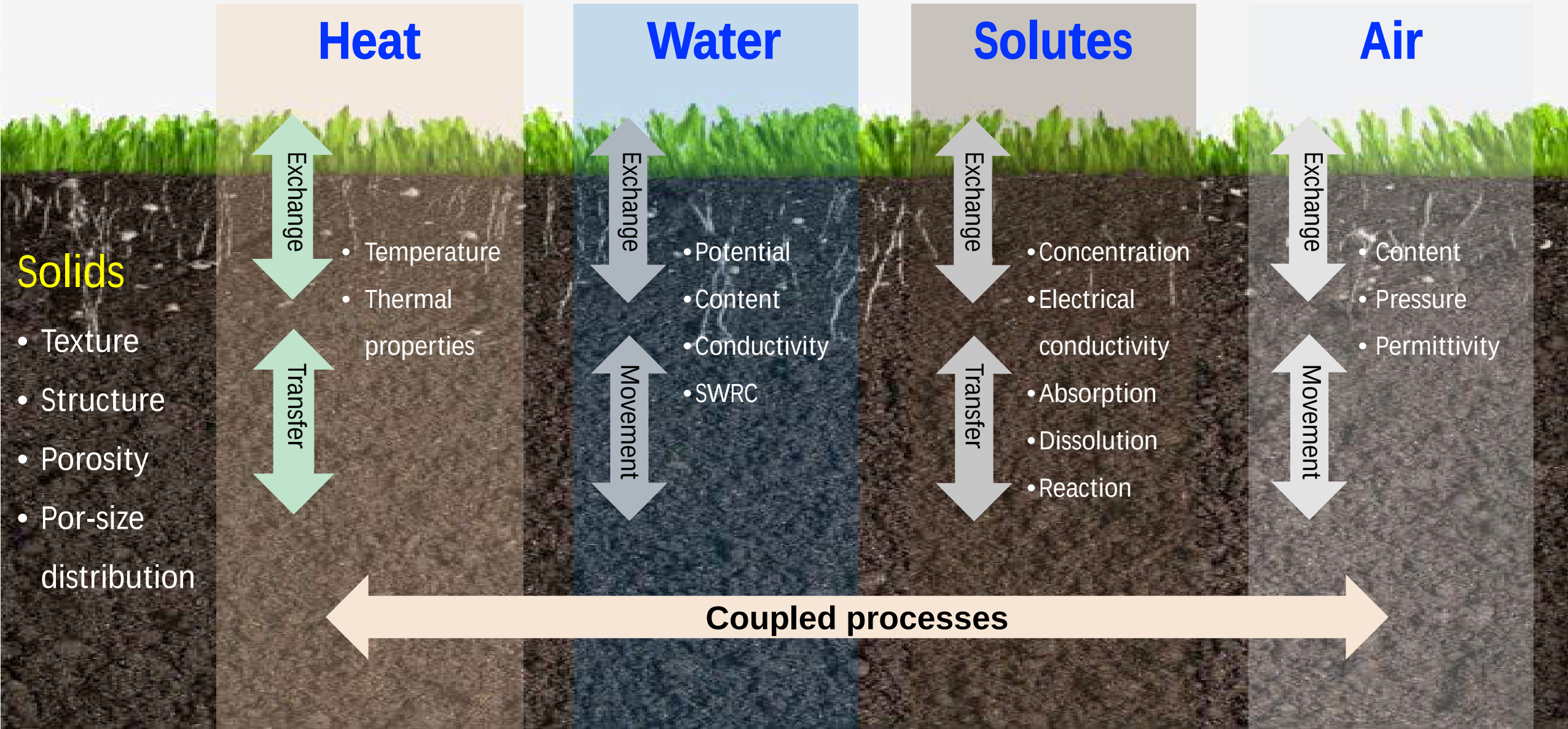


A thermo-TDR approach for simultaneous determination of transport parameters of sandy soils

