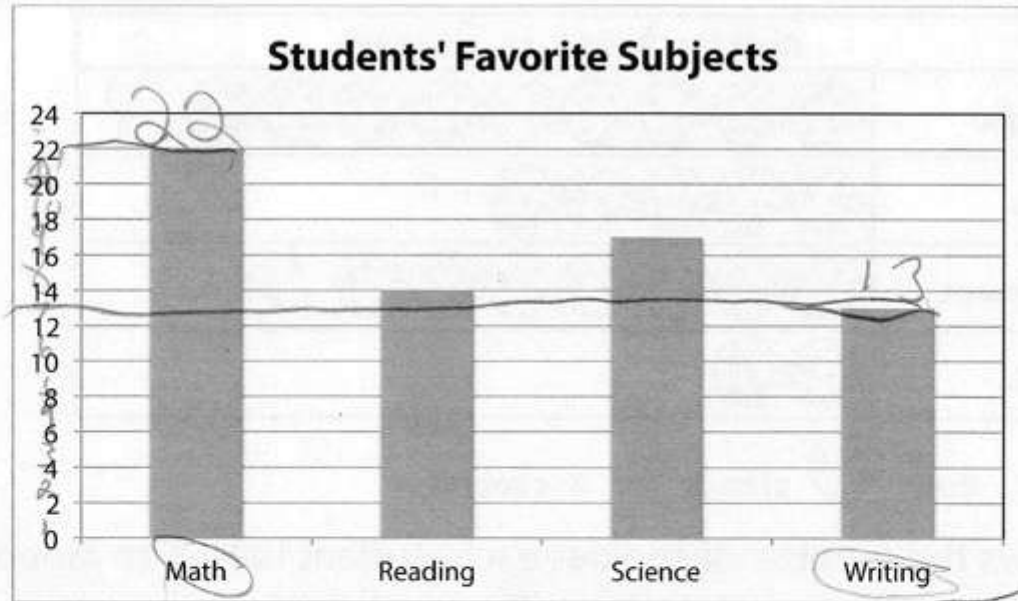
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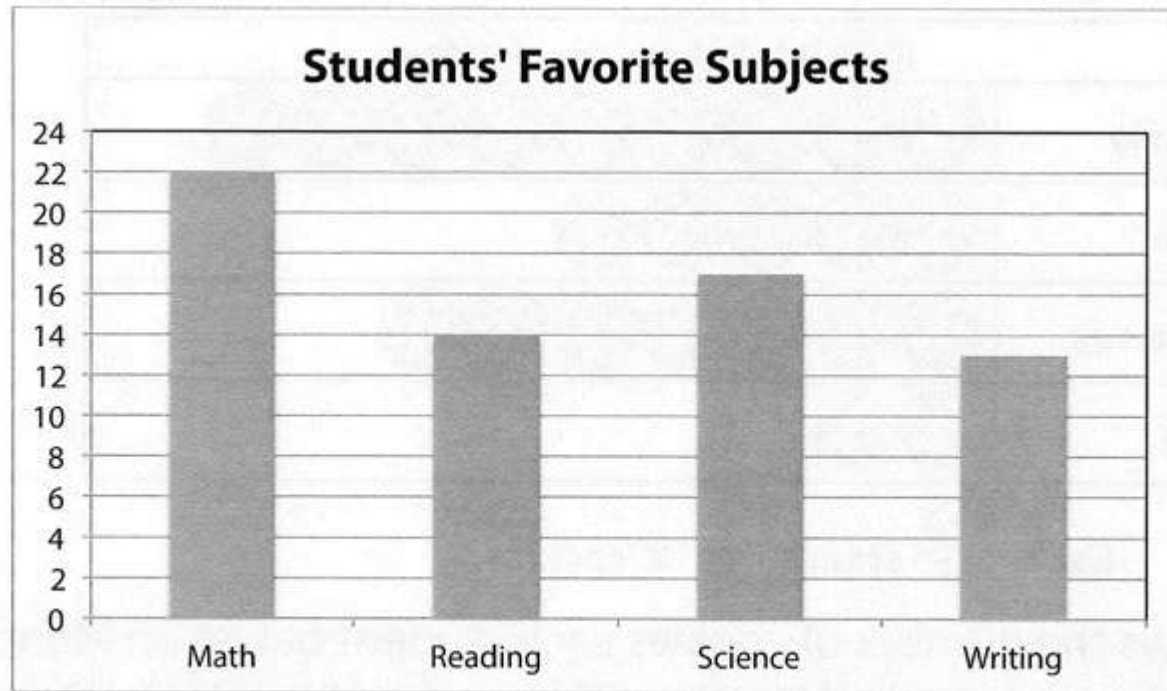


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COMMON

Undetermined

Repeated
information from
problem

Didn't provide
answer

Wrong
information

Wrong
(misapplied)
operation

UNCOMMON

Addition error

Subtraction error



Word Problems? No Problem!

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Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

Solve the problem

What skills are necessary to solve this problem?