Lin Liu, Yili Lu, Tusheng Ren
China Agricultural University

Robert Horton
Iowa State University

Soil Physical Properties & Processes



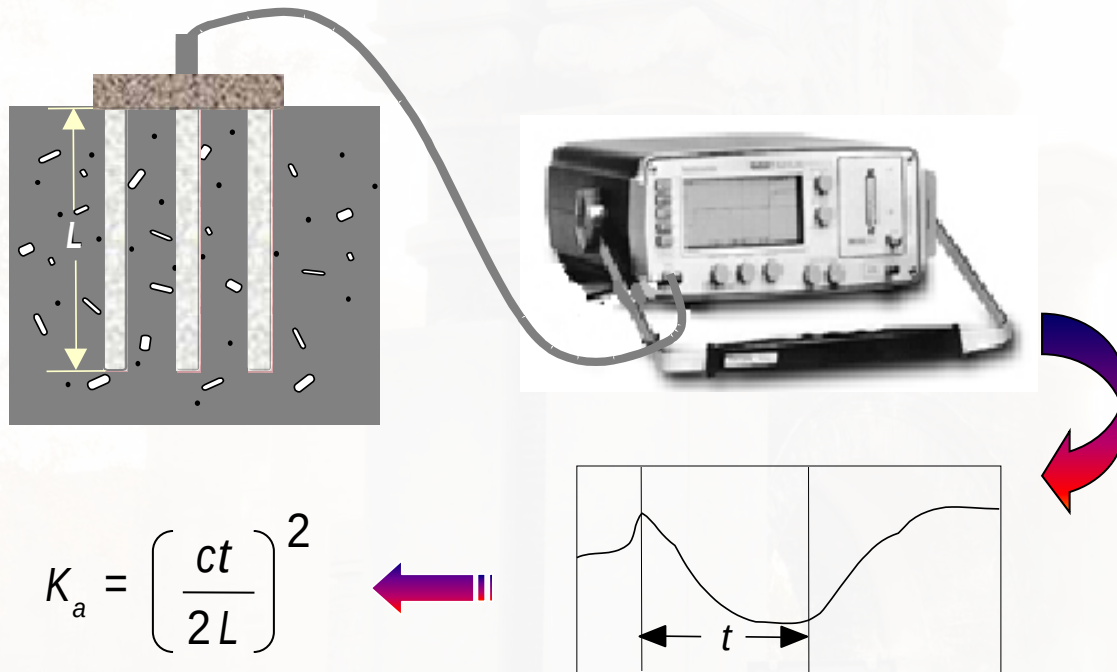


- ü **Thermo-TDR technology**
- ü **Thermo-TDR approach for monitoring soil water retention and gas diffusivity**
 - **Theory**
 - **Validation**



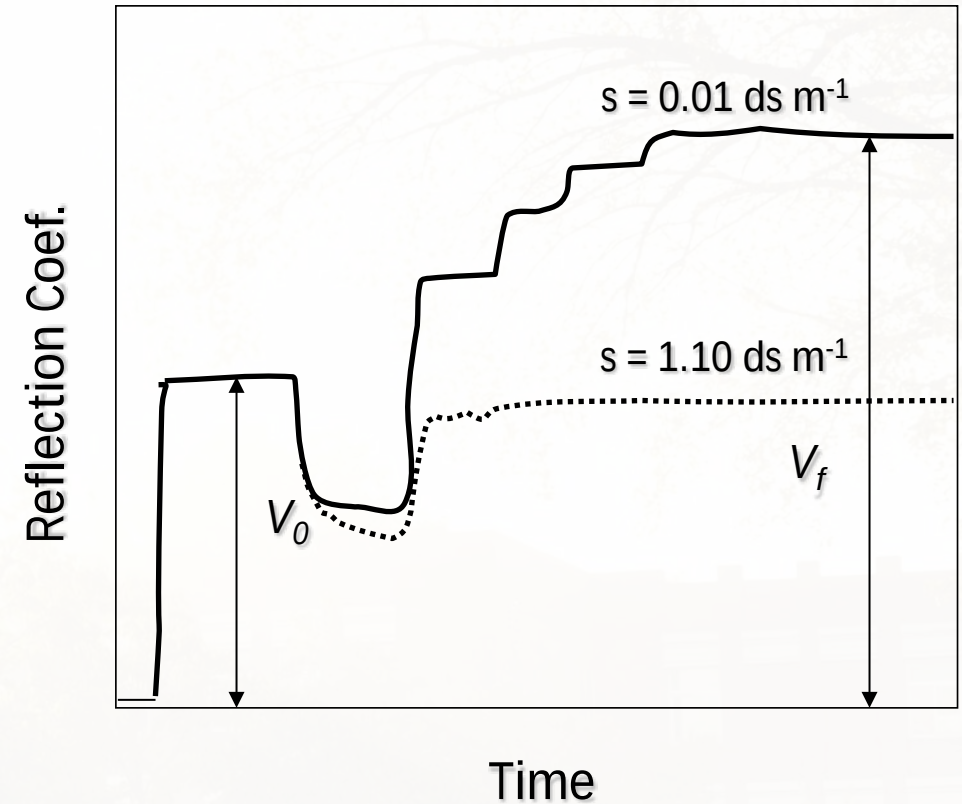
Thermo-TDR technology

TDR: monitoring soil water content and electrical conductivity



$$q = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} K_a - 5.5 \times 10^{-4} K_a^2 + 4.3 \times 10^{-6} K_a^3$$

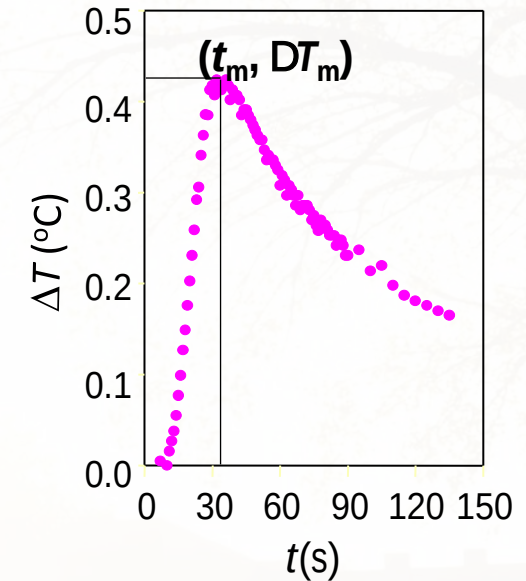
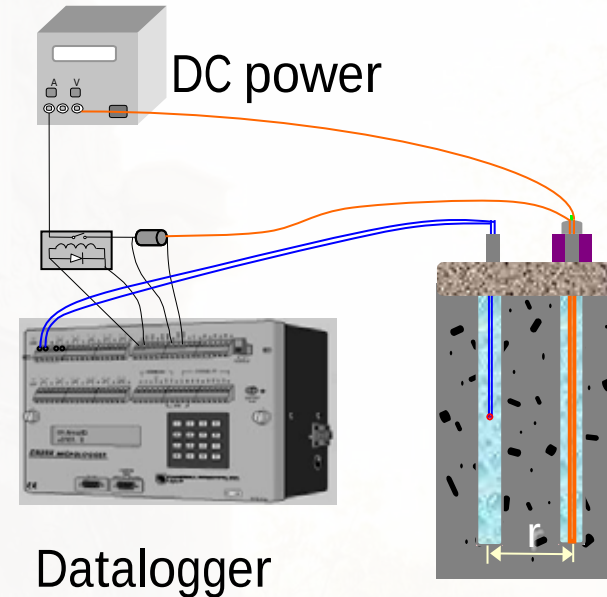
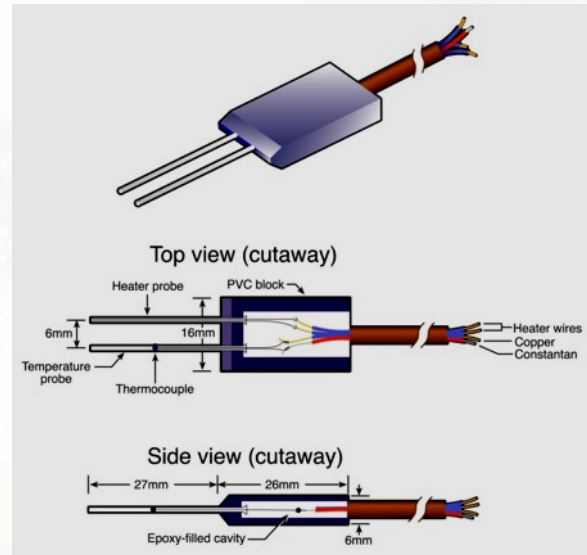
Topp et al. 1980. WRR. 16:579-582



$$\sigma = \frac{K_a}{Z_u} \left[\frac{2V_0}{V_f} - 1 \right]$$

Dalton et al. 1984. Science. 224:989-990

Heat pulse method: measuring soil thermal properties



$$C = \frac{q}{4\pi a T_m} \left[\frac{EiC}{EiC} - \frac{r^2}{4a(t_m - t_0)} \right] - \frac{EiC}{EiC} \frac{r^2}{4a t_m} \frac{\partial}{\partial t} \frac{\partial}{\partial t}$$

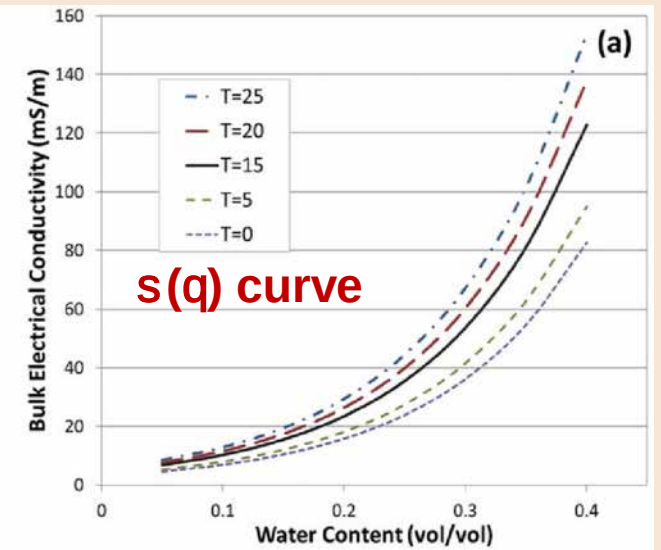
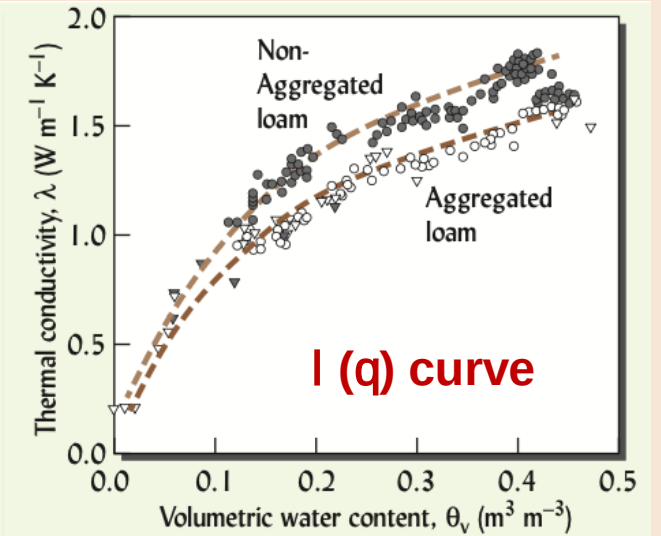
C: Volumetric heat capacity