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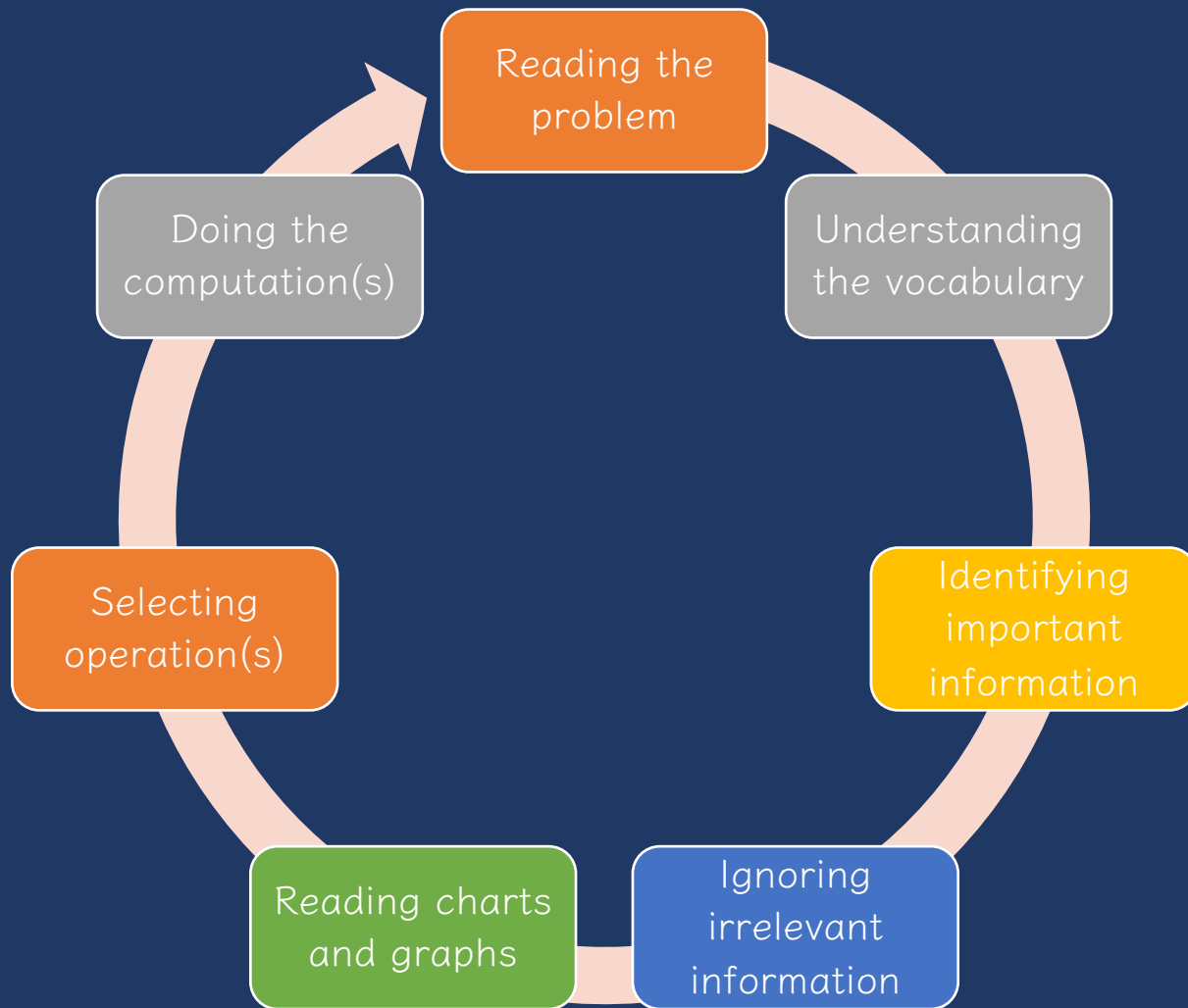


How would you solve this problem?
What skills are necessary to solve this problem?



Word-Problem Solving

Teaching Problem Solving



Ineffective Strategies





1. Keywords tied to operations



Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> add all together or altogether and both combined how many in all how much in all increased by plus sum together total 	<ul style="list-style-type: none"> change decreased by difference fewer or fewer than how many are left (or have left) how many did not have how many how much taller, less less or less lost minus need to reduce remain subtract take away

Multiplication Words	Division Words
<ul style="list-style-type: none"> by (dimension) double each group every factor of increased by multiplied by of product times triple 	<ul style="list-style-type: none"> as much cut up each group equal share half (or of) how many parts per percent quotient ratio of separated share son

OPERATION clue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> and total join more than in all altogether sum increased 	<ul style="list-style-type: none"> less than remaining difference take away fewer

MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times all by of equal groups 	<ul style="list-style-type: none"> quotient each broken into distribute evenly parts

KEY Words

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> sum total both combined increased by more than plus 	<ul style="list-style-type: none"> product double times per every each

SUBTRACTION	DIVISION
<ul style="list-style-type: none"> difference remain left less than minus how many more 	<ul style="list-style-type: none"> quotient divide by into split out of

Division

Taking apart

Addition

Putting two or more things amounts together

Keywords: Total, Altogether, In all, Sum, more than, increased by

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter left less than minus remain take away

key words

addition	subtraction	multiplication	division
<ul style="list-style-type: none"> sum both more than 	<ul style="list-style-type: none"> in all together perimeter total plus add 	<ul style="list-style-type: none"> factor multiply each per in all multiple area double 	<ul style="list-style-type: none"> quotient split divide half shared equally each average distribute

Key Words for All Operations

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than give/take away decreased by how many more show much longer/smaller/shorter minus remaining 	<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

Math Operation - Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than give/take away decreased by how many more show much longer/smaller/shorter minus remaining 	<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> plus sum add total all together increase more combine 	<ul style="list-style-type: none"> subtract minus difference left left over decrease take away fewer 	<ul style="list-style-type: none"> times product factor double groups each area rows 	<ul style="list-style-type: none"> quotient split share divide separate each average equal groups







Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.



Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.

Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about
keywords
("What does
more than tell
you about?")



But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 3 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 3 more cookies. How many cookies did LaMonte have in all?

Division Word Problems

1. Zookeeper Al has 567 bananas. He has 23 monkeys at the zoo and each monkey gets 24 bananas. How many bananas does each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will go in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 59 pages to each of his 23 students. She wants to give 59 pages to each of his 23 students. How many extra pages will he have left over?
4. Mr. King has 1376 pages of scrap paper. He instead wants to give 59 pages to each of his 23 students. Will there be enough scrap paper for all the students. How much more scrap paper does he need?

Effective Strategies



Teach an attack strategy

Teach about schemas



Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20



What was your process for working through this problem?



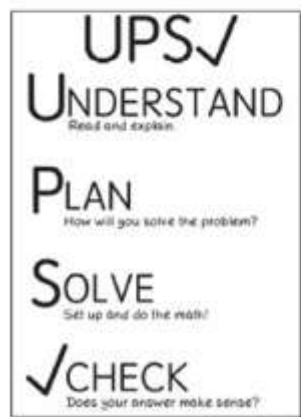
Attack Strategy

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.



SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.



UPS✓
UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?

Created by Sarah Powell (spowell@unicon.net)





Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Total

Additive Word Problems

A.

Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks.
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats
were kayaks, how many were canoes?

Part



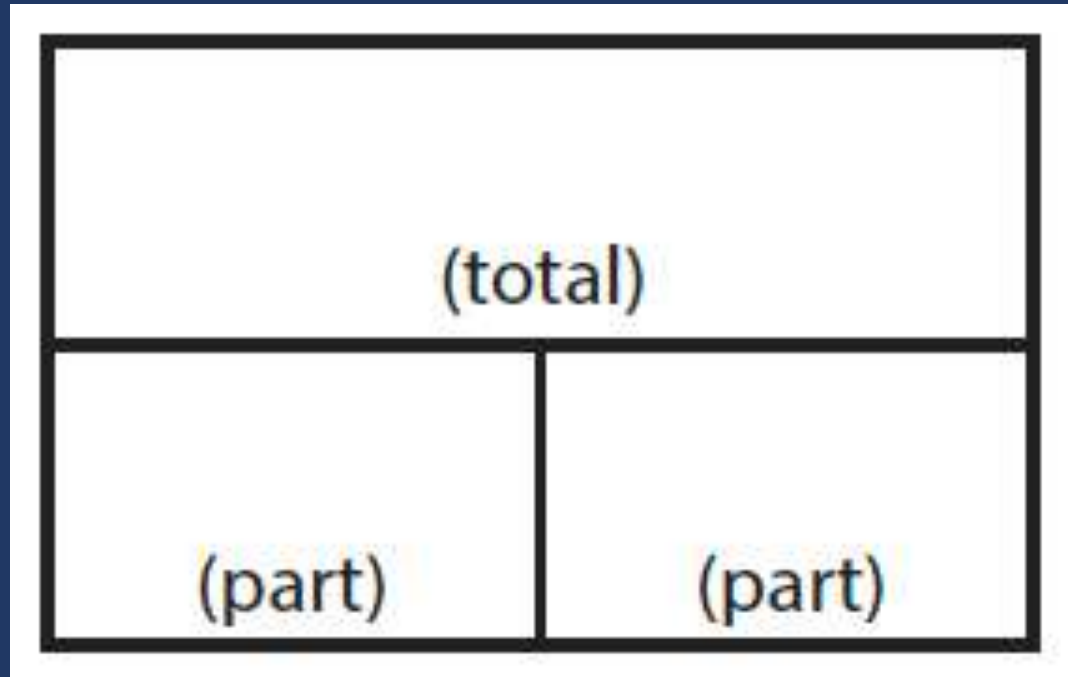
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Total

Additive Word Problems

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NOTES ABOUT TOTAL PROBLEMS:



Total

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

U✓

P✓

S✓

✓✓

$$P1 + P2 = T$$

$$3.9 + ? = 11.4$$

$$? = 7.5 \text{ inches}$$



Total



Share a Total problem.



Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils.
How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater
amount

Tracy has **6** fewer pencils than Adrianna.
Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser
amount



Total

“Are parts put together for a total?”

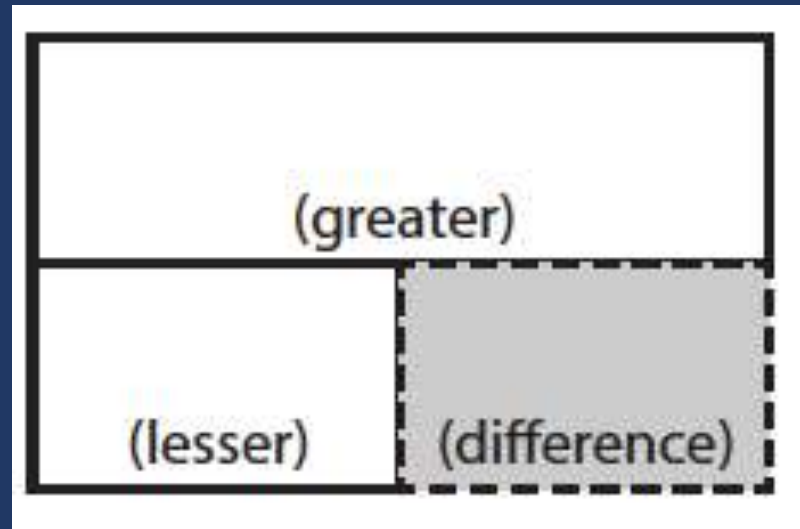
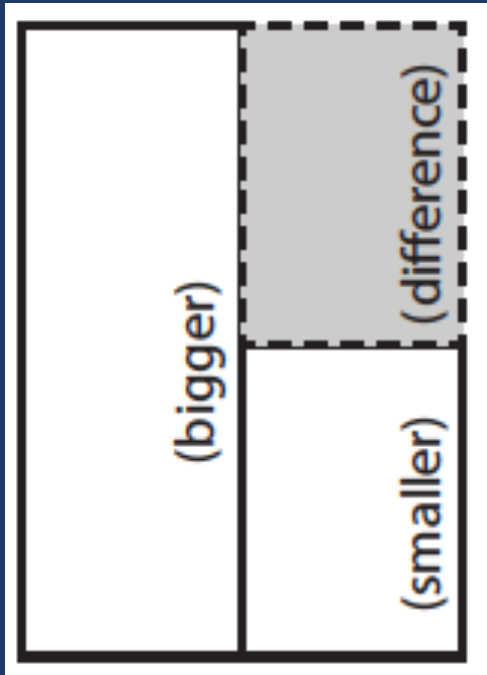
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems	
D. Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?	E. Damian's dog eats $5\frac{1}{2}$ cups of dog food each week. Monte's dog eats $4\frac{1}{2}$ cups more each week than Damian's dog. How much does Monte's dog eat in a week?
F. The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?	NOTES ABOUT DIFFERENCE PROBLEMS:



Difference

Additive Word Problems

D.

Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

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F.

The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

NOTES ABOUT DIFFERENCE PROBLEMS:



Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

Enter your answer in the response box.

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
0	.	$\frac{\Box}{\Box}$

U

P

S

✓

$$G - L = D$$
$$107 - 68 = ?$$

? = 39 more
wooden beads

Difference



Share a Difference problem.

Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change
amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start
amount



Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change
amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start
amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

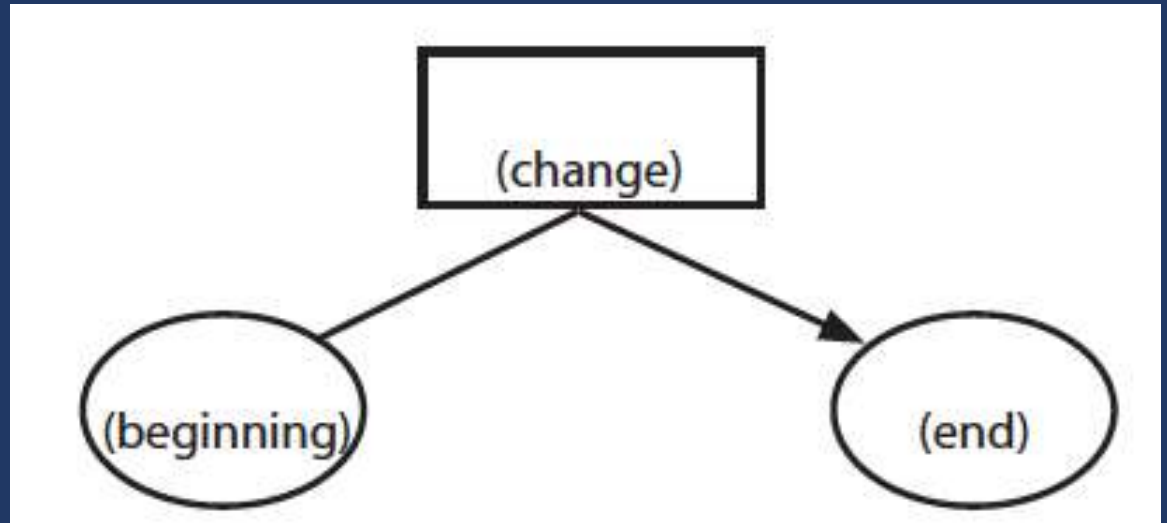
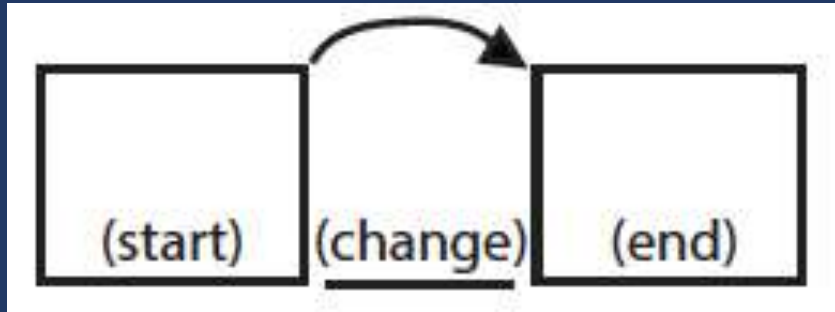
Change

“Does an amount increase or decrease?”



Change

$$ST + / - C = E$$



Change

Additive Word Problems	
<p>G. A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?</p>	<p>H. Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?</p>
<p>I. Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?</p>	<p>NOTES ABOUT CHANGE PROBLEMS:</p>



Change

Additive Word Problems

G.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

H.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

I.

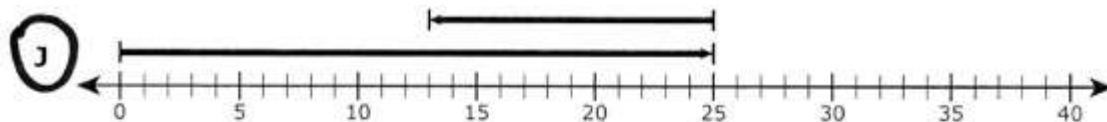
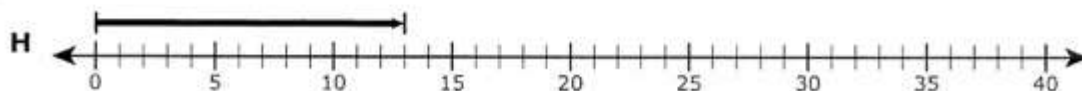
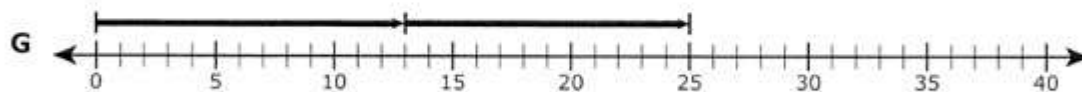
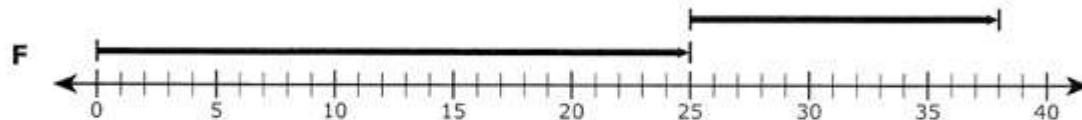
Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

NOTES ABOUT CHANGE PROBLEMS:



Change

- 28 There were 25 people in a library. Some people left the library and went home. Then there were 13 people remaining in the library. Which number line represents one way to determine the number of people who left the library?



U
P
S
✓

$$\boxed{25} - ? = \boxed{13}$$

? = 12 people left

Change



Share a Change problem.



Schema Check!



Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

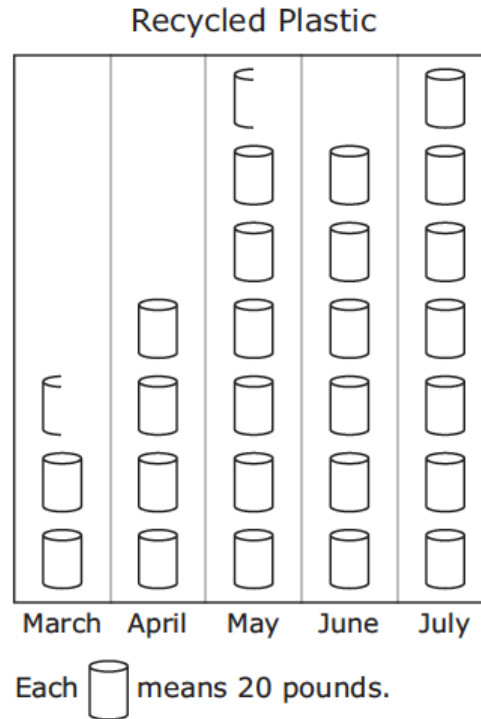
26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Total

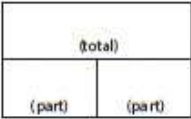
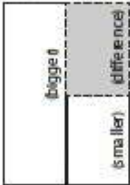
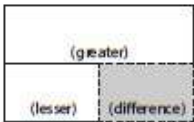

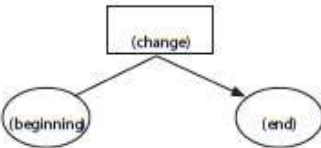
Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- Ⓐ 300
- Ⓑ 340
- Ⓒ 350
- Ⓓ 360

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay, Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST +/- C = E$ (start +/- change = end)  	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	

Total

Difference

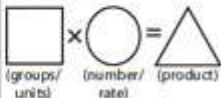
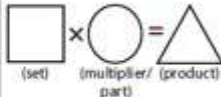
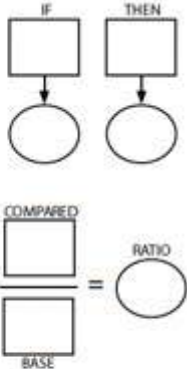
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units	$GR \times N = P$ 	Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	$S \times T = P$ 	Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Ratios/Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPoi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.



Equal Groups

Array
Vary

Groups multiplied by **number in each group**
for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in
each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



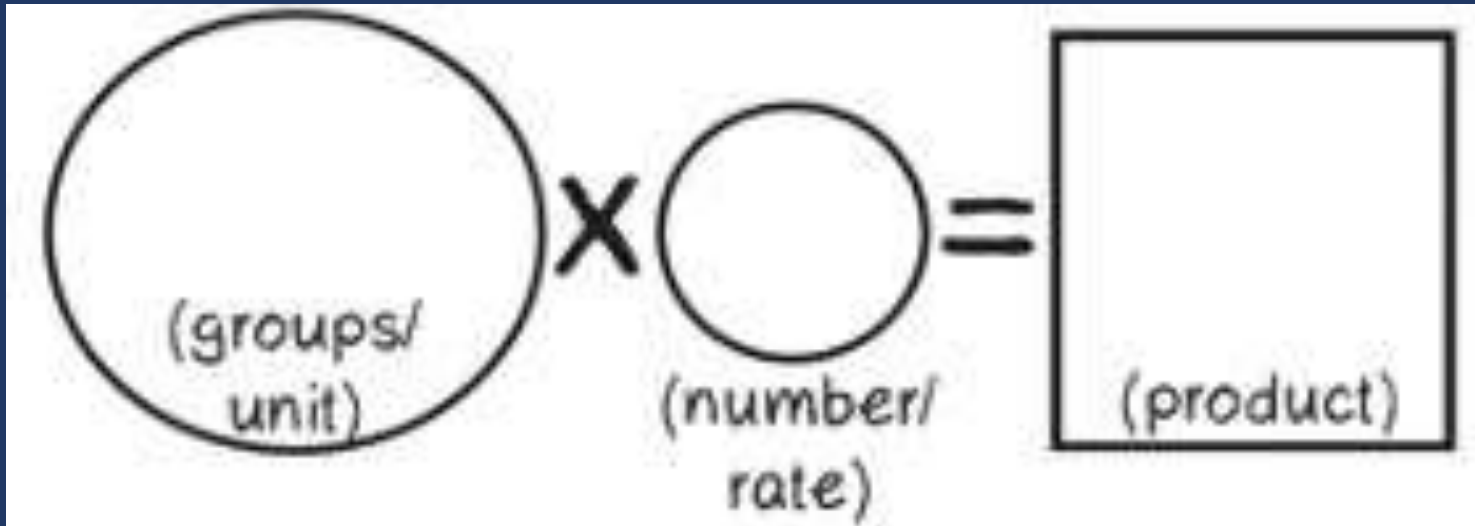
Equal Groups

“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N(E) = P$$



Equal Groups

Multiplicative Word Problems	
A. Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?	B. Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?
C. Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5\frac{1}{4}$ inches of string. How many bracelets could Zachary make?	NOTES ABOUT EQUAL GROUPS PROBLEMS:



Equal Groups

Multiplicative Word Problems

A.

Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.

Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.

Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5 \frac{1}{4}$ inches of string. How many bracelets could Zachary make?

NOTES ABOUT EQUAL GROUPS PROBLEMS:



Equal Groups

Jack has 24 fish. He puts them into 4 bowls. Each bowl has an equal number of fish.

How many fish are in each bowl?

Handwritten student work on a digital math interface:

On the left is a numeric keypad with buttons for 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, a decimal point, and a fraction template button.

Handwritten text on the left side of the work area reads: U, P, S, and a checkmark (✓).