$$\alpha = \frac{r^2}{4} \left[\frac{1}{E(t_m - t_0)} - \frac{1}{t_m} \right] / \ln \left[\frac{t_m}{E(t_m - t_0)} \right] \quad l = a' C$$

a: Thermal diffusivity

l : Thermal conductivity

Thermo-TDR technology: combining two units into one

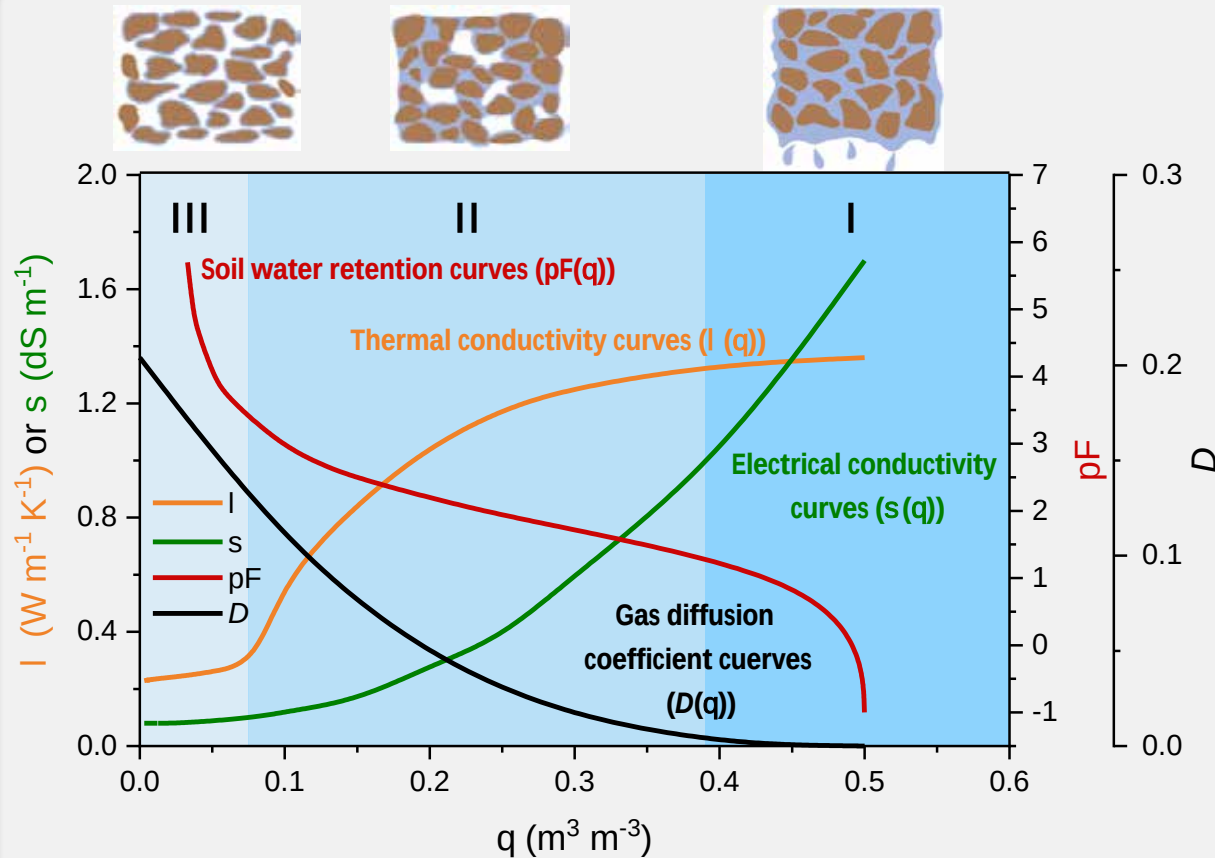


Ren et al (1999), Lu et al (2024)



Determining soil water retention and gas diffusivity: theory

Similarities between soil transport parameters



- The parameters have similarities in curve shapes;
- All curves are governed by water content and pore traits (determined by texture and bulk density).

ü Measuring $pF(q)$ & $D(q)$:

- Time and labor consuming
- Difficult to monitor in the field

ü Measuring $I(q)$ & $s(q)$:

- Relatively quick and simple
- In-situ monitored is possible with thermo-TDR sensors.

? How are the four curves correlated with each other?

? Is it possible to determine $pF(q)$ and $D(q)$ curves indirectly from easily measured $I(q)$ and $s(q)$ curves?

From I (q) curve to pF(q) curve

van Genuchten (1980)
Soil Sci Soc Am J

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{1 + (\alpha pF)^{1/(1-m)}} \bar{\psi}^{-m}$$

θ_s : saturated water content
 θ_r : residual water content
 α, m : Soil specific parameters

Inputs

- Texture
- Bulk density: r_b
- Particle density: r_s
- Measured I (q) curve
- Field capacity (pF_{fc}, q_{fc})

q_s $\theta_s = 1 - r_b / r_s$

q_r $\frac{\lambda - \lambda_{dry}}{\lambda_{sat} - \lambda_{dry}} = 1 - \frac{1}{1 + \frac{\alpha \theta}{\theta_f} \bar{\psi}^{1/(1-p)}} \bar{\psi}^{-p}$

Lu & Dong et al (2015)
J Geotech Geoenviron Eng

a, m $\frac{\theta_{fc} - \theta_r}{\theta_s - \theta_r} = \frac{1}{1 + a (pF_{fc})^{1/(1-m)}} \bar{\psi}^{-m}$

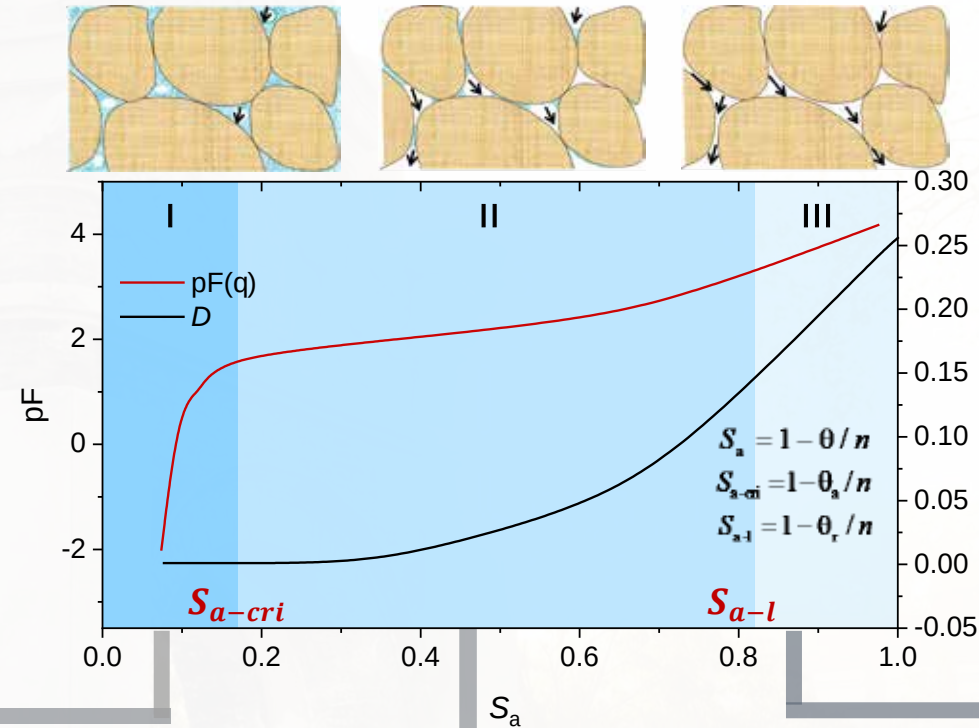
Liu et al (2024)
Soil & Till Res

From $h(q)$ curve to $D(q)$ curve

Section I ($0 < S_a \leq S_{a-cri}$):

- pF increases dramatically with S_a
- Air diffusion strongly restricted in limited pore space

$$D = 0.0001 \text{ (very low, constant)}$$



Section III ($S_a \geq S_{a-l}$):

- Most pore space for air diffusion
- pF & D increases linearly with S_a

$$D = D_{dry} - b(1 - S_a)$$

Section II ($S_{a-cri} < S_a < S_{a-l}$):

- More space for air diffusion
- pF, D increases exponentially with S_a

$$D = D_{dry} - b(1 - S_{a-l}) \left(\frac{S_a - S_{a-cri}}{S_{a-l} - S_{a-cri}} \right)^c$$

Determination of model parameters

$$D = \begin{cases} 0.0001 & \text{Section I} \\ [D_{dry} - b(1 - S_{a-l})] \left(\frac{S_a - S_{a-cri}}{S_{a-l} - S_{a-cri}} \right)^c & \text{Section II} \\ D_{dry} - b(1 - S_a) & \text{Section III} \end{cases}$$

Dividing points of degree of air saturation:

$$S_{a-cri} = 1 - \theta_a/n$$

$$S_{a-l} = 1 - \theta_r/n$$

Air-entry water content (q_a) and residue water content (q_r) are determined from the pF(q) curve.

Diffusion coefficient of dry soils D_{dry} :

$$D_{dry} = n^{X_{dry}}$$

X_{dry} is the particle shape factor, estimated by fitting the D_{dry} model to measured (n, D_{dry}) data.

Parameters b and c :

Determined by fitting the above $D(q)$ model to measured (S_a, D) data.