The main work area contains the following:

- A multiplication equation: $\boxed{4} \times \textcircled{?} = \triangle 24$
- A division problem: $\begin{array}{r} 4 \\ \times ? \\ \hline 24 \end{array}$
- A long division problem: $4 \overline{)24}$
- The final answer: $? = 6 \text{ fish}$

Equal Groups



Share an Equal Groups problem.

Comparison

Set multiplied by a number of **times**
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4** times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

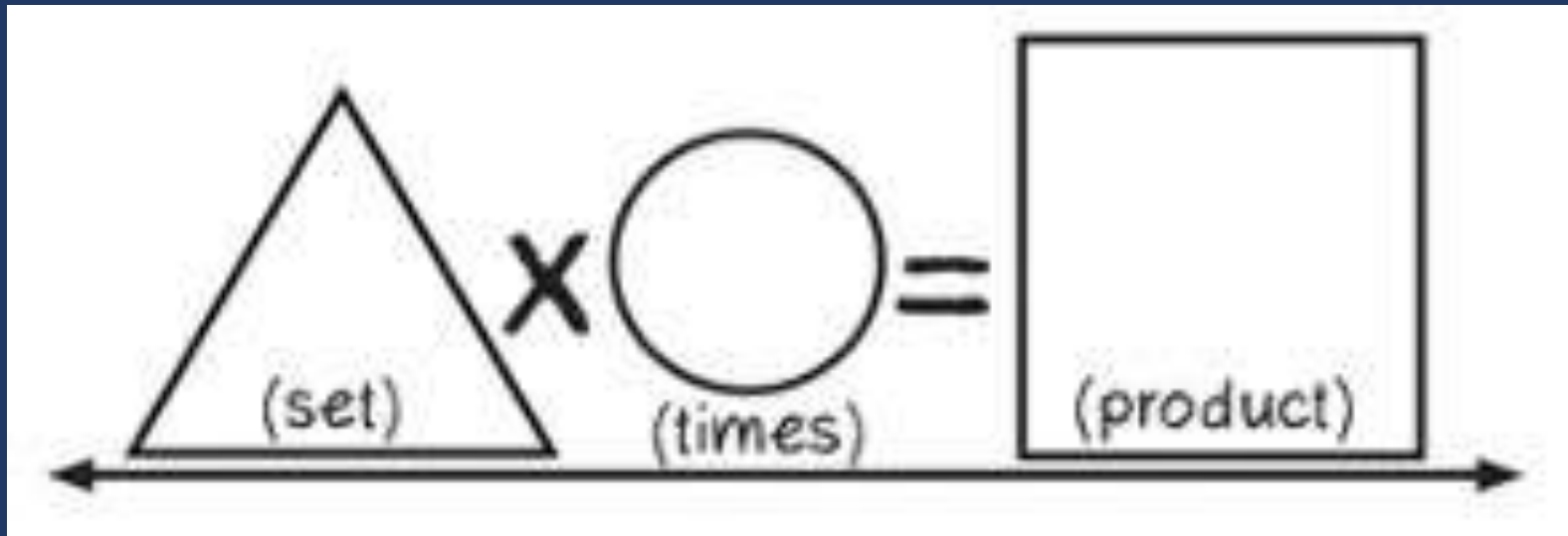
Comparison

“Is a set compared a number of times?”



Comparison

$$S \times T = P$$



Comparison

Multiplicative Word Problems	
D. Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?	E. Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?
	NOTES ABOUT COMPARISON PROBLEMS:
F. Molly ran 2 miles in 18 minutes. At this rate, how many miles could Molly run in 90 minutes?	G. An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

Comparison

Multiplicative Word Problems

D.
Enrique has 2 times as many pencils as Ava.
Ava has 6 pencils. How many pencils does Enrique have?

E.
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?

NOTES ABOUT COMPARISON PROBLEMS:

F.
Molly ran 2 miles in 18 minutes. At this rate, how many miles could Molly run in 90 minutes?

G.
An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?



Comparison

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

(A) $\square - 3 = 18$

(B) $3 + 18 = \square$

(C) $18 \div \square = 3$

(D) $3 \times 18 = \square$

U
P
S
✓

$$\boxed{18} \times \textcircled{3} = \triangle ?$$

$$? = 54 \text{ books}$$

Comparison



Share a Comparison problem.

Ratios/Proportions

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

Equal Groups

“Are there groups with an equal number in each group?”

Comparison

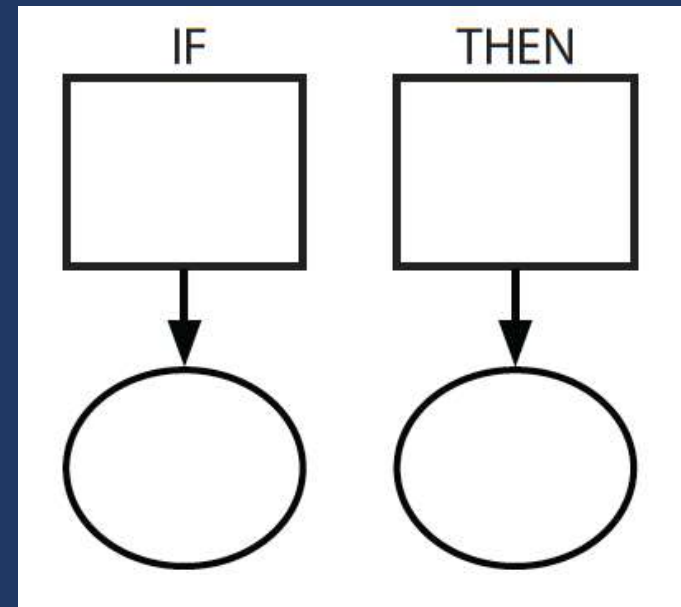
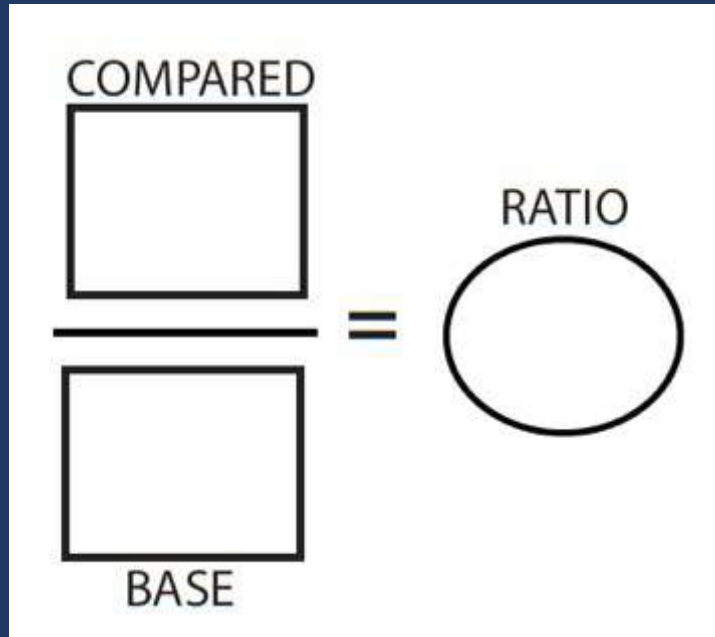
“Is a set compared a number of times?”