Determining soil water retention and gas diffusivity: validation

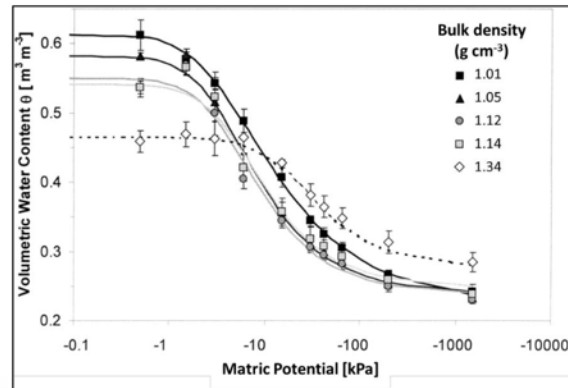
Measurement on repacked soil columns

Sand: sand - 94%, clay - 5%

r_b : 1.40, 1.50, 1.60 Mg m⁻³

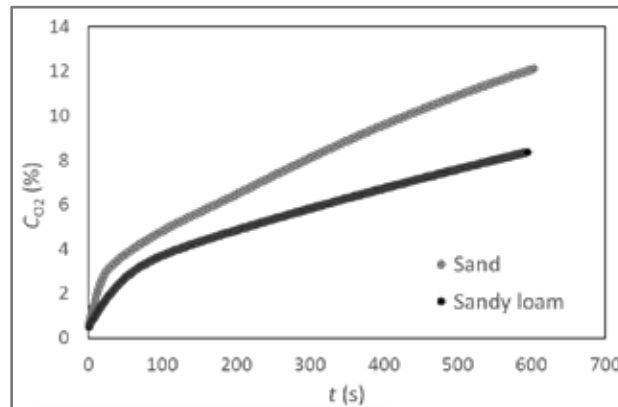
Sandy loam: sand - 39%, clay - 1%

r_b : 1.35, 1.45, 1.55 Mg m⁻³



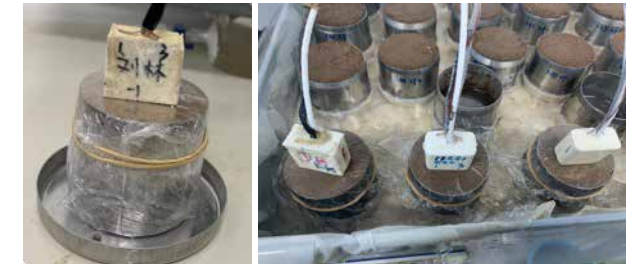
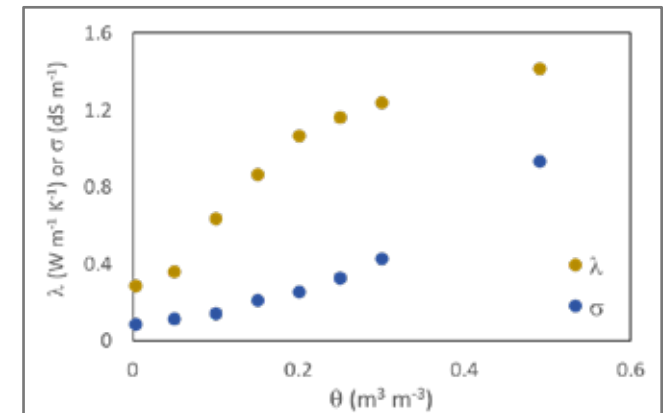
$h(q)$

Sand box, pressure plate



$D(q)$

One-chamber method



$I(q)$, $s(q)$

Thermo-TDR technology

Results: thermo-TDR Measured $I(q)$, $pF(q)$, $D(q)$ curves

Thermal conductivity curve: $I(q)$

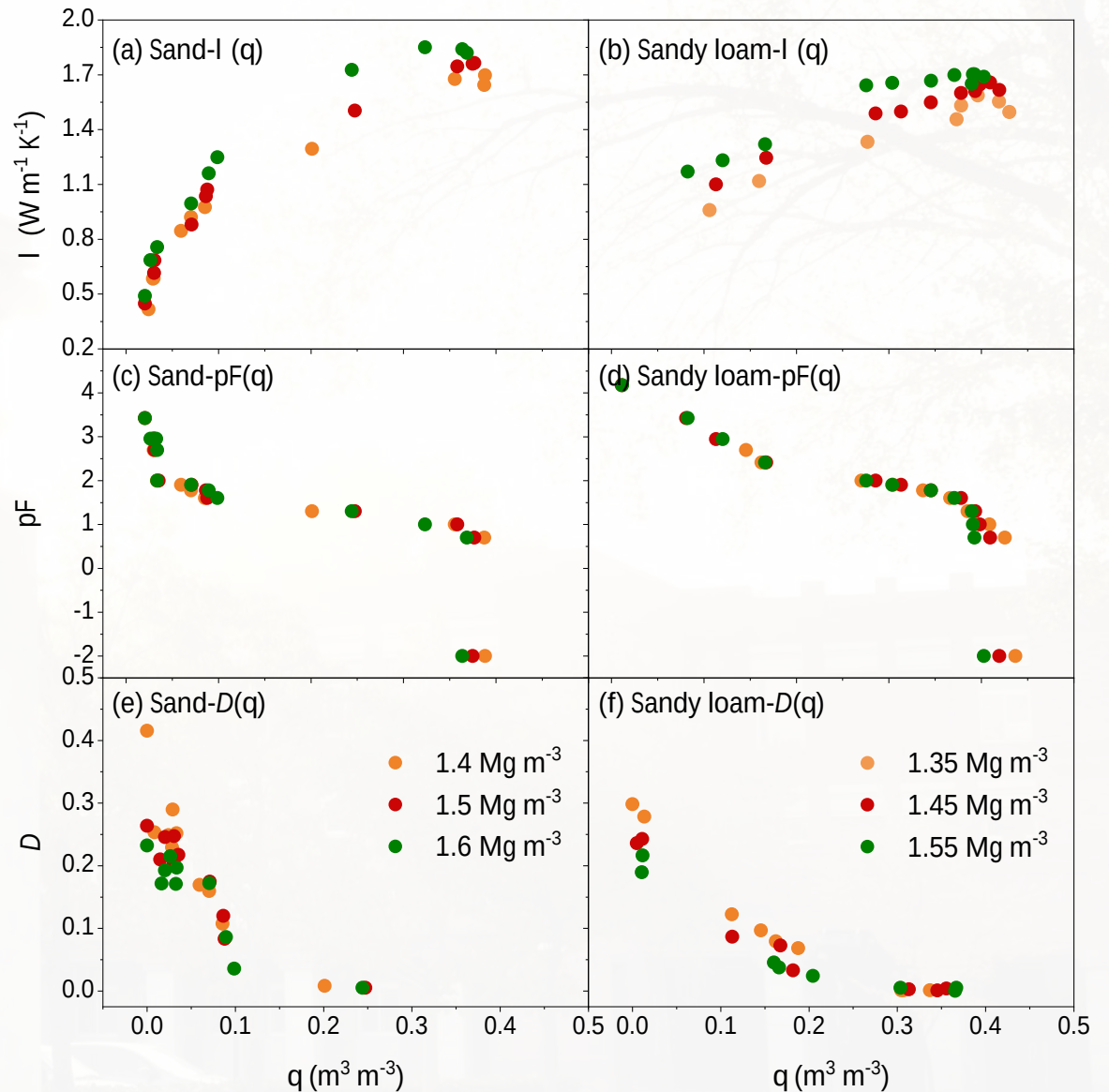
- I increases with both sand content and r_b
- At equivalent q , $I_{\text{sand}} > I_{\text{sandy loam}}$

Water retention curve: $pF(q)$

- Sand shows lower water retention capacity;
- r_b : slightly improves water retention, more pronounced in sandy loam.

Gas Diffusivity: $D(q)$

- Sand requires higher air content (lower q) to activate dominant air-phase diffusion;
- D decreases at greater r_b values, more severe in sandy loam.



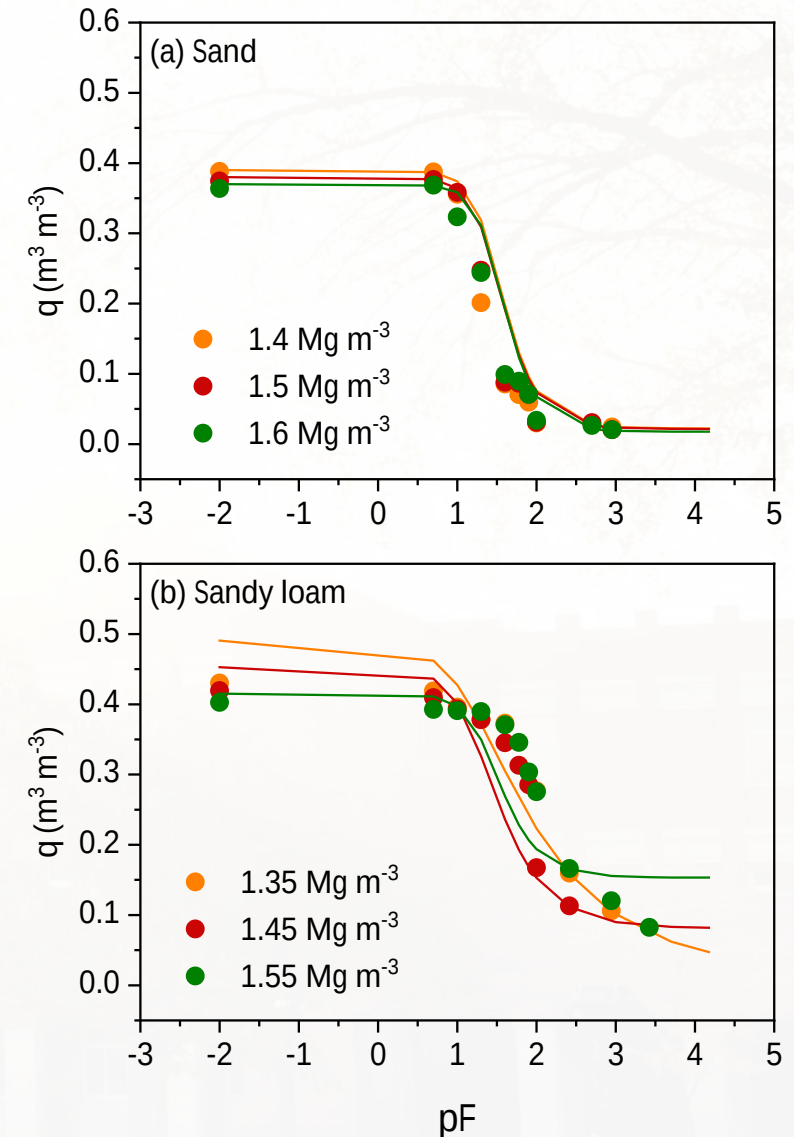
Results: pF(q) curve derived from I (q) curve