Ratios/Proportions

“Are there relationships among quantities – if this, then this?”



Ratios/Proportions



Ratios/Proportions

Multiplicative Word Problems	
<p>H. The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?</p>	<p>I. Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.</p> <p>NOTES ABOUT RATIOS/PROPORTIONS PROBLEMS:</p>
<p>J. Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?</p>	<p>K. Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?</p>



Ratios/Proportions

Multiplicative Word Problems

H.

The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

I.

Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

NOTES ABOUT RATIOS/PROPORTIONS PROBLEMS:

J.

Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?

K.

Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?



Ratios/Proportions



Share a Ratios or
Proportions problem.



Schema Check!



Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?	

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Teach an attack strategy

Teach about schemas



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?



Revisit this problem.
Discuss the schemas in
the problem.

Teach an attack strategy

Teach about schemas





Pirate Math Equation Quest

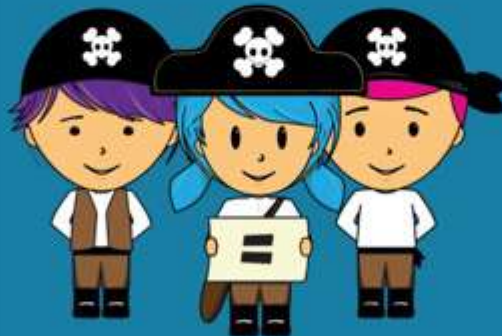
[About](#)[Research](#)[Individual](#)[Small Group](#)[STAAR](#)[Videos](#)

Welcome to Pirate Math Equation Quest!

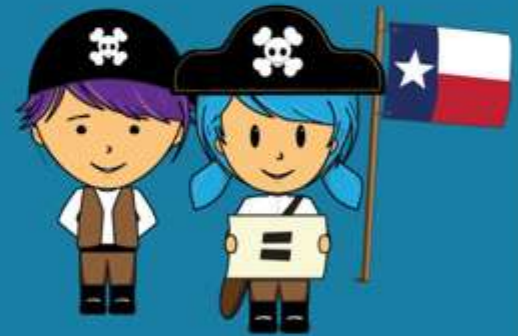
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

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Intervention ▾

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Implementation
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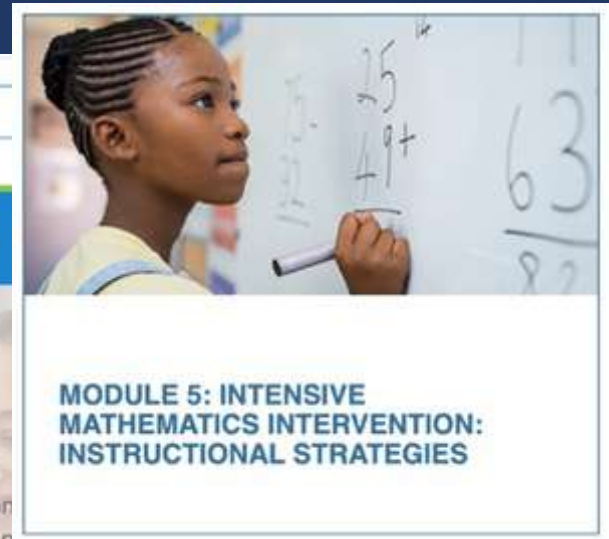
Information
For... ▾