ü Model performance:

- Model fits the measured data well for both soils;
- Model is applicable for predicting water retention behavior under various bulk density and texture scenarios.

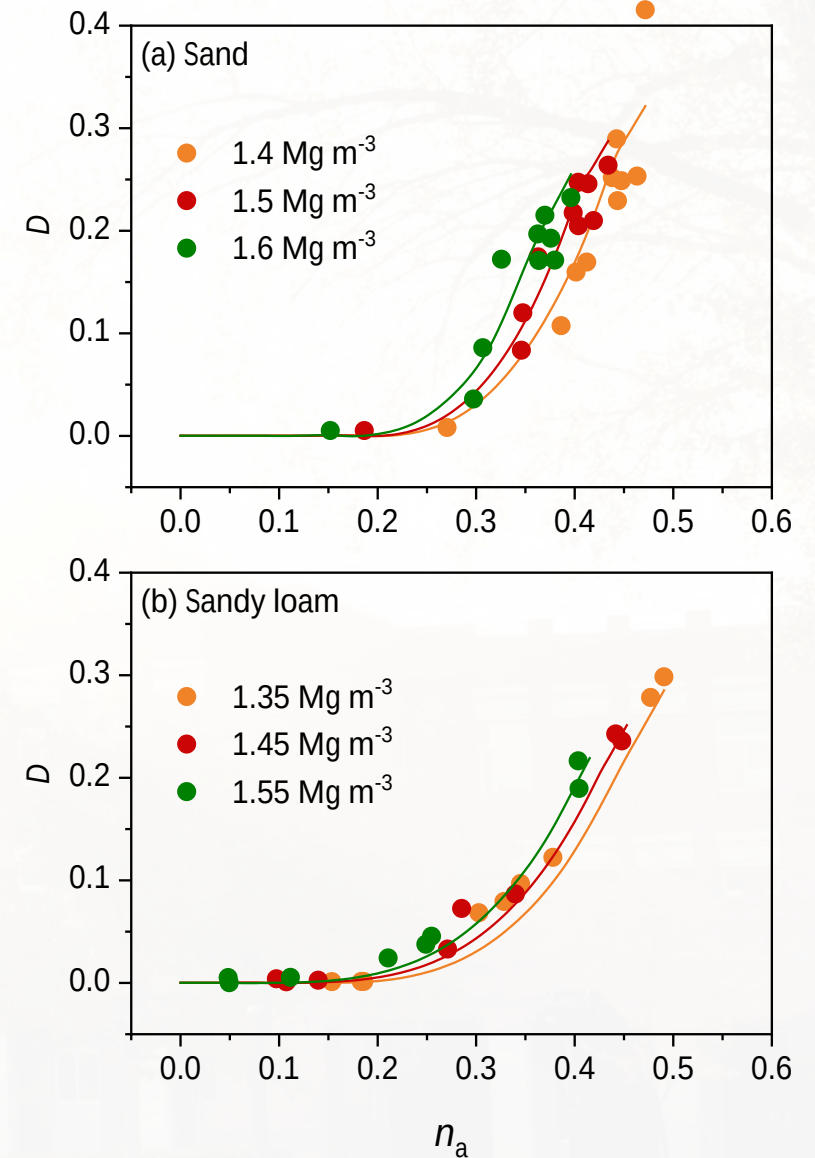
ü Model errors:

- Acceptable: RMSEs in estimated q : $< 0.055 \text{ m}^3 \text{ m}^{-3}$;
- Slight overestimation near saturation.

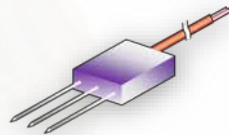


Results: $D(q)$ curve derived from $pF(q)$ curve

- ü **Performance:** Worked well for both soils across full range of air-filled porosities.
- ü **Errors:** RMSE of estimated D is low: 0.034 for sand and 0.017 for sandy loam.
- ü **Robustness:** Accurately captured the effect of bulk density on air diffusivity, as evidenced by the consistent downward shift of the curves with increasing bulk density.



Thermo-TDR technique



State variables

T, q, K_a

Thermal & electrical properties

$\theta(q), \alpha(q), C(q), K_a(q), s(q)$

Fractions of solid, water, air

r_b, V_s, n_a, S

Soil processes

- Heat transfer
- Water movement
- Water evaporation
- Air movement
- Heat-water-solute-air processes

In situ soil properties

$\theta(q), s(q) \longrightarrow pF(q), D(q)$



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