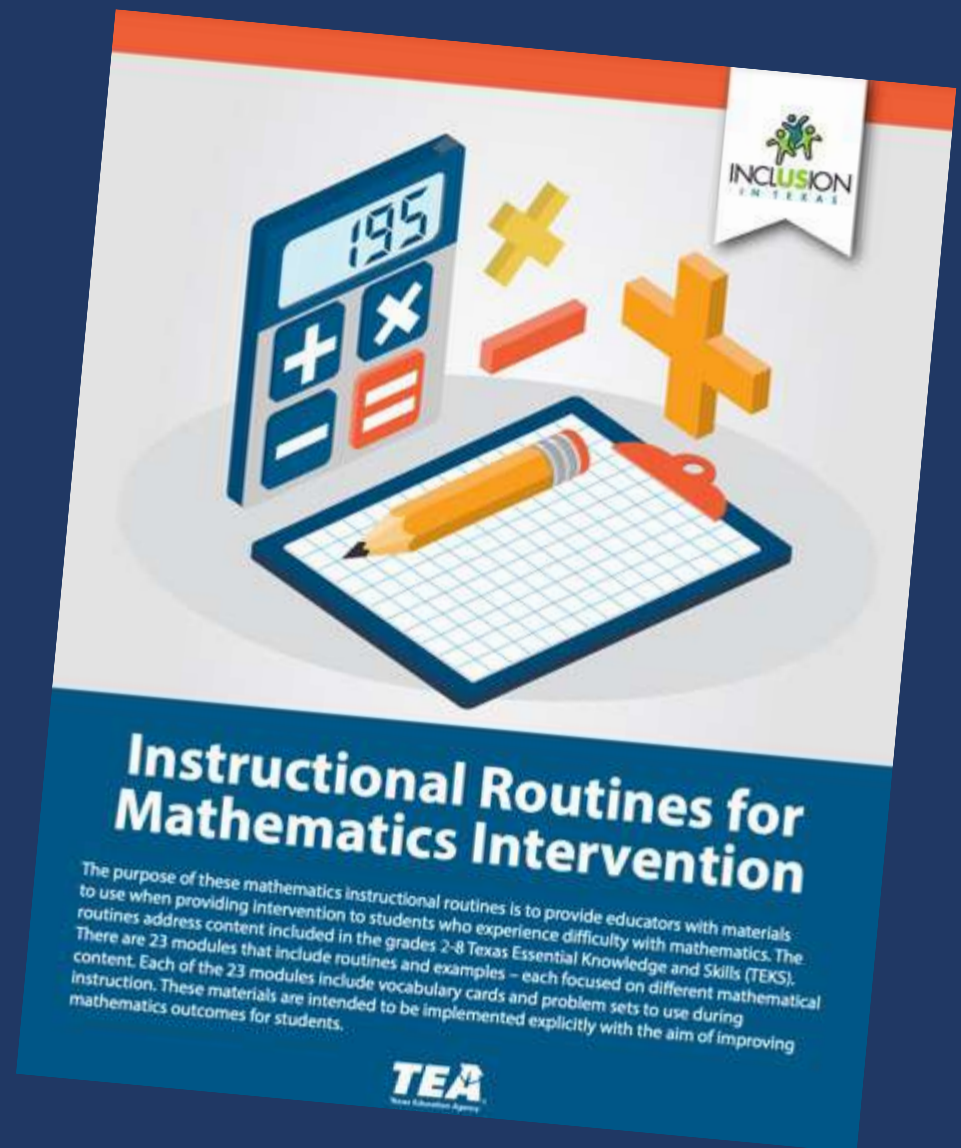
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

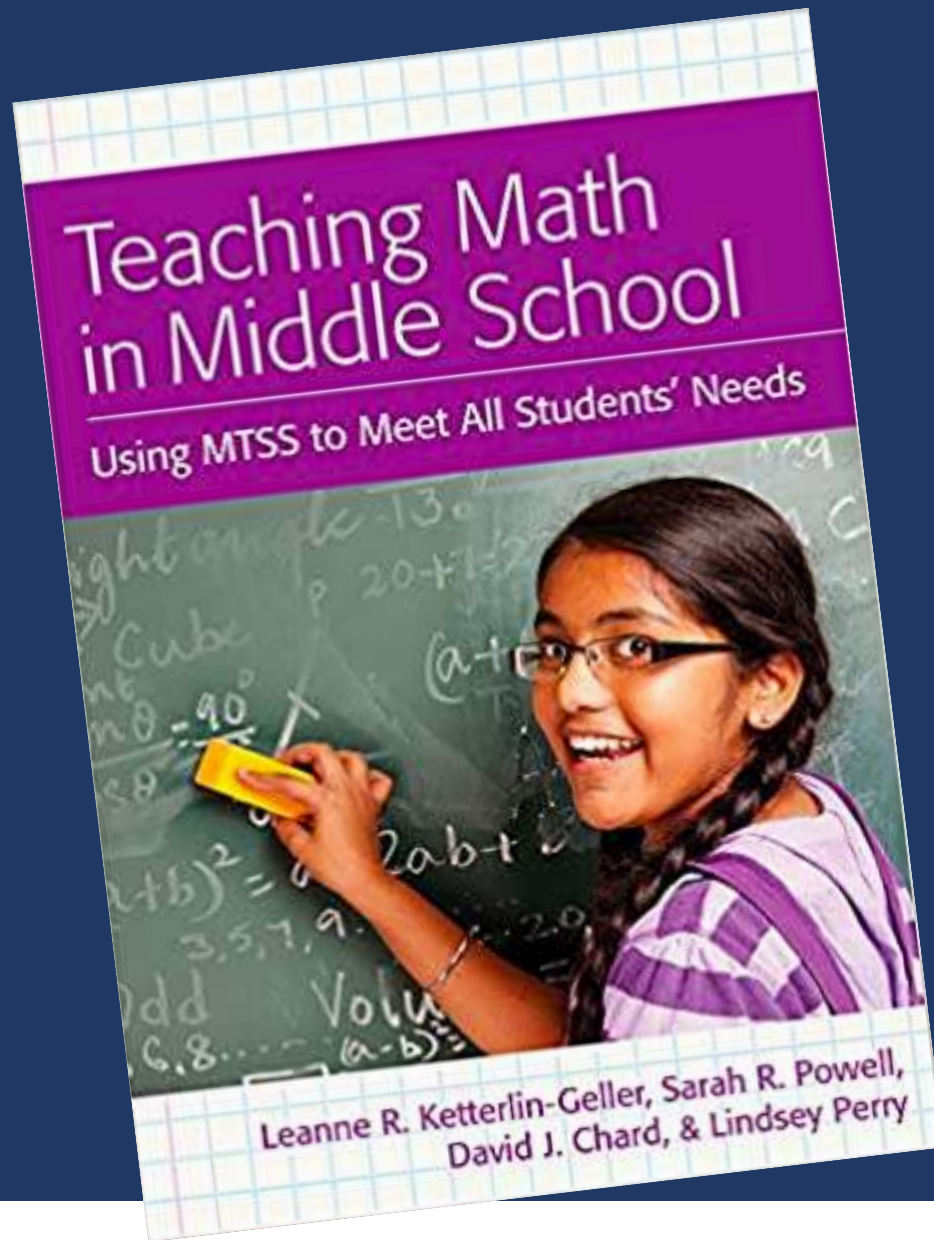
NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.





https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859





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